Luttinger-Liquid Behavior in Tunneling through a Quantum Dot at Zero Magnetic Field

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Thermodynamic and transport properties of a two-dimensional circular quantum dot are studied theoretically at zero magnetic field. In the limit of a large confining potential, where the dot spectrum exhibits a shell structure, it is argued that both spectral and transport properties should exhibit Luttinger liquid behavior. These predictions are verified by direct numerical diagonalization. The experimental implications of such Luttinger liquid characteristics are discussed.

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Non-Fermi liquid systems—electronic systems whose elementary excitations cannot be described by electrons—have always fascinated physicists due to their unusual properties (such as superconductivity and magnetism). Luttinger liquid (LL), describing interacting electrons in one dimension, is one of the most studied models of such a non-Fermi liquid system, since it has been solved a long time ago [1]. The non-Fermi-liquid characteristics are expected to play a major role in transport through a LL or tunneling into it [2], giving rise to a power-law dependence of the current or the tunneling density of states on energy, voltage, or temperature. Accordingly, there have been many theoretical suggestions of physical systems that should exhibit LL behavior and numerous experimental attempts to observe this behavior in such systems. These attempts have not been very successful, due to the sensitivity of LL to disorder, and the difficulty of making clean one-dimensional systems. Only recently, several decades after the model was originally introduced, experimental observation of Luttinger liquid behavior in one-dimensional systems such as nanotubes [3] and semiconductor systems [4] has been reported. Two-dimensional systems are also expected to exhibit LL behavior in strong magnetic fields, due to the formation of one-dimensional edge states near the edge of the systems [5]. However, tunneling experiments into the edge of a two-dimensional electron gas in strong magnetic fields [6] have not been conclusive.

Quantum dots have been used in the last several years to study various aspects of strongly correlated systems, most notably the Kondo effect [7,8]. The experimental control over the properties of the dot allow a more detailed investigation of the strongly correlated states than that accessible in bulk systems. An experimental study of LL behavior in quantum dots has been suggested by Kinaret *et al.* [9] who studied theoretically tunneling into a quantum dot in a strong magnetic field. Here, due to the finite number of electrons in the dot N, the power-law behavior of the tunneling density of states as a function of energy, characteristic of LL, will be manifested in the power-law dependence of the tunneling probability on N.

Technological progress allows now a construction of circularly symmetric quantum dots [10]. The addition spectrum of these dots displays shell structure expected from such symmetry, which has been numerically reproduced using a density-functional analysis of interacting electrons in circularly symmetric harmonic potential [10]. In this Letter, we propose and demonstrate a new type of Luttinger liquid behavior in such circular two-dimensional quantum dots at zero magnetic field, which is manifested in the spectral and transport properties of the dot. The main idea in this work is that because of the shell structure, one of the two quantum numbers needed to characterize an electronic state in two dimensions is frozen (for excitations within the shell), and thus these excitations are described by an effective one-dimensional Hamiltonian. We explicitly solve the quantum dot Hamiltonian with long-range electron-electron interactions and obtained the low-energy spectrum and wave functions of this many-body system. The spectrum and the tunneling amplitude indeed follow the predictions of Luttinger liquid behavior.

We describe a quantum dot by a system of $\mathcal{N}$ interacting spinless electrons in two dimensions, confined by a parabolic potential,

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{\mathcal{N}} \boldsymbol{\nabla}_i^2 + \frac{1}{2}m\omega_0^2 \sum_{i=1}^{\mathcal{N}} \mathbf{r}_i^2 + \sum_{i<j} V(|\mathbf{r_i}-\mathbf{r_j}|), \quad (1)$$

with the interaction potential $V(r)$. The noninteracting part of the Hamiltonian is trivially diagonalized, and its eigenstates and energies are given by

$$\Psi_{n,\ell}(r) = A r^{|\ell|} e^{-r^2/2+\imath\theta\ell}\, {}_1F_1[-(n-|\ell|-1)/2, |\ell|+1, r^2],$$
$$E_{n,\ell} = n\hbar\omega_0, \quad (2)$$

with $n = 1, 2, \ldots$ and $\ell = -(n-1), -(n-3)\ldots, n-1$, and where ${}_1F_1$ is the confluent hypergeometric function.

r and θ are, respectively, the radial and azimuthal coordinates of the two-dimensional vector $\mathbf{r}$, and r is in units of $\sqrt{\hbar/m\omega_0}$. Thus the spectrum consists of equally spaced shells. The energy of the nth shell is $n\hbar\omega_0$, and its degeneracy is n. Within a given shell the states are characterized by a single quantum number, e.g., the angular momentum ℓ.

The average mean-field Coulomb interactions between electrons leads to the usual Coulomb blockade, but does not affect the spectral properties for a given electron number $\mathcal{N}$. For the rest of the paper, while including the full Coulomb interaction [the last term in (1)], we make the assumption that the intershell gap $\hbar\omega_0$ is significantly larger than the *fluctuations* in the Coulomb interactions between electrons in different levels. Thus the Coulomb interactions cannot excite electrons from one shell to another. (This assumption is appropriate for the lowest three shells in the experiments [10], where these fluctuations are at most a few percent of the intershell addition energy.) Consider now a quantum dot containing $\mathcal{N}$ electrons occupying $n_{\rm sh}$ shells. All shells except the last will be fully occupied, leaving $N = \mathcal{N} - n_{\rm sh}(n_{\rm sh}-1)/2$ electrons in the partially occupied shell (POS). Since the Coulomb interactions do not excite electrons between the shells, then the ground state and the lowest excited states will belong to a subspace containing $\binom{n_{\rm sh}}{N}$ states, each consists of $\mathcal{N}-N$ electrons fully occupying the lower $n_{\rm sh}-1$ shells and N electrons occupying N of the $n_{\rm sh}$ states in the POS. All these states are degenerate in the noninteracting limit. Our aim is to diagonalize the full interacting Hamiltonian (1) in this subspace.

In principle, the Coulomb interactions scatter two electrons from two initial states onto two final states, with the conservation of angular momentum. In the absence of intershell excitations, these terms can be divided into three contributions—(a) those where the two initial states (and thus the final states) are in the filled shells, (b) those where one of the initial states is in the filled shell (and is necessarily one of the final states), and (c) those where the two states are in the POS. In the relevant subspace the first contribution is a constant, and thus is disregarded. The second type of terms, the effective potential due to the filled shells, contribute to the diagonal of the Hamiltonian matrix, leading to the removal of the energy degeneracy between the different angular momenta states (angular momenta $\pm\ell$ are still degenerate). The resulting dispersion turns out to be very close to an inverse parabola.

Thus, the effective Hamiltonian in the relevant subspace can be written as

$$\mathcal{H}_{\rm eff} = \sum_{\ell} \epsilon_\ell c_\ell^\dagger c_\ell + \sum_{j,k,\ell} V_{k,j,\ell} c_{\ell-k}^\dagger c_{j+k}^\dagger c_j c_\ell, \qquad (3)$$

with

$$\epsilon_\ell = \sum_{n<n_{\rm sh},j} \iint d^2r' d^2r |\Psi_{n,j}(r)|^2 V(r-r') |\Psi_{n_{\rm sh},\ell}(r')|^2,$$

$$V_{k,j,\ell} = \iint d^2r d^2r' \Psi^*_{n_{\rm sh},\ell-k}(r') \Psi^*_{n_{\rm sh},j+k}(r) \times V(r-r') \Psi_{n_{\rm sh},j}(r) \Psi_{n_{\rm sh},\ell}(r'), \qquad (4)$$

where c_ℓ annihilates a particle with angular momentum ℓ in the POS, and the summations in $\mathcal{H}_{\rm eff}$ are over all angular momenta states in the POS.

This effective Hamiltonian in the POS resembles the Hamiltonian describing interacting electrons in one dimension,

$$\mathcal{H}_{1d} = \sum_k \epsilon_k c_k^\dagger c_k + \sum_k V_k \rho_k \rho_{-k}, \qquad (5)$$

where $\rho_k \equiv \sum_j c_{k+j}^\dagger c_k$, $\epsilon_k = v_F k$, with k the momentum quantum number and v_F the Fermi velocity. Comparing the effective Hamiltonian (3) to (5) we note that in our case the interaction V depends on three quantum numbers, and not only on their difference. Thus unlike the LL model, the interaction is not merely a product of densities. This difference arises because the system is not strictly one dimensional, but the electronic wave functions in the $n_{\rm sh}$th shell, while having the same value of $\langle r^2\rangle \propto n_{\rm sh}$, do have a nontrivial dependence on r. Additionally, the number of the electrons in the POS, which are described by the effective Hamiltonian (3), is finite (and could be small).

Since the one-dimensional Hamiltonian (5) is exactly solvable and is described by LL theory, it is tempting to check whether the two-dimensional behavior of a symmetric quantum dot is governed by LL physics. In order to investigate the relevance of LL behavior to the spectral and transport properties of the dot, we diagonalized numerically the Hamiltonian (3) for several values of $n_{\rm sh}$ and N, and for a particular choice of interaction, $V(r) = \log(1/|r|)$. This choice allows us to evaluate all the matrix elements ϵ_ℓ and $V_{j,k,\ell}$ analytically. To investigate the particular role of the interaction term [the second term in (3)], we multiplied it by a prefactor α and carry out our study for several values of α. The results shown here are for $n_{\rm sh} = 20$. For each value of total angular momentum L, the Hamiltonian was diagonalized in the subspace of the lowest six eigenstates [11], in order to determine the lowest energy state with angular momentum L.

For an even number of electrons, the lowest energy state is of total angular momentum $L = 0$, the first excited states carry angular momentum $L = \pm 2$, then $L = 0, \pm 4$, etc., According to LL theory (for spinless electrons), the low-energy spectrum of (5) is given by [12]

$$E_p = p\sqrt{v_F^2 + 2v_F V_p/\pi}. \qquad (6)$$

In order to check the spectrum of the two-dimensional quantum dot (4) against the predictions of LL theory, we

compare in Fig. 1 the evaluated gap, $\Delta \equiv (E_{L=2} - E_{L=0})/2$, to the expectation from LL theory (6). As can be clearly seen from the figure, the nontrivial dependence of the gap on the interaction strength, α, is very well fitted by the functional form expected from LL theory, with a single fitting parameter $V_{p=2} = 0.055$. (Note that in our case the Fermi velocity, v_F, is also α dependent, due to the diagonal interaction terms.) Numerically we indeed find that the interaction integrals $V_{2,j,\ell}$ are about 0.05 ± 0.01 for several values of k and ℓ (independent of what branch k and l are on). The interaction integrals for higher values of p are much smaller, e.g., $V_{4,j,\ell} \simeq 0.01$. For $\alpha = 20$, however, we see deviations from the theoretical predictions, indicating perhaps that such strong interactions involve states beyond the linear dispersion regime.

LL characteristics are manifested not only in the spectrum, but also in the matrix elements, or the tunneling probabilities. The amplitudes of the Coulomb blockade peaks, T_N, in the linear response conductance through the quantum dot are proportional, when the temperature is smaller than the gap in the excitation spectrum, to [13]

$$T_N \propto |\langle \Psi_N | d_\ell | \Psi_{N+1} \rangle|^2, \tag{7}$$

where Ψ_N is the many-particle ground state of a system with N particles is the POS, and d_ℓ annihilated a particle with the corresponding angular momentum (namely, the difference in total angular momentum between Ψ_N and Ψ_{N+1}). This quantity was studied extensively for the case of quantum dot in the fractional quantum Hall regime by Kinaret *et al.* [9]. Using the explicit form of the edge-state propagators they showed that $T_N \propto N^{-(1/\nu-1)/2}$, where ν is the filling factor. This dependence was also verified numerically. The fractional quantum Hall liquid turned out to be a special case of the general theory of tunneling into a LL [2], where it was shown that the tunneling probability depends on energy in a power-law manner,

$$T_N \propto \epsilon^{\beta-1}, \qquad \beta = (g + g^{-1})/2, \tag{8}$$

where g characterizes the strength of interactions in LL theory ($g = 1$ no interactions, $g < 1$ repulsive interactions). The power-law dependence on ϵ is directly translated into a power-law dependence of T_N on N, $T_N \propto N^{-(\beta-1)/2}$ (the factor 2 comes from the two-dimensionality, $N \sim L^2 \sim k^{-2} \sim \epsilon^{-2}$).

To check these LL predictions we have evaluated numerically the matrix elements $|\langle \Psi_N | d_\ell | \Psi_{N+1} \rangle|^2$, for the same set of parameters mentioned above, and for different values of the interaction strength α. As can be seen in Fig. 2, these matrix elements exhibit power-law dependence on the number of particles in the POS, in agreement with LL theory. Surprisingly, the power-law behavior is already manifested in tunneling into a quantum dot with a small number of electrons in the POS.

To further check the relevance of LL theory to the tunneling matrix elements, we employ the functional dependence of the LL parameter g on the interaction strength, which can be explicitly evaluated for a one-dimensional model with long-range interactions [only $V_0 \neq 0$ in (5)]. For this case [12],

$$g = \frac{1}{\sqrt{1 + V_0/\pi v_F}}. \tag{9}$$

In Fig. 3 we compare the resulting exponent to the prediction of LL theory with such long-range interactions. We see an excellent agreement between the numerically obtained exponent and the theoretical prediction, with a

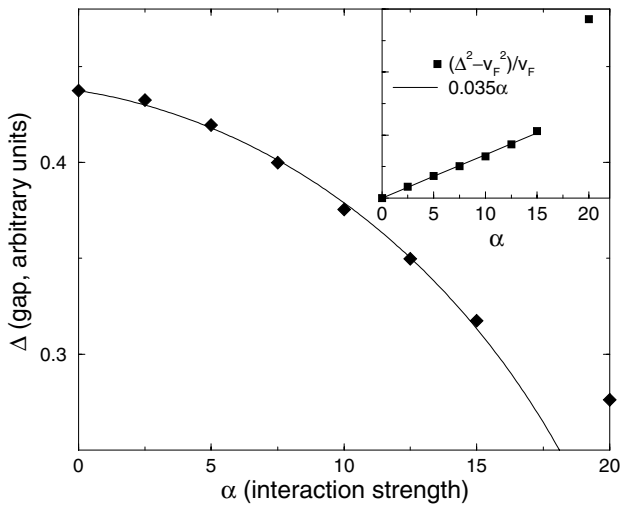

FIG. 1. Comparison of the numerically evaluated gap Δ in the quantum dot spectrum to the prediction of Luttinger liquid theory [Eq. (6)]. There is a single fitting parameter, $V_{p=2} = 0.055$ (see text). Inset: Plot of $(\Delta^2 - v_F^2)/v_F$ vs the interaction strength. Luttinger liquid theory predicts a straight line, of slope $2V_2/\pi$.

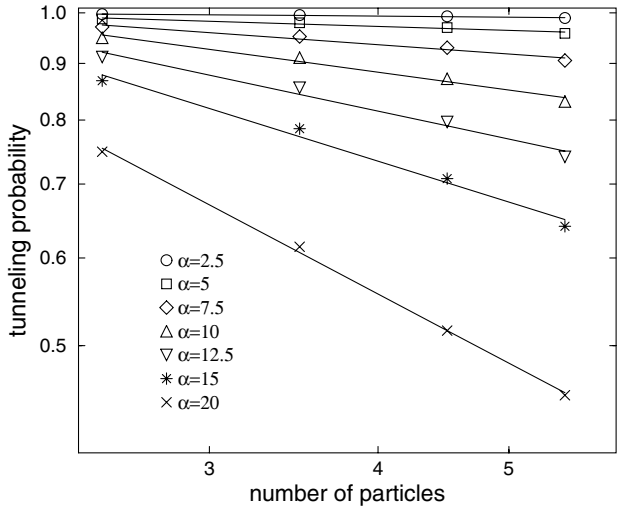

FIG. 2. The numerically evaluated tunneling matrix elements T_N, which are proportional to the Coulomb blockade peak amplitudes, for several values of the interaction strength α. In agreement with Luttinger liquid theory, a power-law dependence of T_N on N is observed, with an exponent that increases with the strength of the interactions.

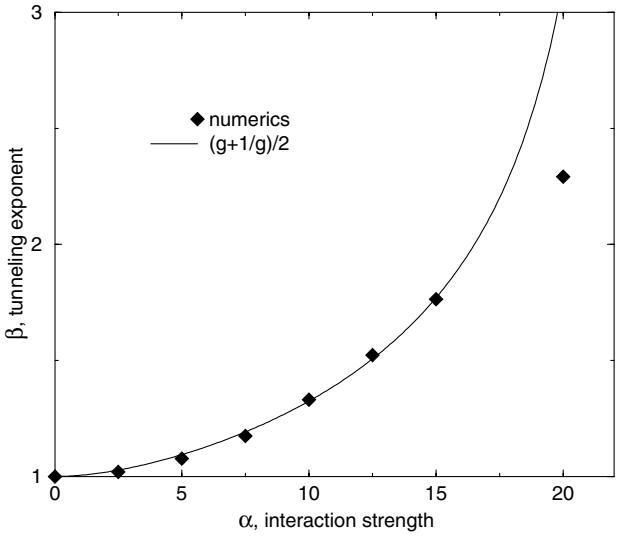

FIG. 3. The numerically evaluated tunneling matrix exponent β, derived from Fig. 2, as a function of the interaction strength α. The dependence upon α exhibits a good agreement with the predictions of Luttinger liquid theory, Eqs. (8) and (9).

single fitting parameter $V_0 \simeq 0.33$ (with again a deviation at very large interaction strength, $\alpha = 20$). The agreement between the theoretical prediction and the numerical calculation also indicates that the logarithmic interactions used in the numerics mimic the long-range nature ($V_p = 0$, $p > 0$) of the prediction (9).

To conclude, we have demonstrated, by heuristic arguments and by numerical calculations, that the excitations in a circular quantum dot at zero magnetic field are described by Luttinger liquid theory. These should have an observable influence on transport through quantum dots. We predict that in a given shell the amplitude of the Coulomb blockade peak decreases in a power-law fashion. Taking into account that this power law was observed numerically for a quantum dot with a small number of electrons in the outer shell (see Fig. 2), this prediction can be tested with existing structures. Indeed, the peak amplitudes seem to be decreasing as a function of the electron number within the third shell in circular dots [10]. Intriguingly, nanotube quantum dots, where intrinsic symmetry leads to an observable shell structure [14], also exhibit decreasing peak amplitudes within a given shell. The relevance of our theory to these observations will be studied in the future.

An even more intriguing possibility is the relevance of this work to transport through a system of interacting electrons in the presence of potential disorder, which is one of the most challenging problems of condensed matter physics. When the disorder potential is large enough to break the electron liquid system into puddles, or effective quantum dots, the overall dissipative resistance may be determined by transport through such a random quantum dot array [15]. Thus the present calculation suggests that transport in a two-dimensional disordered interacting system may be dominated by LL behavior, similar to transport in such a system in a strong magnetic field [16].

In this paper the spin of the electrons was ignored. It would be quite interesting to investigate the role of spin in such devices and explore the possibility of, e.g., spin-charge separation and its relevance to transport through a quantum dot. Another avenue worth exploring is the question of whether other systems exhibiting shell structure, such as nuclei and atoms, can also, under some circumstances, exhibit Luttinger liquid behavior. These questions will be investigated in a future publication.

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