# Percolation-Type Description of the Metal-Insulator Transition in Two Dimensions

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A simple noninteracting-electron model, combining local quantum tunneling and global classical percolation (due to a finite dephasing time at low temperatures), is introduced to describe a metal-insulator transition in two dimensions. It is shown that many features of the experiments, such as the exponential dependence of the resistance on temperature in the metallic phase, the linear dependence of the exponent on density, the $e^2/h$ scale of the critical resistance, the quenching of the metallic phase by a parallel magnetic field, and the nonmonotonic dependence of the critical density on a perpendicular magnetic field, can be explained by the model.

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The experimental observation of a metal-insulator transition in two dimensions [1–5] has been a subject of extensive investigation, since it is in disagreement with the predictions of single-parameter scaling theory for noninteracting electrons [6]. Several theories, based on the treatment of disorder and electron-electron interactions by Finkelstein [7], have been put forward [8]. Other approaches considered spin-orbit scattering [9], percolation of electron-hole liquid [10], or scattering by impurities [11]. To date there is no acceptable microscopic theory that describes quantitatively the observed data.

Here we present a simple noninteracting electron model, combining local quantum tunneling and global classical percolation, to explain several features of the experimental observations. The main observations we want to understand are the following: (1) As the system is cooled down, the resistance of samples with density higher than some critical density extrapolates to a finite value at zero temperature, while that of samples with lower density diverges. (2) The resistance of the sample with the critical density does not depend on temperature (at least for a limited range of low temperatures). (3) The conductance of the critical-density sample is of the order of $e^2/h$. (4) On the metallic side the functional dependence of the resistance is of the form $R(T) = R_0 + R_1 \exp(-A/T)$. The parameter $A$ varies linearly with the density and vanishes at the transition. (5) In perpendicular magnetic fields this transition is continuously connected with the quantum Hall–insulator transition [12]. The critical density varies nonmonotonically with magnetic field, with a minimum around $\nu = 1$. (6) Parallel magnetic fields destroy the metallic phase, at least for densities near the transition [13,3].

Before we introduce the model, let us mention three other experimental observations: (a) The strong disorder is crucial to see the transition. In GaAs the transition is seen only at samples with low mobility (even with the same density). In fact, Ribeiro *et al.* [14] have recently observed a zero-field metal-insulator transition in high-density $n$-type GaAs sample with strong enough disorder (which was introduced by a matrix of randomly distributed quantum dots). (b) There is additional experimental indication that the transition is not driven by interactions—Yaish and Sivan [5] studied a system of two parallel gases, one of electrons and one of holes. The observed metal-insulator transition in the hole gas depended only slightly on the electron density, even though one expects that increasing electron density will screen the interactions between holes and suppress the metallic phase in the hole gas. On the contrary, increasing electron density led to increasing conductance in the hole gas, indicating that its main role is to screen the impurity potentials in the hole gas. (c) There is a growing experimental evidence that even at the lowest available temperatures, the dephasing time is still finite [15,16].

Based on all these observations we now suggest the following scenario to explain the experimental observations: the potential fluctuations due to the disorder define density puddles. (Density separation into puddles in gated GaAs was indeed observed experimentally by Eytan *et al.* [17], using near-field spectroscopy.) Locally, between these puddles, transport is via quantum tunneling through saddle points, or quantum point contacts (QPCs), while between such tunneling events it is assumed that dephasing takes place. Thus the conductance of the system will be determined by adding classically these quantum resistors. A related model was introduced by Shimshoni *et al.* [18] to describe successfully transport in the quantum Hall (QH) regime. The percolative nature of the system in the QH regime was indeed verified experimentally [19]. We will return to the QH regime below.

We characterize each saddle point by its critical energy $\epsilon_c$, such that the transmission through it is given by $T(\epsilon) = \Theta(\epsilon - \epsilon_c)$. Thus the conductance through each QPC is given by the Landauer formula,

$$G(\mu, T) = \frac{2e^2}{h} \int d\epsilon \left( -\frac{\partial f_{\mathrm{FD}}(\epsilon)}{\partial \epsilon} \right) T(\epsilon) = \frac{2e^2}{h} \frac{1}{1 + \exp[(\epsilon_c - \mu)/kT]}, \quad (1)$$

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where $\mu$ is the chemical potential and $f_{\mathrm{FD}}$ is the Fermi-Dirac distribution function.

The system is now composed of classical resistors, where the resistance of each one of them is given by Eq. (1), with random QPC energies. In the numerical data presented below, we solved a $20 \times 20$ system of QPCs (which, for simplicity, has the topology of a square lattice), each averaged over 1000 realizations of disorder, where the QPC energies were taken from a square distribution of width $W$. At zero temperature the conductors have either zero conductance or a conductance equal to $2e^2/h$ and one has the usual second-order percolation transition. The critical conductance exponent $t$ is known at two dimensions and is equal to $\simeq 1.3$ [20]. In Fig. 1 we fit the experimental data of [4] and of [5] to the expected critical dependence. Clearly, the agreement with the classical percolation prediction is excellent.

As temperature increases, the Fermi-Dirac distribution is broadened. Consequently, the conductance of the insulating QPCs ($\epsilon_c > \mu$) increases exponentially, while that of the transparent ones ($\epsilon_c < \mu$) decreases exponentially. Thus we expect to see rather dramatic effects as a function of temperature. This is indeed depicted in Fig. 2. As temperature is lowered systems with slightly different resistance at high temperatures will diverge exponentially with decreasing temperatures. The resistance of systems on the metallic side ($n > n_c$) will saturate at zero temperature, while that of insulating samples will diverge. Note that there is an upward turn even on the metallic side of the transition. We will come back to this point below.

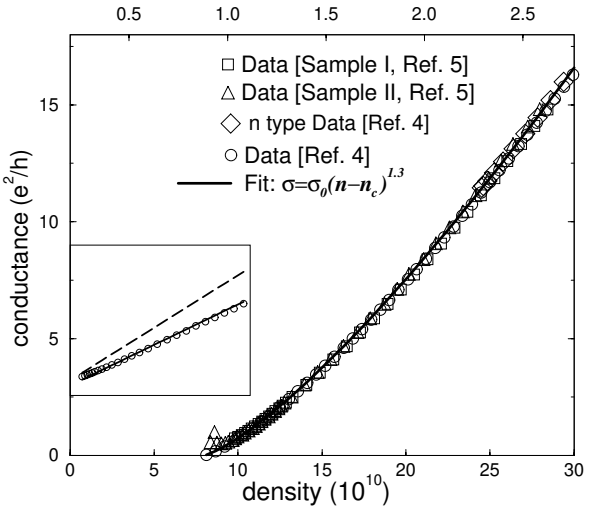


FIG. 1. Comparison of the lowest temperature data of [5] (two sets of data, triangles and squares, 330 mK, density given by the lower axis) and of [4] (circles, 57 mK, density given by the upper axis), and of the $n$-type data [4] (diamonds) to the prediction of percolation theory (solid line). Inset: Logarithmic derivative of the data [5] which gives a line whose slope is the inverse of the critical exponent. The percolation prediction ($t = 1.3$) is given by the solid line. For comparison a $t = 1$ slope is also shown (broken line).

The high-temperature resistance of the critical density network is naturally around $h/e^2$, the only resistance scale in this model.

For systems of exponentially distributed resistors the resistance of the whole circuit is determined by the critical resistor, the worse resistor in the minimal percolating network [21]. Then the resistance of the network would be equal to the inverse of the Fermi-Dirac function, $\{1 + \exp[(\epsilon_c - \mu)/kT]\}h/e^2$, where $\epsilon_c$ is its threshold energy. Then clearly the overall resistance will be of the form observed experimentally, $R = R_0 + R_1 \exp(-A/T)$, with $A$ varying linearly with the density and vanishing at the transition. In our case, the resistors on the metallic side have a bound distribution, and accordingly there will be other resistors, in parallel and in series, that will contribute to the overall resistance of the circuit. This will not change the functional form, but will renormalize the parameters $R_0$, $R_1$, and $A$. Such a functional dependence on the metallic side is indeed found numerically and displayed in the inset. In fact, close to the transition, on the metallic side, as temperature increases, some QPCs that before had zero conductance start to conduct and add to the overall conductance. Since the critical percolation cluster is very ramified (in fact, of fractal dimension), there will be many such resistors in parallel to the main conducting network, and the effect of improving these resistors will overcome the fact that resistors on the conducting network itself become worse. This leads to a downward turn of the resistance with increasing temperature even on the metallic side, the details of which may depend sensitively on the geometry. Only deeper into the metallic regime, as seen in Fig. 2, the overall resistance increases with increasing temperature. This also suggests that the density at which the resistance is

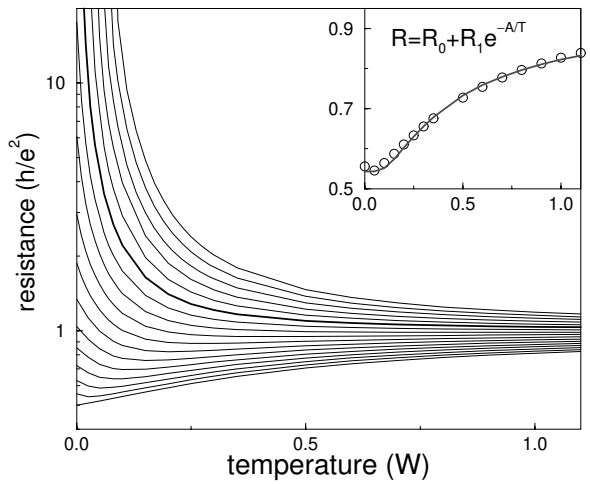


FIG. 2. Temperature dependence of the resistance for systems of different densities. Below the critical line (bold curve) all curves saturate at zero temperature, while above it the resistance diverges. The resistance of the more metallic samples decreases exponentially (inset).

approximately temperature independent is not the true critical point, but rather deeper on the metallic side. This is clearly seen in Fig. 3, where one can see a point where all the low-temperature curves nearly cross, well inside the metallic regime. The above discussion suggests that one should be cautious in associating the critical point with the "temperature-independent" point, as done routinely in the experiment interpretations.

We turn next to the effects of a magnetic field. The effect of a parallel field is straightforward to understand, as there have been several studies of transport through a QPC in parallel fields [22]. These experimental and theoretical studies demonstrated that the threshold density where the QPC opens up *increases* parabolically with the in-plane magnetic field. This effect was attributed to coupling of the in-plane motion to the strong confinement in the vertical direction, leading to an increase in the energy levels. Since this increase occurs for all QPCs in the sample, such a field in our case will strongly inhibit the metallic behavior: QPCs which were conducting at zero field will have exponentially small conductance with increasing field. Once the density of conducting QPCs falls below the critical density, the system becomes an insulator, in agreement with experimental observations.

The situation in perpendicular magnetic fields is more interesting as QH states are formed. Transport through a single QPC in perpendicular field and the crossover between the zero-field limit and the QH limit have been studied in detail [23]. As expected, one finds that the critical energy oscillates with magnetic field due to the depopulation of Landau levels. In our case, we expect the oscillations to be smoothed out by the disorder and by the averaging over many QPCs. Thus only the strongest oscillation, near $\nu = 1$, may survive, leading to a single dip in the critical density vs magnetic field plot, as was observed experimentally. This is indeed in agreement with our numerical calculation. We studied the energy levels of one puddle of electrons, which we modeled by a circular disk, in the presence of disorder [24]. In Fig. 4 we plot the "critical density"—the number of electrons that need to occupy the puddle, so that the energy of the highest-energy electron will be enough to transverse the QPC [25], equivalent in the bulk system to the critical density—as a function of magnetic field. Indeed, we see a dip near $\nu = 1$ with all other oscillations smoothed out by the disorder. This curve has a strong resemblance to the experimental data [12] (inset). In addition, we expect that as the magnetic field is lowered below the $\nu = 1$ minimum more than one channel will transverse some QPCs, leading to an increase in the critical conductance, as indeed reported experimentally.

All the above results and discussion demonstrated that many of the experimental observations can be explained in the context of the simple model introduced here. Nevertheless, there are clearly other physical effects that need to be included in order to have a full picture of the experiments. In particular, electron-electron interactions are expected to play an important role in these low densities. As we can regard the metallic puddles described above as quantum dots, one can use the abundant information about the role of interactions in such structures [26] to gain additional understanding of the characteristics of the puddles and the phase separation. Other effects, including the energy dependence of the transmission coefficient and the possibility of more than one channel through the QPCs, the temperature dependence of the dephasing length, the role of interband scattering [5], and temperature-dependent impurities [11] may also be important to understand quantitative aspects of the data. Nevertheless, the fact that several important aspects of the

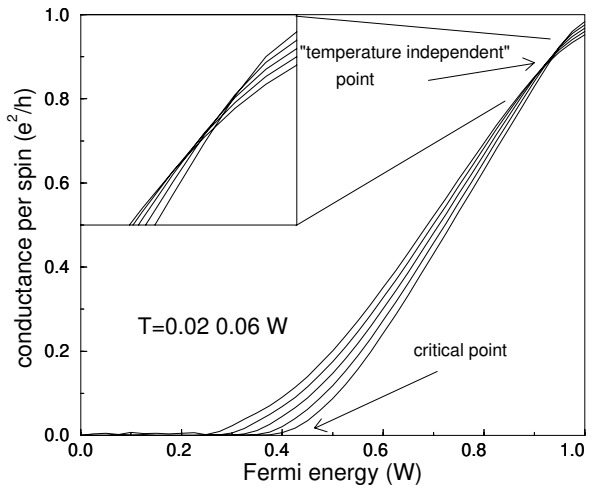


FIG. 3. Conductance vs density (Fermi energy) for several temperatures. There is a density, well above the true critical point, where the curves seem to cross each other.

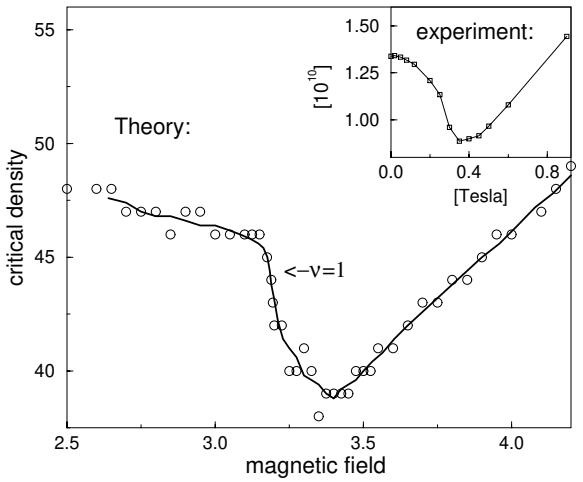


FIG. 4. The critical density—the number of electrons in the puddle, so that the topmost energy will allow transport through the point contact—as a function of magnetic field, in the presence of a finite disorder. The continuous curve is an averaged fit through the (necessarily integer) data points. Inset: the corresponding experimental data [12].

experimental data can be explained in the context of a simple model is quite encouraging. Moreover, while in the degenerate electron gas limit ($T \ll E_F$) the conductance on the metallic side can decrease by only a factor of 2 with decreasing temperature, the model predicts arbitrary reduction in the conductance in the nondegenerate case ($T \sim E_F$), relevant to the Si systems.

The picture described above can be checked experimentally. The experiments verifying the percolative structure in the QH regime [19] can be extended to the zero-field systems. (An experimental evidence for phase separation was observed at zero field in [17].) An even more direct evidence of the percolative nature of the system will be local probes [27]. In fact, an enhancement in the fluctuations of the local chemical potential has already been observed [28] as the system enters the "insulating" phase, in a fashion similar to the enhancement of chemical potential fluctuations with closing of the barriers forming a quantum dot [29]. In fact, a "smoking gun" verification of the picture presented here will be periodic oscillations of the local chemical potential on the insulating side, due to depopulation of the Landau levels, as was observed in quantum dots [30].

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[1] S. V. Kravchenko *et al.,* Phys. Rev. B **50**, 8039 (1994); **51**, 7038 (1995); Phys. Rev. Lett. **77**, 4938 (1996); V. M. Pudalov *et al.,* JETP Lett. **65**, 932 (1997); D. Popović *et al.,* Phys. Rev. Lett. **79**, 1543 (1997); K. Ismail *et al.,* cond-mat/9707061; J. Lam *et al.,* Phys. Rev. B **56**, R12 741 (1997).
[2] P. T. Coleridge *et al.,* Phys. Rev. B **56**, R12 764 (1997).
[3] M. Y. Simmons *et al.,* Phys. Rev. Lett. **80**, 1292 (1998).
[4] Y. Hanein *et al.,* Phys. Rev. Lett. **80**, 1288 (1998); Phys. Rev. B **58**, R7520 (1998); **58**, R13 338 (1998).
[5] Y. Yaish *et al.,* cond-mat/9904324; (unpublished).
[6] E. Abrahams *et al.,* Phys. Rev. Lett. **42**, 673 (1979).
[7] A. M. Finkelstein, Zh. Eksp. Teor. Fiz. **84**, 168 (1983); Z. Phys. **56**, 189 (1984); C. Castellani *et al.,* Phys. Rev. B **30**, 527 (1984); **30**, 1596 (1984).
[8] V. Dobroslavljević *et al.,* Phys. Rev. Lett. **79**, 455 (1997); C. Castellani, C. Di Castro, and P. A. Lee, Phys. Rev. B **57**, R9381 (1998); D. Belitz and T. R. Kirkpatrick, Phys. Rev. B **58**, 8214 (1998); P. Phillips *et al.,* Nature (London) **395**, 253 (1998); F.-C. Zhang and T. M. Rice, cond-mat/9708050; S. Chakravarty *et al.,* Philos. Mag. B **79**, 859 (1999); G. Benenti, X. Waintal, and J.-L. Pichard, Phys. Rev. Lett. **83**, 1826 (1999).
[9] V. M. Pudalov, JETP Lett. **66**, 175 (1997).
[10] Song He and X. C. Xie, Phys. Rev. Lett. **80**, 3324 (1998).
[11] B. L. Altshuler and D. L. Maslov, Phys. Rev. Lett. **82**, 145 (1999).
[12] Y. Hanein *et al.,* cond-mat/9901186.
[13] V. M. Pudalov *et al.,* JETP Lett. **65**, 932 (1997); D. Simonian *et al.,* Phys. Rev. Lett. **79**, 2304 (1997); V. M. Pudalov *et al.,* Physica (Amsterdam) **249-251B**, 697 (1998).
[14] E. Ribeiro *et al.,* Phys. Rev. Lett. **82**, 996 (1999).
[15] P. Mohanty, E. M. Jariwala, and R. A. Webb, Phys. Rev. Lett. **78**, 3366 (1997); Phys. Rev. B **55**, R13 452 (1997).
[16] D. P. Pivin *et al.,* Phys. Rev. Lett. **82**, 4687 (1999); A. G. Huibers *et al.,* cond-mat/9904274.
[17] G. Eytan *et al.,* Phys. Rev. Lett. **81**, 1666 (1998).
[18] E. Shimshoni and A. Auerbach, Phys. Rev. B **55**, 9817 (1997); E. Shimshoni, A. Auerbach, and A. Kapitulnik, Phys. Rev. Lett. **80**, 3352 (1998).
[19] A. A. Shashkin *et al.,* Phys. Rev. Lett. **73**, 3141 (1994); V. T. Dolgapolov *et al.,* JETP Lett. **62**, 162 (1995); I. V. Kukuskin *et al.,* Phys. Rev. B **53**, R13 260 (1996).
[20] For a review of percolation theory and for values of the critical exponents, see, for example, D. Stauffer and A. Aharaony, *Introduction to Percolation Theory* (Taylor & Francis, London, 1992).
[21] V. Ambegaokar, B. I. Halperin, and J. S. Langer, Phys. Rev. B **4**, 2612 (1971).
[22] T. P. Smith *et al.,* Phys. Rev. Lett. **61**, 585 (1988); K. J. Thomas *et al.,* Phys. Rev. Lett. **77**, 135 (1996); Phys. Rev. B **58**, 4846 (1998).
[23] See, e.g., B. J. van Wees, Phys. Rev. B **43**, 12 431 (1991).
[24] To simplify the numerics, we chose random potentials with azimuthal symmetry.
[25] H. A. Fertig and B. I. Haleprin, Phys. Rev. B **36**, 7969 (1987).
[26] For recent reviews on quantum dots see, e.g., M. A. Kastner, Comments Condens. Matter Phys. **17**, 349 (1996); R. Ashoori, Nature (London) **379**, 413 (1996).
[27] See, e.g., S. H. Tessmer *et al.,* Nature (London) **392**, 6671 (1998); H. F. Hess *et al.,* Solid State Commun. **107**, 657 (1998).
[28] A. Yacoby (unpublished).
[29] C. Pasquier *et al.,* Phys. Rev. Lett. **70**, 69 (1993).
[30] P. L. McEuen *et al.,* Phys. Rev. Lett. **66**, 1926 (1991).