Theory of the magnetoresistance of disordered superconducting films

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Recent experimental studies of magnetoresistance in disordered superconducting thin films on the insulating side of the superconductor-insulator transition reveal a huge peak (about 5 orders of magnitude compared with the resistance at the transition). While it may be expected that magnetic field destroys superconductivity, leading to an enhanced resistance, attenuation of the resistance at higher magnetic fields is surprising. We propose a model which accounts for the experimental results in the entire range of magnetic fields, based on the formation of superconducting islands due to fluctuations in the superconducting order parameter amplitude in the disordered sample. At strong magnetic fields, due to Coulomb blockade in these islands, transport is mainly through the normal areas, and thus a decrease is the size and density of the superconducting islands leads to an enhanced conductance and a negative magnetoresistance. As the magnetic field is reduced and the size and density of these islands increase, the conductance is eventually dominated by transport through the superconducting islands and the magnetoresistance changes sign. Numerical calculations show a good qualitative agreement with experimental data.

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I. INTRODUCTION

The interplay between superconductivity and disorder is a long-standing problem, dating back to the late 1950's,[1,2] resulting in the common wisdom that weak disorder has no dramatic effect on superconductivity. Strong disorder, however, may have a profound effect, driving the system from a superconducting (SC) to an insulating state. Such a SC-insulator transition (SIT) was observed in two-dimensional amorphous superconducting films.[3] Reducing the film thickness or increasing the magnetic field drives these films (which are held below their bulk critical temperature) from a SC state, characterized by a vanishing resistance as $T \to T_c$, to an insulating state, characterized by a diverging resistance as $T \to 0$. The possibility of tuning the system continuously between these two phases is a manifestation of a quantum phase transition.[4]

The origin of this transition is still in debate. While some theories[5] claim that it may be understood in terms of Cooper-pair scattering out of the SC condensate into a Bose-glass state (so-called "dirty boson" models), there is evidence, both experimental[6–9] and theoretical,[10] that a percolation description of the SIT is more adequate for, at least, some of these samples.

Further insight into the nature of the SIT may be gained by looking at the magnetoresistance (MR) on its insulating side. Decade-old experiments observed nonmonotonic MR, exhibiting a shallow peak at some magnetic field $B_{\max}$.[11,12] Recent experiments[13] show, however, that in some samples the effect is dramatic, with the resistance value at the peak $R_{\max}$ reaching as high as a few orders of magnitude its value at the SIT. As the magnetic field is further increased the resistance drops back a few orders of magnitude (inset of Fig. 3). Further investigations of this effect[14] reveal that disorder also has a major influence. With increasing disorder strength, the critical field for the SIT B_c diminishes, while $R_{\max}$ increases. The temperature dependence of the MR at high temperatures fits an activationlike behavior $R \propto \exp(T_0/T)$, with a magnetic field dependent T_0 as seen in the inset of Fig. 4(a). At lower temperatures there is a deviation from this behavior towards some weaker temperature dependence.

While an enhancement of the resistance with increasing magnetic field may be understood in terms of decreasing SC order, the suppression of the resistance at higher magnetic field remains a puzzle. In this paper we propose a phenomenological model for clarifying this puzzle, based on simple physical assumptions. The model successfully describes the peak in the MR together with its temperature and disorder dependence.

The rest of the paper is organized as follows. In Sec. II the model and the mechanism leading to negative MR are described. The numerical calculations are presented in Sec. III. In Sec. IV we discuss the effects of disorder and Sec. V is devoted to summary and conclusions.

II. THE MODEL

In this section we propose a model for the MR in the entire range of magnetic fields. The model is based on three assumptions. The first is that disorder induces formation of SC islands (SCIs) due to fluctuations in the amplitude of the SC order parameter. This concept has already been used to interpret some experiments.[11,13] It was directly observed in other systems by STM measurements[15] and further corroborated by numerical simulations.[16] The second assumption is that as the magnetic field is increased, the concentration and size of these SCIs decrease. Preliminary numerical results support this picture and will be reported elsewhere. The third assumption is that the SCIs have a charging energy and, thus, a Cooper pair entering a SCI (via an Andreev tunneling process) has to overcome it. This charging energy is expected to be inversely proportional to the island size, and thus to in-

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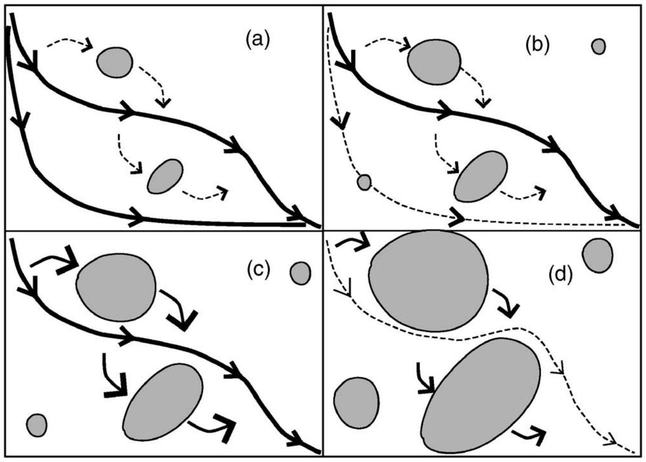

FIG. 1. Schematic representation of the model. Solid lines present the dominant mode of transport compared to the broken lines. (a) At strong magnetic fields, $B \gg B_{\max}$. The system is composed of small superconducting islands with large charging energy. In this regime transport through normal paths (solid lines) is always preferable than transport through the superconducting islands (SCIs) (see text). (b) As the magnetic field is decreased, but still $B > B_{\max}$, more SCIs appear, resulting in a decrease in available trajectories for transport [bottom solid line in (a) and bottom dashed line in (b)] and hence negative magnetoresistance. (c) At a certain field $B_{\max}$ the resistance of normal paths and paths that include SCIs is comparable, resulting in a peak in the resistance. (d) For even lower fields, $B_c < B < B_{\max}$, transport through SCIs is always favored, resulting in positive magnetoresistance.

crease with increasing magnetic field (but this latter assumption is not crucial to the observation of the MR peak, see below).

In order to elucidate the mechanism by which the MR can be negative, consider such a system in the strong magnetic field regime $B \gg B_{\max}$ [Fig. 1(a)]. Due to the strong magnetic field the SCIs are small and have a large charging energy. There are two types of trajectories available for electron transport: those which follow normal areas of the sample ["normal paths," solid lines in Fig. 1(a)] and those in which an electron tunnels into a SCI via the Andreev channel ["island paths," dashed lines in Fig. 1(a)]. The resistance of the normal paths R_N has some value (which may depend on, e.g., length, temperature, disorder, etc.) and is assumed to be weakly affected by magnetic field. Due to Coulomb blockade, transport through the SCI paths is thermally activated, and hence the resistance of the island paths is of the form $R_I \sim \exp(E_c/T)$, where E_c is the charging energy of the island. If E_c is large then the main contribution to the conductance is due to transport along the normal paths. Consistent with experiment, the MR in this regime is small.

As the magnetic field is decreased [but still in the regime $B > B_{\max}$, Fig. 1(b)], more SCIs are created and their size increases, but they are still small enough such that transport along normal paths is favorable. However, some paths which were normal at higher fields [e.g., bottom solid line in Fig. 1(a)] now become island paths and hence unavailable for electron transport [bottom dashed line in Fig. 1(b)]. Thus, the effective phase space available for electron transport diminishes, resulting in a negative MR. Eventually, at a certain magnetic field $B = B_{\max}$ [Fig. 1(c)] some SCIs are large enough so that their charging energy is small and the resistance through them is comparable to the resistance through normal paths, i.e.,

$$R_N \approx R_I. \tag{1}$$

At this point the resistance reaches its maximum value, since as the magnetic field is further decreased [Fig. 1(d)] the SCIs are so large that transport through them is always preferred over transport through normal paths. An increase in density and size of the SCIs will thus result in a decrease in the resistance. At the critical field B_c the SCIs percolate through the system, resulting in an insulator-to-SC transition.

Putting it in other words, conductance on the left side of the peak is dominated by transport through the SC islands, and as their density and size decrease with magnetic field, the resistance increases, while on on the right side of the peak transport is mainly through the normal areas and here the reduction in the density and size of the SC island with increasing magnetic field leads to a lower resistance.

We note that if the temperature is lower than the Josephson coupling between two islands then SC correlations will extend between them, effectively joining them into a single island. Thus, we expect that at low temperatures the geometry will be temperature dependent, resulting in a temperature dependent critical point as is indeed seen in some experiments. However, as the magnetic field rises the distance between the SCIs increases, leading to a reduction in the Josephson coupling between them, rendering it irrelevant to the physics near the MR peak.

III. NUMERICAL RESULTS

In order to substantiate these heuristic arguments we model the thin film by a square lattice, where each site can be either normal, with probability p, or SC, with probability $1-p$, corresponding to the concentration of normal areas and SCIs in the sample. The probability p is a function of the magnetic field B, and we may assume that $p(B)$ is an increasing monotonic function. To describe the disordered system at strong magnetic fields, in the absence of SCIs, we follow Ref. 18 and assign a resistance between any two normal sites of the form

$$R_{ij} = R_0 \exp\left(\frac{2r_{ij}}{\xi_{\text{loc}}} + \frac{|\epsilon_i| + |\epsilon_j| + |\epsilon_i - \epsilon_j|}{2kT}\right), \tag{2}$$

where R_0 is a constant, r_{ij} is the distance between sites i and j, ξ_{loc} is the localization length, ϵ_i is the energy of the ith site measured from the chemical potential (taken from a uniform distribution $[-W/2, W/2]$), and T is the temperature. The localization length ξ_{loc} is taken to be small (in units of lattice constant), effectively allowing only nearest- and next-nearest-neighbor hopping. The resistance between two (neighboring) SC sites R_{SS} is taken to be very small

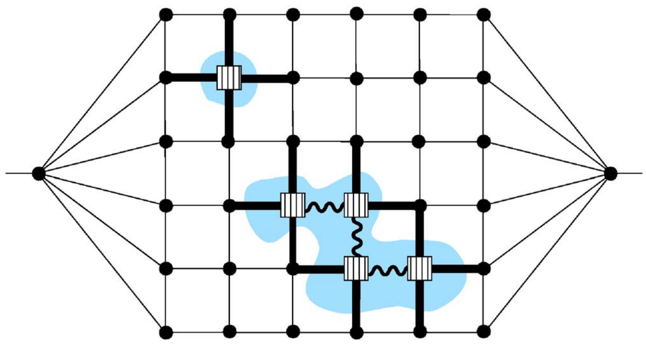

FIG. 2. (Color online) The lattice model is composed of regular sites and superconducting sites. Clusters of the latter form SCIs (shaded islands). The resistance between two normal sites (small circles) is given by Eq. (2). The resistance between two SC sites (squares) is very small (and becomes smaller with decreasing temperature, see text) for neighboring sites (wavy lines) and is infinite (no link) between non-neighboring sites. The resistance between neighboring normal and SC sites (thick lines) is much higher than the resistance between two normal sites, and is exponential in the charging energy of the SC island [Eq. (3)].

compared with that of Eq. (2), but still not zero and temperature dependent, in such a way that it vanishes as $T \to 0$ (distant SC sites are disconnected). The calculations of resistance were conducted with several functional forms for $R_{SS}(T)$ (power law, exponential dependence, etc.) and no qualitative difference between them was found. The resistance between a normal site and a SC site (local N-S junction) is taken to be

$$R_{NS} \propto \exp(E_c/kT), \tag{3}$$

where E_c is the charging energy of the island. For simplicity, and to avoid additional parameters, the charging energies of all islands were taken to be identical, independent of island size. It is demonstrated below that the main experimental observations can be well understood even under such an assumption.[19] This is because the important Coulomb blockade energy is that associated with typical island size near B_{max}, where transport through the SCIs becomes relevant. In actual physical systems, however, the charging energy depends on the island size, and hence on p, and this effect is expected to further enhance the competition between normal and island paths described above. Repeating the calculation with a p-dependent charging energy for various dependencies yields no qualitative change in the results.

In our calculations the lattice is connected to electrodes (Fig. 2) and the resistance is calculated numerically using Kirchhoff's laws. The left-most and right-most links in Fig. 2 are taken to be SC, thus avoiding a dependence of the resistance on the properties of the edge sites.

In Fig. 3 we plot the calculated resistance (on a log scale) as a function of the probability p for different temperatures. The calculations were conducted on lattices of size 25×25 (and repeated for different lattice sizes, with no qualitative change in the results) and log-averaged over 100 realizations of disorder. A peak in the MR is observed at $p_{max}=0.5$, with

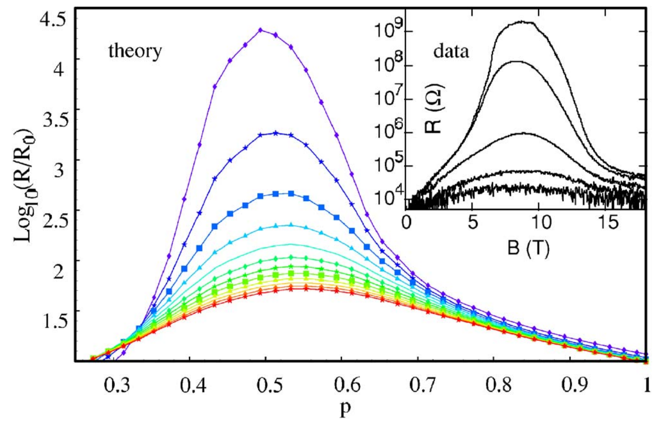

FIG. 3. (Color online) Numerical results: resistance (on a log scale) as a function of probability p for different temperatures, with the parameters $W=0.4, E_c=4, \xi_{loc}=0.1, T=0.1, 0.2, \ldots, 4$. This is to be (qualitatively) compared with the experimental data (inset) of Ref. 13.

peak resistance four orders of magnitude higher than the resistance at the transition. The results are compared with the experimental data of Ref. 13 (inset of Fig. 3), and a qualitative agreement is evident. Notice that the critical probability p_c defined as the probability at which the resistance is temperature independent, is shifted from the percolation critical probability. The reason for this is that the resistance of the SC links R_{SS} is finite. As the temperature is decreased the critical probability moves towards the percolation critical probability, eventually reaching it at $T=0$.

When fitting the resistance as a function of temperature to an activationlike behavior, $R \propto \exp(T_0/T)$, we find a nonmonotonic dependence of T_0 on the probability p (Fig. 4), resembling the experimental data of Ref. 13 (upper inset of Fig. 4) and of Ref. 14. The activation temperature rises from $T_0 \approx W$ at $p=1$ to $T_0 \approx E_c$ for $p=p_{max}$. It then drops back again due to increasing weight of SC areas in the sample, eventually reaching $T_0=0$ at the transition.

At high temperatures the activation fit is excellent for all values of p. For low temperatures, on the other hand, the fit

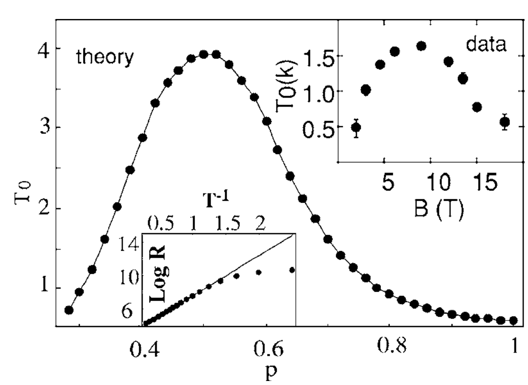

FIG. 4. The activation energy T_0 as extracted from fitting the resistance to an activation behavior, obtained from the numerical calculation and from the experimental data of Ref. 13 (inset). Lower inset: an Arrhenius plot of the resistance as a function of temperature for $p=p_{max}=0.5$. A deviation from an activated behavior is clearly seen.

becomes worse (lower inset of Fig. 4). Similar results were presented in the experimental data of Ref. 13. The reason for this is that at low enough temperatures tunneling into the SCIs is suppressed, except very close to the SIT. Since the resistance through normal areas is activated not by E_c but rather by W, the slope of the Arrhenius plot changes, as is also evident in the experiment. Thus, we predict that the low-temperature behavior near B_c will be the same as in higher temperatures in strong magnetic fields $B \gg B_{\mathrm{max}}$.

The fact that the temperature dependence is not a pure activation in the whole magnetic field and temperature range is crucial to the observation of such a peak in the MR. If one assumes solely a magnetic-field-dependent activation energy $T_0(B)$, then for different temperatures T_1 and T_2 the ratio

$$\frac{\ln[R(B,T1)/R_0]}{\ln[R(B,T2)/R_0]} = \frac{T_1/T_0(B)}{T_2/T_0(B)} = \frac{T_1}{T_2}, \tag{4}$$

would be independent of field, in contrast with the experimental observation.

IV. EFFECT OF DISORDER

The amount of disorder affects the parameters in our model in several ways. First, the width of the energy distribution W increases, though this has a minor effect on the behavior near the MR peak. Second, the initial concentration of normal islands, p_0, increases with disorder. If one assumes $p(B) \simeq p_0 + \alpha B^x$ (where α is some constant and x is probably equal to 2 for small fields, as the system is symmetric under reversal of the magnetic field direction), then $B_{\mathrm{max}} = (p_{\mathrm{max}} - p_0)^{1/x}/\alpha$, namely, B_{max} decreases with increasing disorder. Finally, the typical size of the SCIs decreases, leading to an enhancement of the Coulomb charging energy and an exponential increase of R_{max}. These latter two points are consistent with the experimental observations,[14] though quantitative predictions require a detailed study of the distribution of the SCIs with disorder and magnetic field, in order to know, e.g., how p_0 depends on B. Another effect of increasing disorder is a decrease in B_c.[14] The same reasoning as for the decrease in B_{max} may be applied for B_c as well leading to a shift of B_c with disorder.

According to the results of the previous section, the activation energy T_0 is determined by the Coulomb blockade energy E_c (or by W away from the resistance peak). However, in Ref. 13 the authors point out an inverse correlation between $T_{0\,\mathrm{max}}$, the activation energy at the resistance peak, and the samples critical temperature T_c, which is determined by the amount of disorder, i.e., a reduction in T_c leads to an enhanced $T_{0\,\mathrm{max}}$. This may be readily understood by recalling that T_c is reduced by increasing disorder, leading to an increase in the normal area resistance. Thus, for the condition for the peak resistance, Eq. (1), to hold, the charging energy of the typical island should increase, and hence the higher $T_{0\,\mathrm{max}}$. In other words, for worse normal resistances, the condition (1) holds for smaller size islands, and thus larger charging energy and larger activation energy at the peak.

Interestingly, the two temperatures $T_{0\,\mathrm{max}}$ and T_c are found to be of the same order of magnitude.[13] In order to see why the charging energy and T_c could be of the same order of magnitude, one should note that for a SCI participating in the transport, E_c cannot exceed Δ. If this is the case, the transport through the island will happen by activation of quasiparticles above the gap and thus this island will not play the role of a SCI as defined in the model. As the activation energy is determined by the largest possible Coulomb blockade energy in the electron path, it is bound from above by the local SC gap which, as has been shown numerically,[16,17] may exceed the bulk SC gap. Thus, with increasing disorder and decreasing T_c, E_c increases but cannot exceed the highest local gap and thus remains within that order of magnitude. On the other hand, for weak disorder the SCIs can be rather large, leading to $E_c \ll T_c$. These arguments are supported by the results of Ref. 14, where the authors show that for weakly disordered samples $T_{0\,\mathrm{max}}$ is much lower than T_c (down to less than 10%).

V. DISCUSSION AND SUMMARY

To summarize, we have demonstrated that competition between normal electron and Cooper-pair transport, generated by Coulomb blockade of superconducting islands and driven by perpendicular magnetic field, may yield nonmonotonic magnetoresistance. This is accompanied by a change in the temperature dependence of the resistance, resulting from a crossover from nearest-neighbor hopping in normal areas to tunneling into the SC islands. This crossover may lead to variation of the MR by several orders of magnitude, as was indeed observed in experiments.

The theory presented here may also account for the MR in the presence of a tilted magnetic field. In recent experiments[20] a nonmonotonic MR was observed, similar to the case of perpendicular field. The location of the MR peak was found to be higher for parallel fields.

In order to address this phenomena, we consider a sample with finite width. Taking this into account, a tilted magnetic field suppresses the SCIs both in-plane and in the "thin" direction. Due to the finite width, the electron trajectories are now three dimensional, and thus variation in the SCI size due to parallel magnetic field affects the resistance in a similar way as described above, leading to the same non-monotonic MR for parallel fields. This also explains the shift in the location of the MR peak towards higher fields.

Another experimental observation[14] is that at very strong fields (up to 30 T) the resistance of some samples saturates at values somewhat higher than the resistance just above the (temperature-driven) superconducting transition. Our model provides a natural explanation for this. At strong magnetic fields all vestiges of SC correlations are gone, and one is left with an insulator (or a bad metal) whose higher resistance is hardly affected by the magnetic field).[21]

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