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Universal spin-induced magnetoresistance in the variable-range hopping regime

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Abstract. – The magnetoresistance in the variable-range hopping regime due to Zeeman spin-splitting and intra-impurity interactions is calculated analytically and shown to be a universal function of $\mu H/kT \log R$. Good agreement with numerical calculations in one and two dimensions is observed. With the inclusion of quantum interference effects, excellent agreement with recent experiments is obtained.

Magnetic field may have dramatic effects on the resistance of strongly disordered materials. In this regime, where the resistance is described by Mott law, $R = R_0 \exp[(T_0/T)^{1/(d+1)}]$, with $T_0 \sim 1/\rho\xi^d$, ρ the density of states, and ξ the localization length, the effects of the magnetic field can be divided into orbital effects and spin effects. In weak orbital fields, of the order of one quantum flux through the hopping area, $\sqrt{L^3\xi}$, where L the hopping length, is typically $(\xi/T\rho)^{1/(d+1)}$, the prefactor of the exponent changes due to quantum interference [1]-[4], leading usually to negative magnetoresistance (MR). In higher fields, of the order of one quantum flux through the area defined by the localization length, ξ is enhanced [4], [5], leading to an exponential decrease in the resistance. In yet higher fields, the impurity wave function shrinks, resulting in an exponential enhancement of the resistance [6].

As the resistance does not depend explicitly on spin, one may not expect sensitivity of the resistance to spin-splitting due to magnetic field. However, when intra-impurity interactions are taken into account, it was pointed out [7] that the polarization of the electron spins will block some of the hopping processes, leading to an exponentially increased resistance.

It is non-trivial experimentally to separate the contributions of all these mechanisms to the MR. This is easier for more disordered samples and at lower temperatures, where the magnetic-field scales for these effects become more and more separated. More naturally, the technological progress in growing strongly disordered thin layers makes it possible to study only the spin-effects of a parallel magnetic field [8], [9]. It is thus imperative to develop a good understanding of the spin effects, not only in order to understand transport in the presence of parallel field, but also in order to make it possible to separate the spin effects from the orbital effects in more complicated situations. This question became even more important with recent suggestions to use the variable-range MR as a sensitive magnetic sensor [10].

Kamimura *et al.* [7], in addition to suggesting the mechanism and deriving the limit of high fields, have performed a detailed numerical study of the dependence of the resistance on temperature and magnetic field. More recently, Clarke *et al.* [11] have proposed that the resistance may be written

$$R(H) = R_0 \exp\left[(T_0/T)^{\frac{1}{d+1}} F(y)\right], \tag{1}$$

with F a universal function of $y \equiv \mu H/T(T_0/T)^{1/(d+1)}$. They went on to obtain $F(y)$ in limiting situations, and then calculated it numerically as a function of the dimensionless parameter y.

Here I report an analytic calculation of the MR due to Zeeman splitting. I find that $R(H)$ is indeed given by (1), with $F(y)$ obtained analytically in arbitrary dimensions. This function agrees very well with the numerical data in one and two dimensions, and with available experimental data [9], [12].

The starting point of this calculation is the mapping of the resistance problem into a percolation criterion [13] of an equivalent random resistor network [14], consisting of randomly placed sites, of density ρ, with random energies ϵ_i. The resistance between each pair of sites is given by [13] $R_{ij} = \exp[(|\epsilon_i|+|\epsilon_j|+|\epsilon_i-\epsilon_j|)|/2T+2r_{ij}/\xi]$, where all resistances are measured in units of R_0. Since the resistances vary exponentially, the overall resistance will be dominated by the weakest link, which is the largest resistance, R, such that the cluster formed by all resistances (bonds), satisfying $R > R_{ij}$, percolates. Clearly, all states participating in the percolating network (defined as occupied sites) must satisfy $R > \exp[|\epsilon_i|/2T]$. Following [13], the percolation criterion employed here is the following ([1]) —given such an occupied site, the number of bonds attached to it has to be higher than a critical threshold, Z_c, for the system to percolate,

$$\begin{aligned} Z_c &= \frac{1}{4T\log R} \int_{-2T\log R}^{2T\log R} \mathrm{d}\epsilon_1 \int \mathrm{d}\epsilon_2\, \rho \int \mathrm{d}^d r_{12}\, \Theta(R - \exp[(|\epsilon_1| + |\epsilon_2| + |\epsilon_1 - \epsilon_2|)/2T + 2r_{12}/\xi]) = \\ &= 3T \frac{\xi^d \rho \pi^{d/2} (\log R)^{d+1}}{2^{d+1}\Gamma(d/2) d(d+1)(d+2)}, \end{aligned} \tag{2}$$

leading directly to the Mott hopping law.

In the presence of an intra-impurity interaction U, there are two types of impurities participating in the conduction process —those whose energies lie in the vicinity of the Fermi energy, E_F (type A), and those whose energy is around $E_F - U$ (type B). For $U \gg T$, the latter are at least singly occupied. Application of a magnetic field, such that $\mu H > T$, polarizes the singly occupied impurities, thus blocking any process of hopping from one singly occupied impurity to another, leading to a lower effective density, and to an exponential enhancement of the resistance [7]. For arbitrary magnetic field, one can define an equivalent resistor network, consisting of sites of two types A and B, with relative densities ρ_A and ρ_B, and localizations lengths, ξ_A and ξ_B, respectively, such that the resistance between pairs of sites is given by [7], [11], [15]

$$\begin{aligned} R_{ij}^{AA} &= \exp\left[\frac{|\epsilon_i| + |\epsilon_j| + |\epsilon_i - \epsilon_j|}{2T} + \frac{2r_{ij}}{\xi_A}\right], \\ R_{ij}^{AB} = R_{ij}^{BA} &= \exp\left[\frac{|\epsilon_i| + |\epsilon_j| + |\epsilon_i - \epsilon_j - 2\mu H|}{2T} + \frac{\mu H}{T} + \frac{2r_{ij}}{\xi_{AB}}\right], \end{aligned} \tag{3}$$

where $\xi_{AB} = \max \xi_A, \xi_B$. R_{ij}^{BB} is given by an equation identical to R_{ij}^{AA}, with ξ_B replacing ξ_A.

([1]) Note that the percolation criterion employed here is different from that of ref. [2], as the invariance assumption is more appropriate for the problem at hand.

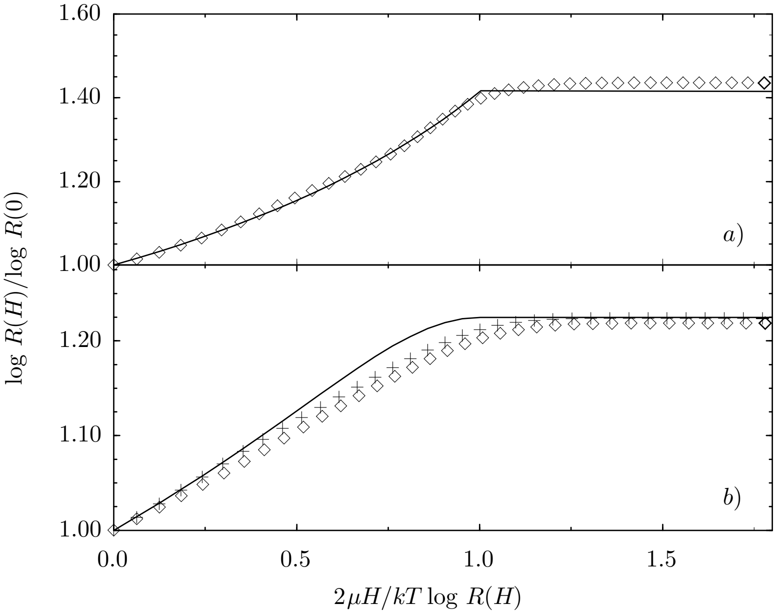

Fig. 1. – Comparison of the numerical calculation (symbols) of $\log R(H)/\log R(0)$ in one dimension to the analytic $F(x)$ (continuous line), with *no free parameters*. *a*) ◇ $\rho_A/\rho_B = 1$, *b*) ◇ $\rho_A/\rho_B = 2$, + $\rho_A/\rho_B = 1/2$.

The percolation condition (2) becomes (2)

$$Z_{\rm c} = \frac{\rho}{4T\log R}\int_{-2T\log R}^{2T\log R} {\rm d}\epsilon_1 \int {\rm d}\epsilon_2 \int {\rm d}^d r_{12}\left[p_A\rho_A\Theta(R-R_{ij}^{AA}) + p_B\rho_B\Theta(R-R_{ij}^{BB})\right] +$$

$$+\frac{\rho}{4T\log R-4\mu H}\int_{-2T\log R+2\mu H}^{2T\log R-2\mu H} {\rm d}\epsilon_1 \int {\rm d}\epsilon_2 \int {\rm d}^d r_{12}\left[p_A\rho_B\Theta(R-R_{ij}^{AB}) + p_B\rho_A\Theta(R-R_{ij}^{BA})\right], \quad (4)$$

where p_A and p_B are the fractions of the sites of types A and B, respectively, on the percolating cluster. From (3) and (4) one finds that one can write $R(H)$ in the form (1) with $F(x) = \{\xi_{AB}^d/[p_a\rho_A\xi_A^d + p_B\rho_B\xi_B^d + \xi_{AB}^d(p_a\rho_B + p_B\rho_A)g(x)]\}^{1/(d+1)}$, with $x = 2\mu H/T\log R$, and $g(x) = \Theta(1-x)(1-x)^d[1+(d-2)x/2+(d-2)(d-1)x^2/12]/(1-x/2)$. An independent equation can be derived for p_A and p_B, using the fact that a site on the percolation cluster has to be connected to either an A-site or a B-site,

$$\frac{p_A}{p_B} = \frac{\rho_A}{\rho_B}\frac{p_A\xi_A^d + p_B\; g(x)\xi_{AB}^d}{p_B\xi_B^d + p_A\; g(x)\xi_{AB}^d}\,, \quad (5)$$

leading to the final expression

$$F(x) = \{2\xi_{AB}^d/[\rho_A\xi_A^d + \rho_B\xi_B^d + \sqrt{(\rho_A\xi_A^d - \rho_B\xi_B^d)^2 + 4g(x)^2\rho_A\rho_B\xi_{AB}^d}]\}^{1/(d+1)}, \quad (6)$$

for small magnetic fields and $\xi_A = \xi_B$, $F(x) \simeq 1 + \rho_A\rho_B x$, in agreement with existing perturbation theory [6], [11]. $F(x)$ saturates at $x = 1$, or $\mu H = T(T_0/T\max\{\rho_A,\rho_B\})^{1/(d+1)}$,

(2) It is assumed that $Z_{\rm c}$ is independent of magnetic field. A different approach to two-colour percolation can be found in [16].

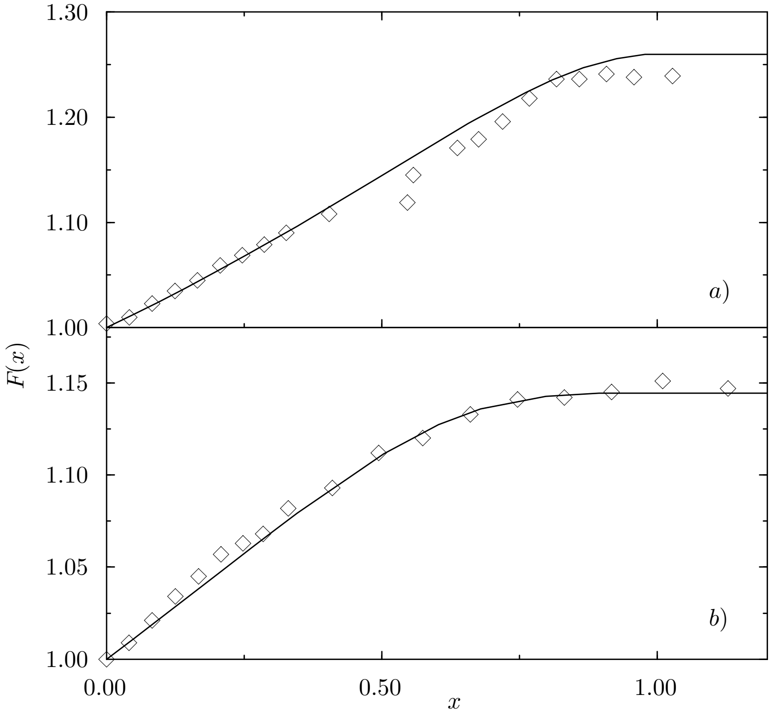

Fig. 2. – Comparison of the analytic $F(x)$ to the numerical data extracted from ref. [11] in two dimensions (no interference effects were included in the calculation). The only free parameter is the x-axis scale. *a)* $\rho_A/\rho_B = 1$, *b)* $\rho_A/\rho_B = 1/2$,

at a value of $(1/\max\{\rho_A, \rho_B\})^{1/(d+1)}$, in agreement with ref. [11]. In the following, for simplicity, I take $\xi_A = \xi_B$.

In fig. 1 we compare the analytic result to the numerical calculation of $\log R(H)/\log R(0) \simeq F(x)$ I have performed in one dimension. In the numerical calculation the resistance of a strongly disordered system of length $L = 50\xi$ and temperature $T = 0.04W$, where W is the width of the energy distribution, has been calculated using the equivalent random resistor network. There are no free parameters in this comparison. In fig. 2 we compare the analytic result to the numerical data in two dimensions reported in [11]. Here the resistance has been calculated using the mapping into the percolation problem. Since the parameter x used in that work differs from ours and the ratio depends sensitively on the value of T_0, we allowed a single fitting parameter —the x-axis scale. In both fig. 1 and 2, data has been presented for *a)* $\rho_A/\rho_B = 1$, and *b)* $\rho_A/\rho_B = 1/2$. Excellent agreement between the numerical calculations and the analytic calculation is observed.

The real test of the theory is comparison to experimental data. In fig. 3 we compare the analytic result (broken curve) in 2d to the experimental data for the MR of an In_2O_{3-x} layer of thickness $d = 110$ nm in a parallel field [9]. In the inset we compare the analytic result in 3d to the experimental data extracted from [12]. In the latter work the magnetic-field dependence of the the exponent (eq. (1)) was attributed to a decrease in ξ (even though theory [5] predicts that ξ increases with field). Here the data was replotted in terms of $F(x)$. In both plots the two fitting parameters used were ρ_A and the scale of the x-axis. While good agreement may be observed in the two figures at high fields, there are clear deviations at low fields. This is due to the contribution of the orbital effects, most importantly the quantum interference effects.

The quantum interference effects —the coherent scattering of the hopping electron by all other impurities— have been taken into account within the percolation approach in [2]. The

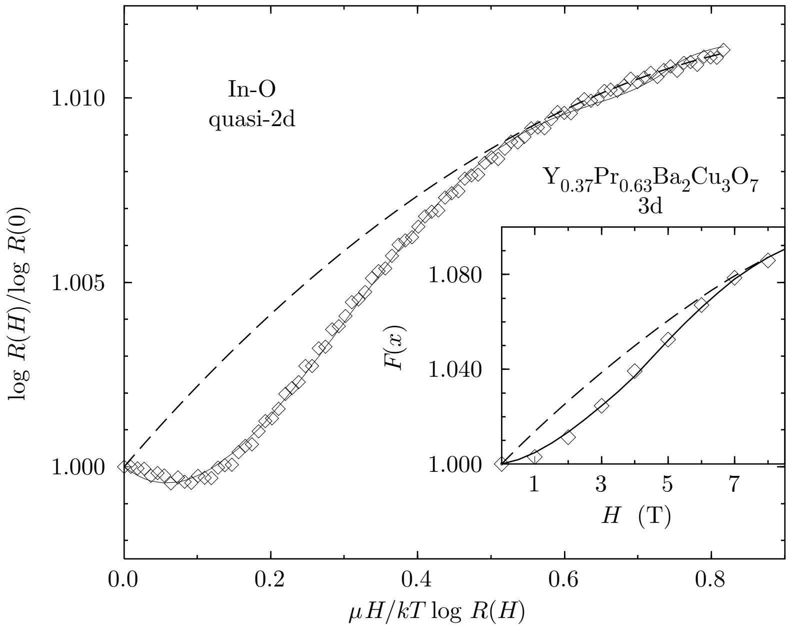

Fig. 3. – Comparison of the analytic result with (solid line) and without (broken line) including interference effects to the experimental data of [9] and (in the inset) of [12]. The fitting parameters are discussed in the text.

percolation condition (2) now also involves an interference probability y^2 multiplying the exponent inside the Θ-function, and an integration over its distribution, which can be derived independently [2], [3]. The resulting integral equation for the MR has to be solved numerically, except for low magnetic fields or deep in the insulating regime. In the latter regime one finds that the resistance is multiplied by $\exp[-\langle\log y^2\rangle]$, where the average is over the amplitude distribution. Using direct integration or random matrix arguments, $\langle\log y^2\rangle$ was found to be [3] $-\gamma - \log 2 + \log(1 + \sqrt{2 - H/H_\phi}H/H_\phi)$, where γ is the Euler constant, and the field H_ϕ corresponds to one flux quantum through the hopping length. A similar procedure can be applied in the presence of spin-split states, as the quantum interference amplitude can be incorporated into the percolation condition (4) and into eq. (5). The calculation does not lead to a change in $F(x)$, but leads to a multiplicative factor in eq. (1), $\exp[-A(x)\langle\log y^2\rangle]$, with $A(x) = 1 - 4\rho_A\rho_B x g(x) g'(x)/(d+1)s(x)(1+s(x))$.

Interestingly, this final result suggests that the prefactor resulting from the quantum interference is changing even for fields larger than the saturation field H_ϕ, due to the change in the effective number of impurities participating in the quantum interference.

The resulting MR is plotted in fig. 3 (solid line). The three fitting parameters used in this plot were $\rho_A = 0.76$, $(T_0/T)^{1/(d+1)} = 8.2$, and $H_\phi = 8.1$ T (no fit was used to scale the x-axis). The latter two values should be compared to the approximate experimental values, $(T_0/T)^{1/3} \sim 10.6$, and $\phi_0/dL \sim 9.1$ T, respectively. An almost perfect agreement with the experimental data is observed. Similar procedure can be successfully applied to fit the data of ref. [12] (inset of fig. 3).

For higher temperatures or perpendicular magnetic fields, the last step in the calculation —the mapping of the percolation problem into a log-averaging procedure, is no longer applicable [17]. The formalism is still valid, but one needs to solve the full integral equation discussed above.

In this work we have reported an analytic calculation of the magnetoresistance due to spin effects in the variable-range-hopping regime. The calculation compares very well with available numerical and experimental data. The mechanism responsible for that effect —the blocking of hopping from a singly occupied impurity to another singly occupied impurity due to spin polarization, suggests that spin-orbit scattering may play an intriguing role in such systems. As the temperature is lowered, the hopping length gets larger and may eventually become larger than the spin-orbit scattering length. When this happens the electron may flip its spin upon hopping from one impurity to another, thus making it possible to hop from one singly occupied impurity to another. Thus it is predicted that spin-orbit scattering may lead to non-trivial temperature effects —a change in the slope of $\log R$ *vs.* $T^{1/(d+1)}$ as the temperature is lowered. As such "turning-on" of spin-orbit scattering with lowering of the temperature has been demonstrated for the magnetoresistance in the weakly localized regime [18], an experimental investigation of this prediction should not be too difficult. We hope that this work will motivate such investigations.

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