

Suppression of Shot Noise in Quantum Point Contacts in the “0.7 Regime”

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(Received 4 May 2006; published 1 November 2006)

Experimental investigations of current shot noise in quantum point contacts show a reduction of the noise near the 0.7 anomaly. It is demonstrated that such a reduction naturally arises in a model proposed recently to explain the characteristics of the 0.7 anomaly in quantum point contacts in terms of a quasibound state, due to the emergence of two conducting channels. We calculate the shot noise as a function of temperature, applied voltage, and magnetic field, and demonstrate an excellent agreement with experiments. It is predicted that, with decreasing temperature, voltage, and magnetic field, the dip in the shot noise is suppressed due to the Kondo effect.

DOI: [10.1103/PhysRevLett.97.186801](https://doi.org/10.1103/PhysRevLett.97.186801)

PACS numbers: 73.61.-r, 71.70.Ej, 73.50.Td, 75.75.+a

The conductance of quantum point contacts (QPCs) is quantized in units of $2e^2/h$ [1,2]. In addition to these integer conductance steps, an extra conductance plateau around $0.7(2e^2/h)$ has been experimentally observed [3–7]. Recently, a generalized single-impurity Anderson model has been invoked to describe transport through QPCs [8]. According to this model, motivated by density-functional calculations that reveal the formation of a quasibound state at the QPC [9], the tunneling of a second electron through that state is suppressed by Coulomb interactions, and is enhanced at low temperatures by the Kondo effect [10]. Thus, at temperatures larger than the Kondo temperature T_K , the conductance will be dominated by transport through the singly occupied level ($G \geq e^2/h$), growing at lower temperature towards the unitarity limit, $G = 2e^2/h$. Kondo physics has indeed been observed at low temperature and voltage bias [7]. The fact that there are effectively two conductance channels affects not only the conductance but also the current shot noise. Around conductance of $G \sim e^2/h$, the model predicts one highly transmitting channel ($T_1 \simeq 1$) and one poorly transmitting channel ($T_2 \simeq 0$). Thus, as the noise is expected to be proportional to the sum of $T_i(1 - T_i)$ over all channels, it should exhibit a dip near that value of the conductance [11], in contrast with the traditional view which associates a conductance of $G \sim e^2/h$ with $T_1 \simeq T_2 \simeq 1/2$ and maximal noise. A reduction in the noise through a QPC near $G \sim e^2/h$ has indeed been observed experimentally [12–14]. The dip was observed to be quite sensitive to magnetic fields. In this Letter, we present a detailed calculation of the noise based on the above model and demonstrate that it reproduces the experimental data. The magnetic field dependence arises from two factors: the dependence of the splitting of the two channels on the field, and the quenching of the Kondo effect. Specific predictions on the disappearance of the dip in the current noise at low temperature,

voltage bias, and magnetic field, due to the unitarity limit of the Anderson model, are made.

The main theoretical difficulty with calculating the noise is that the limit of perfect conductance through a given channel is not accessible via traditional perturbation theory for this interacting problem. Thus, an earlier calculation of the noise through a Kondo impurity [15] had to rely on more elaborate methods in order to be extended to lower temperatures. Because of the additional complexity of the generalized Anderson model, employed to describe QPCs (see below), these methods are not directly applicable. In this work we employ a new type of perturbation theory, starting from the high magnetic field B limit. In this limit spin-flip processes are suppressed, and the current and noise can be exactly (and trivially) calculated, to all orders in the tunneling, giving rise to two separate channels. Perturbation in $1/B$ allows us to follow the contributions and mixing of the two channels. By comparing to the traditional perturbation theory, around zero B , we are able to interpolate the noise between the two regimes [see Eq. (8) below]. This formula, which reduces in the known limits to the obtained perturbative results, allows us to compare to experiment in the whole magnetic field regime, yielding excellent agreement with experiment (Fig. 2) and allowing specific predictions.

Model Hamiltonian.—The extended Anderson Hamiltonian, invoked in [8] to model the QPC, differs from the usual single-impurity Anderson model in two aspects: (i) the tunneling amplitude of the first electron into the quasibound state $V^{(1)}$ is larger than that of the second electron $V^{(2)}$ (see also [16]), and (ii) both couplings increase exponentially as the energy of the incoming electron rises above the QPC barrier, E_{qpc} , defined to be the zero of energy. This Anderson model can be transformed into a Kondo Hamiltonian by performing a Schrieffer-Wolff transformation [17]:

$$H = \sum_{k\sigma \in L,R} \varepsilon_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma} + \sum_{k,k'\sigma \in L,R} (J_{k\sigma;k'\sigma}^{(1)} - J_{k\sigma;k'\sigma}^{(2)}) c_{k\sigma}^\dagger c_{k'\sigma} + 2 \sum_{k,k'\sigma \in L,R} (J_{k\sigma;k'\sigma}^{(1)} + J_{k\sigma;k'\sigma}^{(2)}) c_{k\sigma}^\dagger \vec{\sigma}_{\sigma\sigma'} c_{k'\sigma'} \cdot \vec{S}$$

$$J_{k\sigma;k'\sigma'}^{(i)} = \frac{(-1)^{(i+1)}}{4} \left[\frac{V_{k\sigma}^{(i)} V_{k'\sigma'}^{(i)}}{\varepsilon_{k\sigma} - \varepsilon_{\sigma'}^{(i)}} + \frac{V_{k\sigma}^{(i)} V_{k'\sigma'}^{(i)}}{\varepsilon_{k'\sigma'} - \varepsilon_{\sigma'}^{(i)}} \right], \quad (1)$$

where $c_{k\sigma}^\dagger$ ($c_{k\sigma}$) creates (destroys) an electron with momentum k and spin σ in lead L or R , $\varepsilon_{\sigma}^{(1)} = \varepsilon_{\sigma}$ and $\varepsilon_{\sigma}^{(2)} = \varepsilon_{\sigma} + U$, where ε_{σ} is the energy of local spin state σ and U is the on-site interaction. \vec{S} is the local spin due to the bound state. The potential scattering term (first line), usually ignored in Kondo problems, is crucial here, as it gives rise to the large background conductance at high temperature. The magnetic field B , defining the z -direction, enters the problem via the Zeeman term, $S_z B$. The exponential increase of the couplings is modeled, for simplicity, by a Fermi function $f_{FD}(\varepsilon) = 1/[1 + \exp(\varepsilon)]$, leading to a chemical-potential dependence of the spin-scattering matrix elements,

$$J_{\varepsilon_k \varepsilon_k - B \uparrow}^{(i)} = \frac{(-1)^{(i+1)} V^{(i)2}}{4} \left[\frac{f_{FD}(\frac{-\varepsilon_k}{\delta})}{\varepsilon_k - \varepsilon_{\uparrow}^{(i)}} + \frac{f_{FD}(\frac{B - \varepsilon_k}{\delta})}{\varepsilon_k - B - \varepsilon_{\uparrow}^{(i)}} \right]. \quad (2)$$

(In the above and in the following B and T denote the corresponding energies, $g\mu_B B$ and $k_B T$, respectively, where k_B is the Boltzmann constant, μ_B is the Bohr magneton, and with the appropriate g -factor.) For $B \ll \varepsilon_F$ we can ignore the magnetic field dependence of these matrix elements.

Current noise.—The current noise is defined via the current-current correlation function [18]

$$\hat{S}(t, t') = \frac{1}{2} [\langle I(t)I(t') \rangle + \langle I(t')I(t) \rangle]. \quad (3)$$

Under stationary conditions, the noise is a function of $t - t'$ and here we consider only the steady state, zero frequency component of the noise power $S(\omega = 0)$. The calculation of the noise, detailed below, consists of the following steps: (a) We obtain an exact solution for very large B , where spin-flip processes are suppressed, for the conductance G_{∞} and noise S_{∞} [Eq. (4)]. (b) We expand the noise power to second order in the spin-flip processes, for arbitrary value of the coupling $J^{(1)}$ and small value of $J^{(2)}$, yielding S_B [Eq. (6)] (and G_B , via the fluctuation-dissipation theorem). (c) Since the Kondo terms appear at higher order in perturbation theory, we add the third order terms in $J^{(2)}$, G_3 , and S_3 . (d) We calculate the noise, at small B , using the traditional expansion in $J^{(i)}$ [19]. (e) We derive simple and intuitive interpolation formulas, for both the conductance and the noise, that reduce to the obtained expansions in the two limits of the small and the large magnetic field. The resulting noise S and Fano factor S/I are depicted in Fig. 1 and compared to experiments.

Detailed calculation.—The calculation is carried out using the nonequilibrium Keldysh Green function approach [20]. In this approach there are three independent

Green functions which can be expressed in the terms of the retarded, advanced, and the ‘‘Keldysh’’ Green function, $G^K(\omega)$. For the two leads, the unperturbed Keldysh Green functions are $g_{k \in L,R,\sigma}^K(\omega) = -2\pi i [1 - 2f_{L,R}(\omega)]$, where $f_{L,R}(\omega) = f_{FD}(\omega \pm eV/2)$ are the respective distribution functions in the leads, which depend on the voltage difference V . It is more convenient to work in the symmetric and antisymmetric combinations of the two leads $g_{\pm}^K = g_L^K \pm g_R^K$.

When the magnetic field is large the exchange part of the Kondo Hamiltonian can be neglected. Therefore S_z can be treated as a conserved ‘‘classical’’ parameter. In the Anderson model language, the ground state of the system corresponds to the impurity being occupied by, say, a single spin-up electron. The transport of a spin-up electron from the left lead to the right lead [Fig. 1(a)] is performed by this electron hopping first to the right lead (which involves the hopping amplitude $V^{(1)}$), and then another spin-up electron hopping onto the dot, again with amplitude $V^{(1)}$. Thus the amplitude for transfer of a spin-up electron is proportional to $V^{(1)2}$, or to $J^{(1)}$. On the other hand, transfer of a spin-down electron from left to right [Fig. 1(a)] involves a double occupation of the dot, and its amplitude is thus proportional to $J^{(2)}$. In this case, one can sum up these processes to all orders, and calculate the conductance and noise exactly,

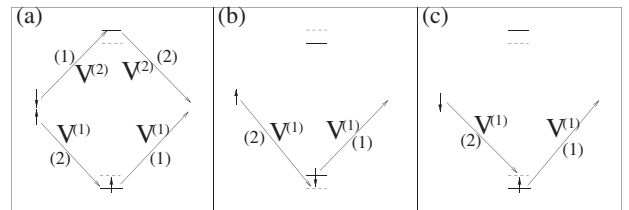


FIG. 1. The processes contributing to electron transport from left to right. (a) In strong magnetic fields, the impurity is occupied by a spin-up electron, and thus the allowed processes are either this electron first hopping to the right, denoted by (1) in the figure, and then another spin-up electron hopping in, denoted by (2), with probability proportional to $V^{(1)2}$, or a spin-down electron hopping in and out, with probability proportional to $V^{(2)2}$, due to double occupation. These processes correspond to the potential scattering term and to the S_z term in the Kondo Hamiltonian. When the magnetic field is reduced, the impurity can be occupied by a spin-down electron either in the beginning of the process (b), or at the end of it (c), corresponding, respectively, to S_+ and S_- in the Kondo language. Both of these processes are proportional to $V^{(1)2}$, since they do not involve double occupation of the impurity.

$$G_\infty = \frac{e^2}{h}(T_1 + T_2)$$

$$S_\infty = \frac{e^2}{h} \sum_i \left[eV \coth\left(\frac{eV}{2T}\right) T_i(1 - T_i) + 2TT_i^2 \right], \quad (4)$$

where $T_{1,2}$, the transmission probabilities for the two channels, are expressed in terms of the coupling constants of Kondo Hamiltonian $g_i = 4\pi\nu J^{(i)}$,

$$T_i = \frac{g_i^2}{1 + g_i^2}. \quad (5)$$

In the large coupling limit, the transmission probabilities go to unity. Since, as a function of energy, g_1 first increases to a large value, while g_2 becomes large only when $\varepsilon_F = \varepsilon_0 + U$, then, for large magnetic fields, as a function of gate voltage, the conductance, in units of $2e^2/h$, will first rise to $\frac{1}{2}$ and then to unity. Concurrently, the shot noise, the first part of S_∞ , will have a dip at the first conductance plateau, in agreement with experiments (Fig. 2).

As the magnetic field decreases, there is a finite probability, proportional to $\exp(-B/T)$, that the impurity is occupied by a spin-down electron. Then, new types of processes can take place [Figs. 1(b) and 1(c)], whereby a spin-down electron occupying the dot hops out and a spin-up electron hops in, or a spin-up electron occupying the dot hops out and a spin-down hops in. The amplitude for both these processes is proportional to $V^{(1)2}$, and thus, in this

order, even if $V^{(2)}$ is negligible, the contribution of the spin-down channel to the transport is finite.

In the Kondo language, the processes described in Figs. 1(b) and 1(c) correspond to spin-flips, and thus the exchange terms in the Hamiltonian have to be taken into account. Since each spin flip involves another factor of $\exp(-B/T)$, we can expand the conductance and noise to second order in the spin-flip processes, still allowing infinite order in $J^{(1)}$. The resulting nonequilibrium noise is a function of applied voltage and also depends on the non-equilibrium magnetization [21] $M(B, T, V)$. The latter is reduced to its equilibrium value $M_{\text{eq}} = \langle S_z \rangle = (-1/2) \times \tanh(B/2T)$ if $B > V$. The resulting additional contributions to the noise S_B and the linear response conductance G_B [obtained from the noise via the fluctuation-dissipation theorem, $G = S(V \rightarrow 0)/(2T)$] in this limit are

$$S_B = \frac{e^2}{2h} (g_1 + g_2)^2 \left[\frac{m_1 + m_2}{2} (A_+ + 4BM) - m_1 m_2 (1 + g_1 g_2) A_- \right] \quad (6)$$

$$G_B = \frac{B}{2T \sinh \frac{B}{T}} (m_1 + m_2) (g_1 + g_2)^2,$$

where

$$A_\pm = B \coth(B/2T) \pm \frac{1}{2} [B_+ \coth(B_+/2T) + B_- \coth(B_-/2T)]. \quad (7)$$

Here $m_{1,2} = 1/(1 + g_{1,2}^2)$ and $B_\pm = B \pm eV$. In Eq. (6) g_2 was considered small. The nonequilibrium magnetization is given by [21] $M = -B/A_+$. In the limit of small $g_{1,2}$, Eq. (6) reduces to the zero frequency current-current correlation function obtained in [21]. Note that the corrections to the infinite field limit, due to spin flips, depend on $\coth(B/2T)$, and thus indeed decrease exponentially with increasing the ratio B/T .

We note that the conductance, to this order, can be written as the expansion of an expression similar to that of Eq. (4), with g_i^2 in Eq. (5) replaced by \tilde{g}_i^2 , with

$$\tilde{g}_i^2 \equiv g_i^2 + \frac{B}{T \sinh \frac{B}{T}} \frac{(g_1 + g_2)^2}{1 + (g_1 + g_2)^2}. \quad (8)$$

As mentioned above, even though g_2^2 is small, \tilde{g}_i^2 can become substantial at smaller magnetic field due to higher order processes involving g_2^2 . Thus, the second channel will also contribute to transport, raising the conductance plateau from its value of $0.5 \times 2e^2/h$ at large magnetic field. This is consistent with the observation that the value of the “0.7 plateau” usually does not drop experimentally below $0.6 \times 2e^2/h$. For simplicity, in order to allow for the contributions of the processes described by Eq. (8) to the noise, we substitute the resulting transmission coefficients in the noise formula (4).

Comparison with experiment.—Figure 2 compares our calculation to the experimental results of Ref. [13] and of

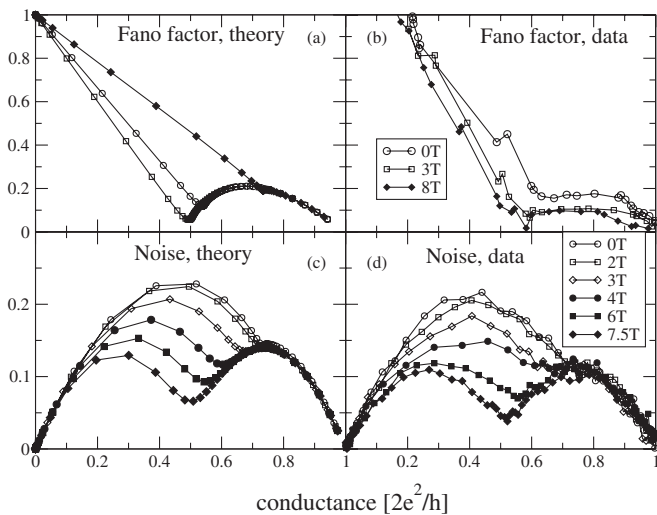


FIG. 2. (a) The Fano factor, calculated from the theory, versus zero-bias conductance at different magnetic fields, $g\mu_B B/k_B T = 0, 4.5, 12$, compared to the experimental results of Ref. [13] (b), for $B = 0, 3$, and 8 T. The parameters used in the theory were $eV = k_B T$, $V^{(1)2}/2\pi = 1$, $V^{(2)2}/2\pi = 0.01$. In (c) the noise is calculated for the same parameters as those corresponding to the data of Ref. [14], depicted at (d), with the magnetic field values denoted in the legend, $k_B T = 280$ mK and $V = 240$ μ V. The values of $V^{(i)2}$ are the same as in (a). A value of g -factor of 0.44 was used.

Ref. [14]. In (a) and (b), we compare the Fano factor, which is obtained, following Ref. [13], by subtracting from the full noise the thermal contribution [the last term in (4) plus $2T[G - (e^2/h)(T_1 + \hat{T}_2)]$], and dividing this difference by the current. Plotting the Fano factor against conductance makes the theoretical plot practically independent of the values of ε_0 , U , and δ , which determine the dependence of the conductance on gate voltage. The ratio of g_2^2/g_1^2 was assumed small ($=0.01$) in the spirit of the model, and the curves for 3 values of magnetic field, in the ratio 0:3:8 as those used in the experiment, are depicted with good agreement with experiment. The noise data of Ref. [14] allow an even more quantitative comparison with experiment, as we used the actual values of magnetic field, voltage, and temperature reported to the experiment, with the bulk GaAs g -factor of 0.44. The small deviations between the practically parameter-free theory and the experimental data probably indicate the accuracy of the approximation. Interestingly, the zero-field dip in the noise is quite small, even though the bare contribution of the second channel g_2 to the conductance is negligible. This is due to the contributions of higher order processes [Fig. 1(b) and 1(c)]. (Comparison to the noninteracting result appears in the supplementary material.) Reilly's phenomenological theory [22], which invokes a linear splitting between the spin directions with increasing gate voltage at zero magnetic field, also displays good agreement with experiment, when appropriate fitting parameters were used.

Kondo enhancement.—While the experiments were carried out outside the Kondo regime, due to the relative high voltage applied, one expects at low temperature, voltage, and magnetic field, that the Kondo effect will enhance the transmission of the low conducting channel, so the total conductance will approach the unitarity limit, $2e^2/h$, as is indeed seen experimentally [7]. Going to third order in J_2 [23], one indeed finds logarithmic divergence, signalling the onset of the Kondo effect below the Kondo temperature $T_K \approx U \exp(-\pi/g_2)$. Using the renormalization-group approach, one can sum up the most divergent logarithms in the higher order Kondo contributions. Separating the contribution to this Kondo series from the leading terms and summing up the series lead to the renormalization of the contribution \tilde{g}_2^2 term to the conductance

$$\tilde{g}_2^2 \rightarrow \tilde{g}_2^2 + g_2^2 \left(\frac{1}{2} - \frac{B}{T \sinh \frac{B}{T}} \right) + G_2^{\text{RG}}, \quad (9)$$

with

$$G_2^{\text{RG}} = \frac{1}{(\ln \frac{\sqrt{B^2 + T^2}}{T_K})^2} \frac{\pi^2}{8} \left(1 + \frac{2B}{T \sinh \frac{B}{T}} \right). \quad (10)$$

A more complicated expression can be derived for the noise [23]. The theory thus predicts that, for temperatures and voltages smaller than the Kondo temperature, the dip in the noise will disappear at zero field, due to the unitary limit of the Kondo effect.

Conclusions.—The excellent agreement between theory and experiment lends even more credence to the relevance of the generalized Anderson model to transport in quantum point contacts. It is interesting to note that a perhaps related dip appears in the measurement of dephasing in a quantum dot [24], as measured by a nearby quantum point contact, when the point contact is in the “0.7 regime.” The present theory suggests a simple explanation of this effect: as the dephasing in the quantum dot is by the current noise in the point contact [25], a dip in the noise will be associated with a dip in the dephasing rate in the quantum dot.

This research has been funded by the Israel Science Foundation, and by the U.S.-Israel Binational Science Foundation. We thank the authors of Ref. [14] for providing us with the data files.

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