Ben-Gurion University of the Negev

Faculty of Natural Sciences

Department of Physics

Diffractive energy spreading and its semiclassical limit

Alexander Stotland

Collaborations:

Doron Cohen

Discussions:

Itamar Sela

Yoav Etzioni

Vladimir Goland

References:

A.Stotland and D.Cohen, J. Phys. A 39, 10703 (2006)

$GIF

Driven Systems

$\mathcal{H} = \mathcal{H}(Q, P; X(t))$

Energy is not a constant of motion!

Moments of the energy distribution:

$$\delta^r E = \int \rho_t(E) E^r dE$$

$r = 1$ expectation value

$r = 2$ variance

Bohr quantum-classical correspondence (QCC):

- Gaussian wavepacket
- Smooth potentials

$\Rightarrow$ The same moments

Restricted versus detailed QCC:

- $r = 1, 2$ restricted QCC (robust)
- $r > 2$ detailed QCC (fragile)

Motivation - the theory of response

"Response" has to do with the $t \to \infty$ dynamics.

Can we expect QCC?

In Linear Response Theory - YES.

- for **short times** - Restricted QCC
- for **long times** - Central limit theorem

The long time behavior is determined by the short time behavior.

Explanation:

Response is expressed using correlation functions.

Restricted QCC is extended to correlation functions.

Stochastic-like behavior is established before QCC breakdown.

Extrapolation using central limit theorem.

Three problems

1. particles pulsed by a step potential -
   the worst case for QCC
2. particles in a box with a moving wall
3. particles in a ring driven by an EMF.

Driving $\Rightarrow$ jumps in energy space.
The route towards QCC is highly non-trivial.

(1) $\rightarrow$ Solved analytically.

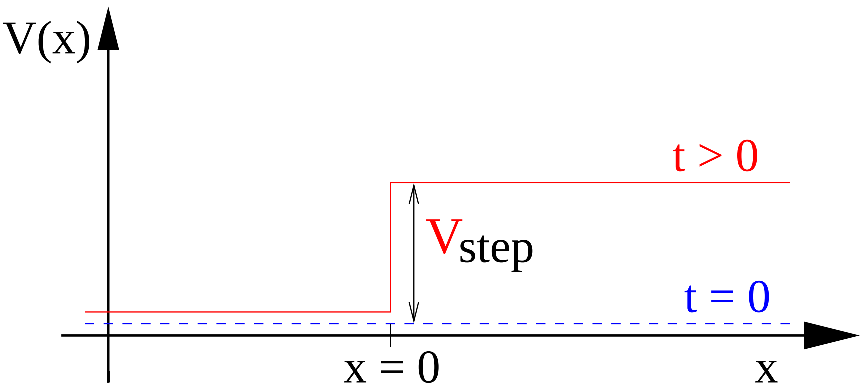
(2),(3) $\rightarrow$ Bloch electrons in a tight binding model:

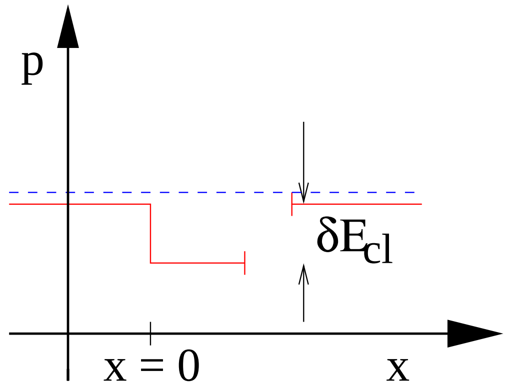
- The energy levels are like sites in a lattice.
- The mean level spacing is like a constant electric field
- long range hopping $\propto 1/(n-m)$.

Solved both analytically and numerically.

Phase space picture

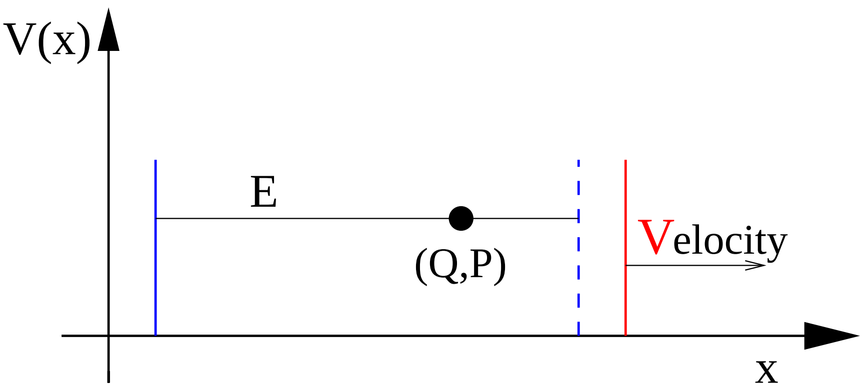
- Step potential

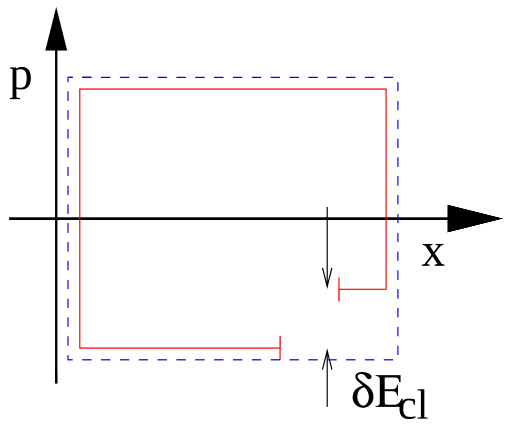




$$\delta E_{\rm cl} = -V_{\rm step}$$

- Moving wall



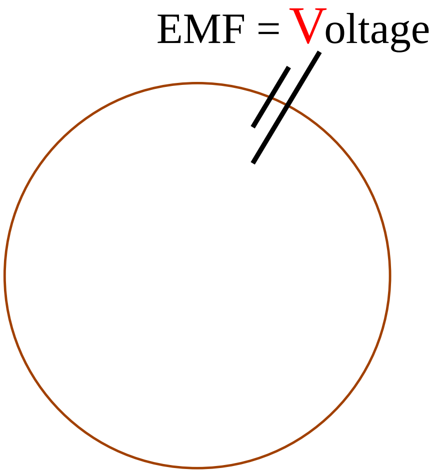


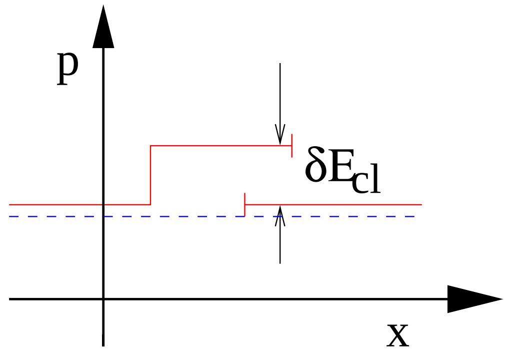
$$\delta E_{\rm cl} = 2 m v_{\rm E} V_{\rm wall}$$

semiclassical regime: $\boldsymbol{\delta E_{\rm cl} \gg \Delta}$

Phase space picture (cont.)

- Ring





$$\delta E_{\rm cl} = eV_{\rm EMF}$$

semiclassical regime: $\delta \mathbf{E}_{\rm cl} \gg \mathbf{\Delta}$

Can we gauge away the EMF non-uniformity?

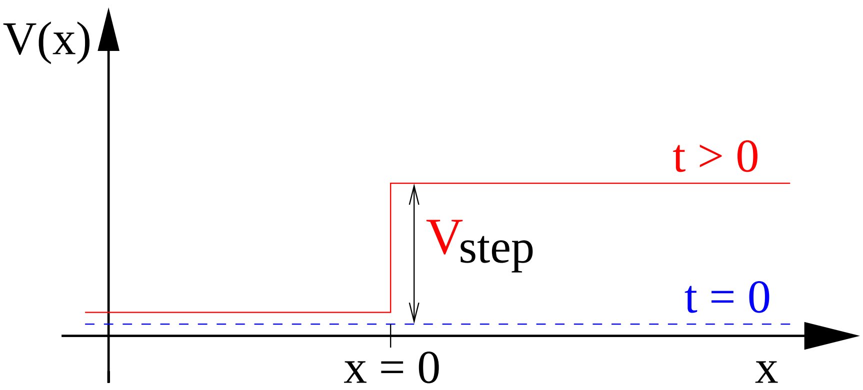
$A(x;t) = \Phi(t)\delta(x - x_0)$ = the vector potential.

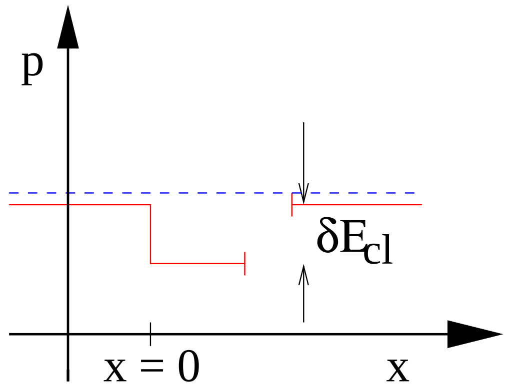
$\mathcal{E}(x) = -\frac{1}{c}\dot{\Phi}\delta(x - x_0)$ = the electric field.

Gauge would imply a step potential!

$$A' = A - \frac{\partial \Lambda}{\partial x} \qquad U' = U + \frac{1}{c}\frac{\partial \Lambda}{\partial t}$$

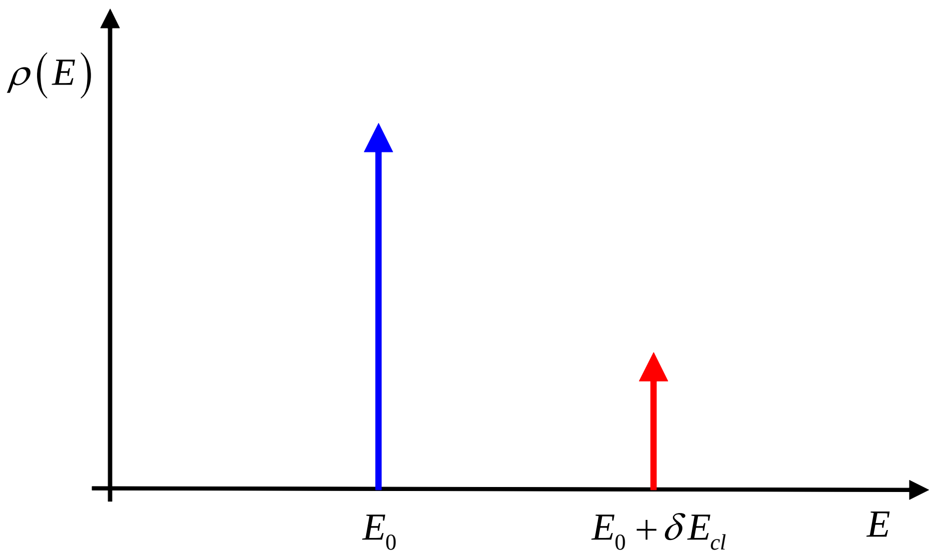
Wavepacket dynamics with the step potential



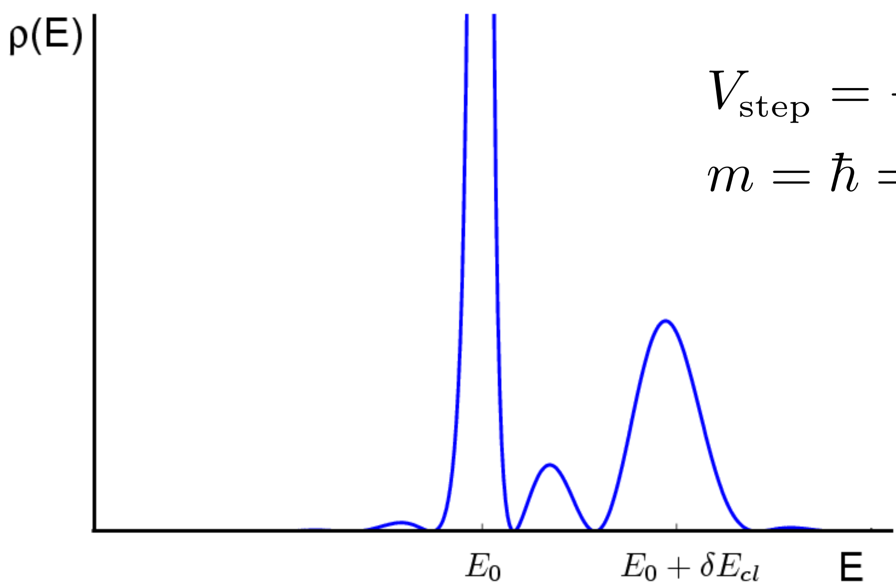


$$\delta E_{\rm cl} = -V_{\rm step}$$

Classical energy distribution



QM energy distribution



$$V_{\rm step} = -1$$

$$m = \hbar = v_{\rm E} = 1$$

Step problem - analysis

Classical moments:

$\delta p_{\rm cl} = -V_{\rm step}/v_{\rm E} \;=\;$ jump in the momentum

$$\langle (p-p_0)^r \rangle \;=\; \delta p_{\rm cl}^r \times v_{\rm E} t$$

Quantum moments:

$$|\langle p_2|\mathcal{U}|p_1\rangle|^2 = \left[\frac{\delta p_{\rm cl}\; v_{\rm E} t}{(p_2-p_1)} \mathrm{sinc}\left(\frac{(p_2-p_1-\delta p_{\rm cl}) v_{\rm E} t}{2}\right)\right]^2$$

- $r = 1$

  $$\langle (p_2-p_1) \rangle \; = \delta p_{\rm cl} \times v_{\rm E} t - \sin(\delta p_{\rm cl} \times v_{\rm E} t)$$

- $r = 2$

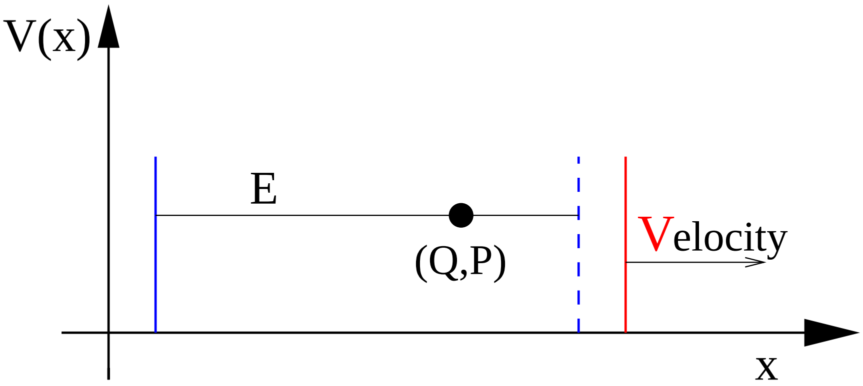
  $$\langle (p_2-p_1)^2 \rangle \; = \delta p_{\rm cl}^2 \times v_{\rm E} t$$

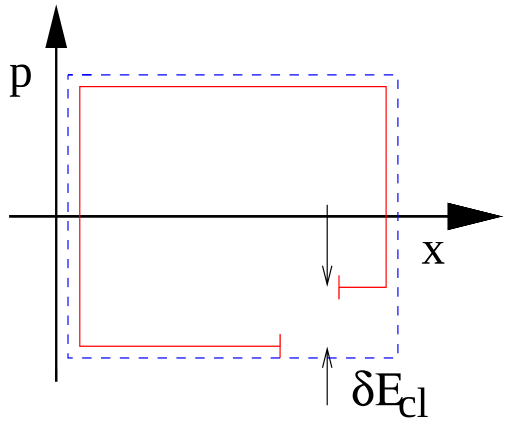
- $r > 2$

  $$\langle (p_2-p_1)^r \rangle = \infty$$

Restricted QCC ($r = 2$) is preserved.
Detailed QCC ($r > 2$) is destroyed.

Moving Wall problem - Analysis

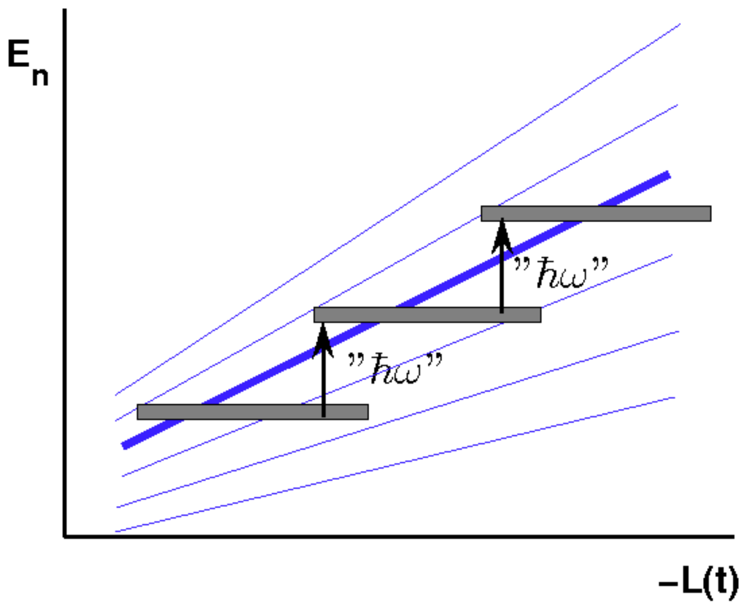




$$\delta E_{\rm cl} = 2\mathsf{m} v_{\rm E} V_{\rm wall}$$

Adiabatic regime: $\delta E_{\rm cl} \ll \Delta \iff V \ll \frac{\hbar}{\mathsf{m}L}$

Semiclassical regime: $\delta E_{\rm cl} \gg \Delta$



$|E_n - E_m| \approx$ "$\hbar\omega$" ??? "$\hbar\omega$" $= \delta E_{\rm cl} = 2\mathsf{m} v_{\rm E} V$

In our problem there is no AC driving!

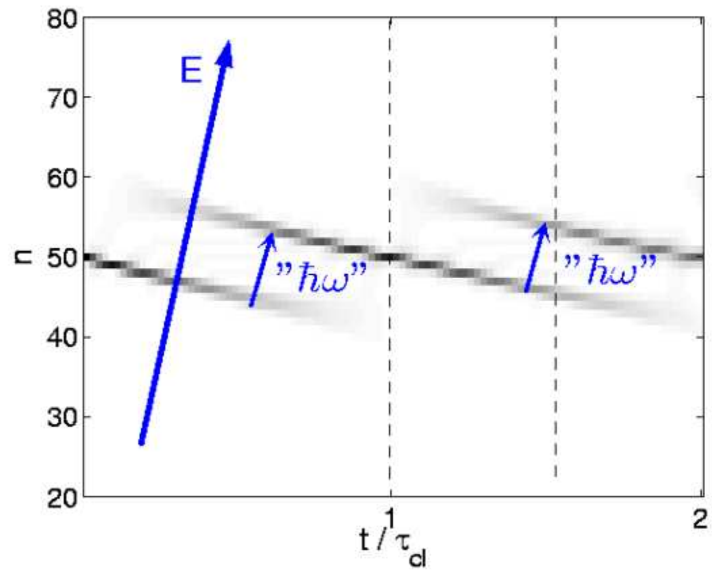
Is there a self-generated ω ??? **YES!**

Moving Wall problem - Numerics

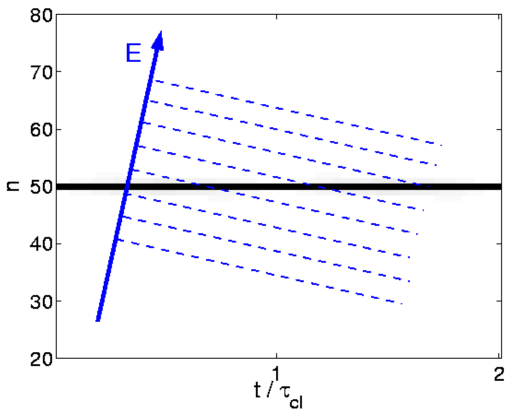
$$\frac{da_n}{dt} = -\frac{i}{\hbar}E_n a_n - \frac{V}{L}\sum_{m(\neq n)} \frac{2nm}{n^2 - m^2}\, a_m$$

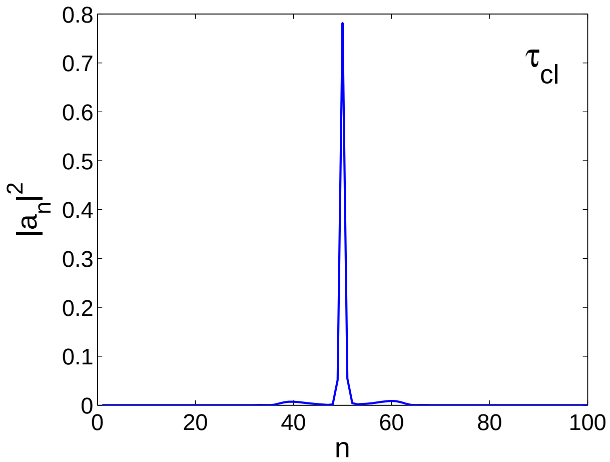
Density plots of $|a_n(t)|^2$:

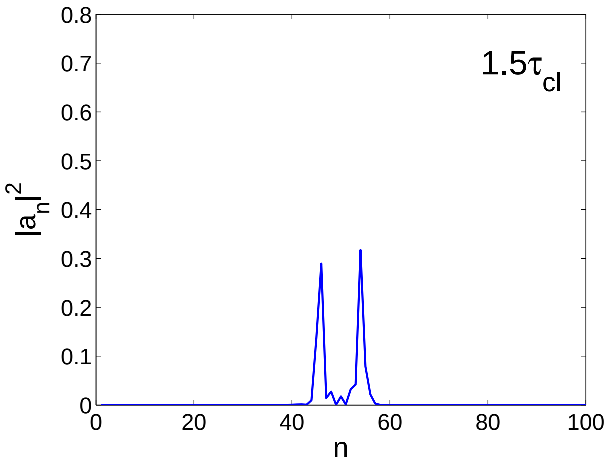
Semiclassical:



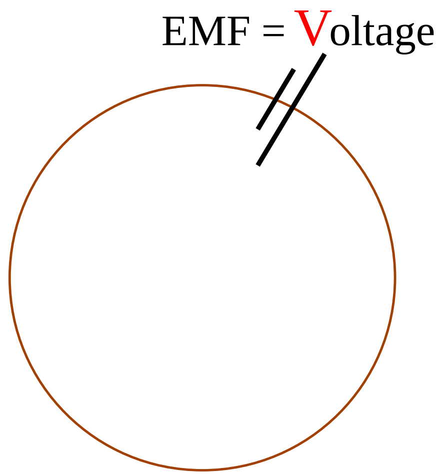
Adiabatic:

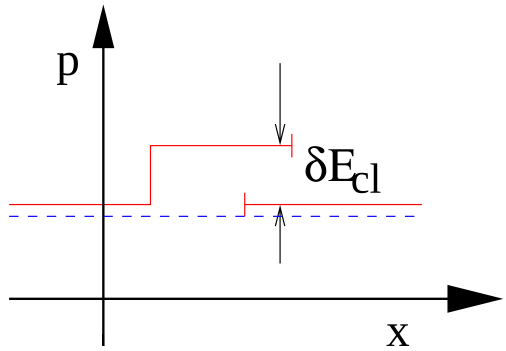






The Ring problem





$$\delta E_{\rm cl} = eV_{\rm EMF}$$

$$\mathcal{H} = \frac{1}{2\mathsf{m}}\left(p - \frac{e}{c}A(t)\right)^2$$

$$A(x;t) = \Phi(t)\delta(x - x_0)$$

$$\mathcal{E}(x) = -\tfrac{1}{c}\dot{\Phi}\delta(x - x_0)$$

$$E_n = \frac{1}{2\mathsf{m}}\left(\frac{2\pi\hbar}{L}\right)^2\left(n - \frac{\Phi(t)}{\Phi_0}\right)^2$$

$$\frac{da_n}{dt} = -\frac{\imath}{\hbar}E_n a_n - \frac{\dot{\Phi}}{\Phi_0}\sum_{m(\neq n)}\frac{1}{n-m}a_m$$

Ring $\Longleftrightarrow$ Bloch

$$\frac{da_n}{dt} = -\frac{\imath}{\hbar} E_n a_n + \alpha \sum_{m(\neq n)} \frac{1}{n-m} a_m$$

$$E_n = \varepsilon n$$

Bloch electrons in a tight-binding model:

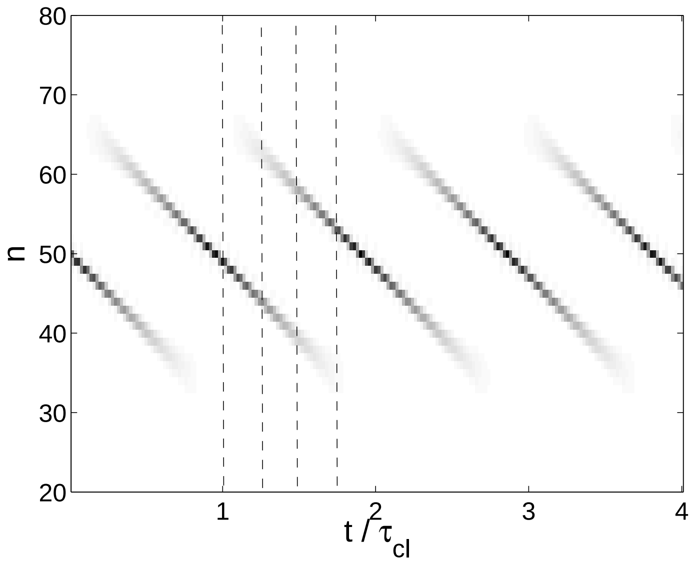
n	site index
E_n	on site energy
ε - mean level spacing	electric field
$\frac{\alpha}{n-m}$	hopping

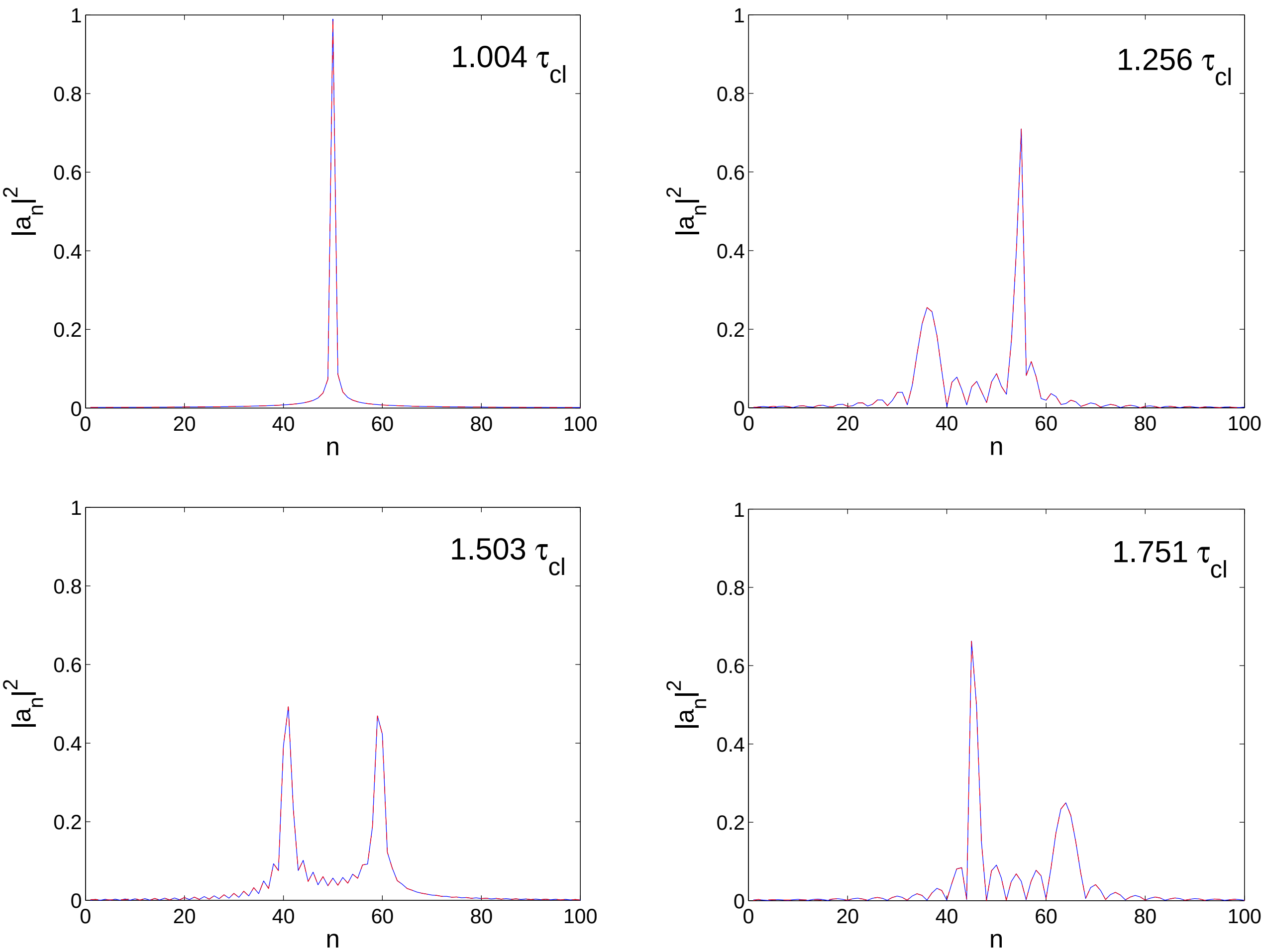
- $\varepsilon = 0 \Rightarrow$ ballistic motion
- $\varepsilon \to \infty \Leftrightarrow \alpha \to 0 \Leftrightarrow$ "stuck"
- But what happens in between?

Analytical solution:

$$|a_t(n)|^2 = \left(2\frac{\alpha}{\varepsilon}\right)^2 \frac{\sin^2\left(\frac{1}{2}\varepsilon t\left(n - n_0 + \alpha(t - \frac{2\pi}{|\varepsilon|})\right)\right)}{(n-n_0+\alpha t)^2 (n - n_0 + \alpha(t - \frac{2\pi}{|\varepsilon|}))^2}$$

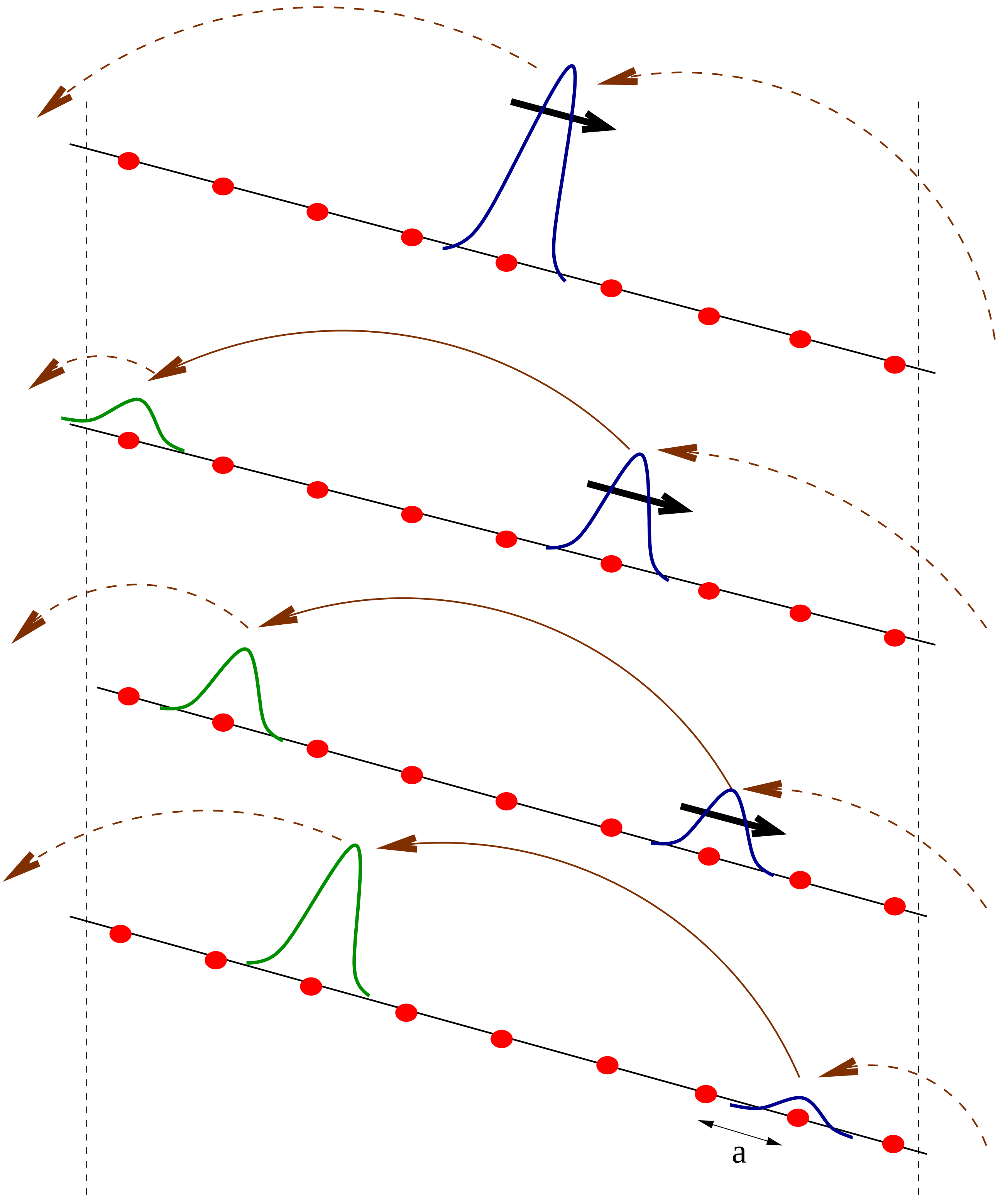
The ring / bloch problem - Solution





MOVIE

Bloch electrons - Dynamics



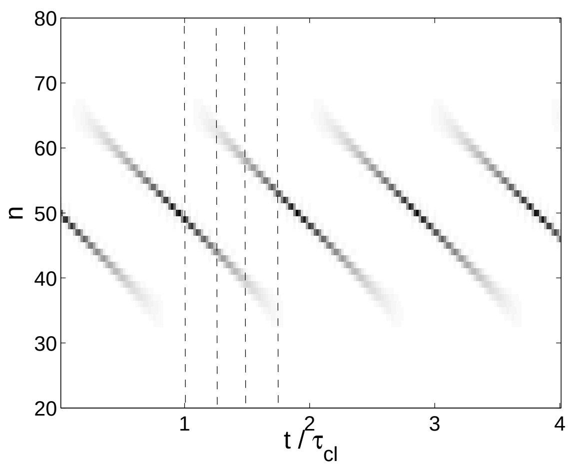
Is the $\propto 1/(n-m)$ hopping significant ???

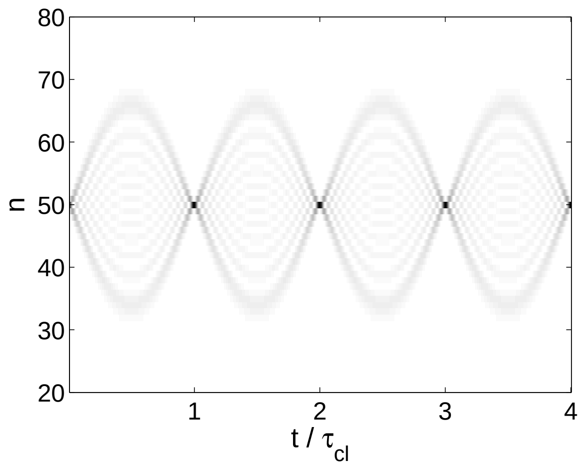
Bloch electrons - Nearest Neighbors

$$\frac{da_n}{dt} = -i\varepsilon n a_n + \frac{\alpha}{2}[a_{n+1} - a_{n-1}]$$

Analytical solution:

$$|a_t(n)|^2 = \left| J_{n-n_0}\left(\frac{2\alpha}{\varepsilon}\sin\left(\frac{1}{2}\varepsilon t\right)\right)\right|^2$$



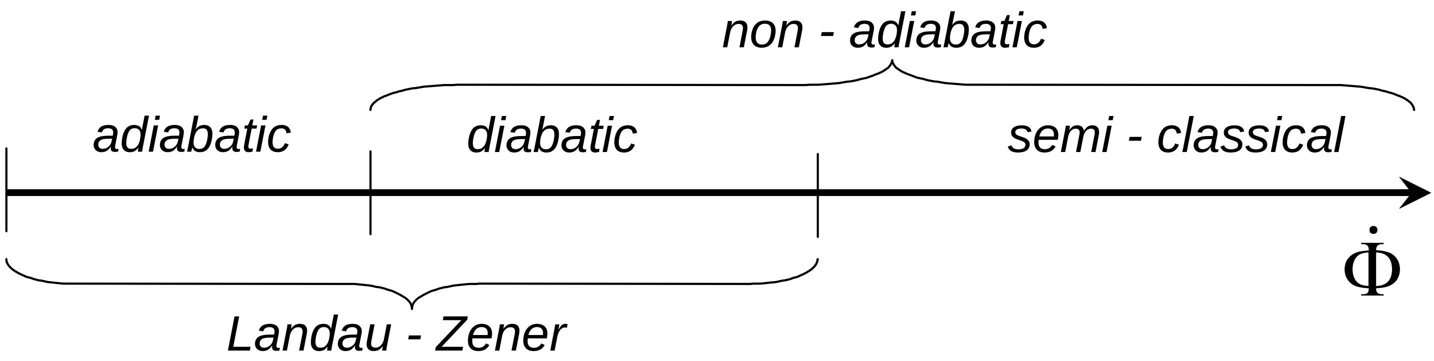


$\propto 1/(n-m)$ hopping — n.n hopping

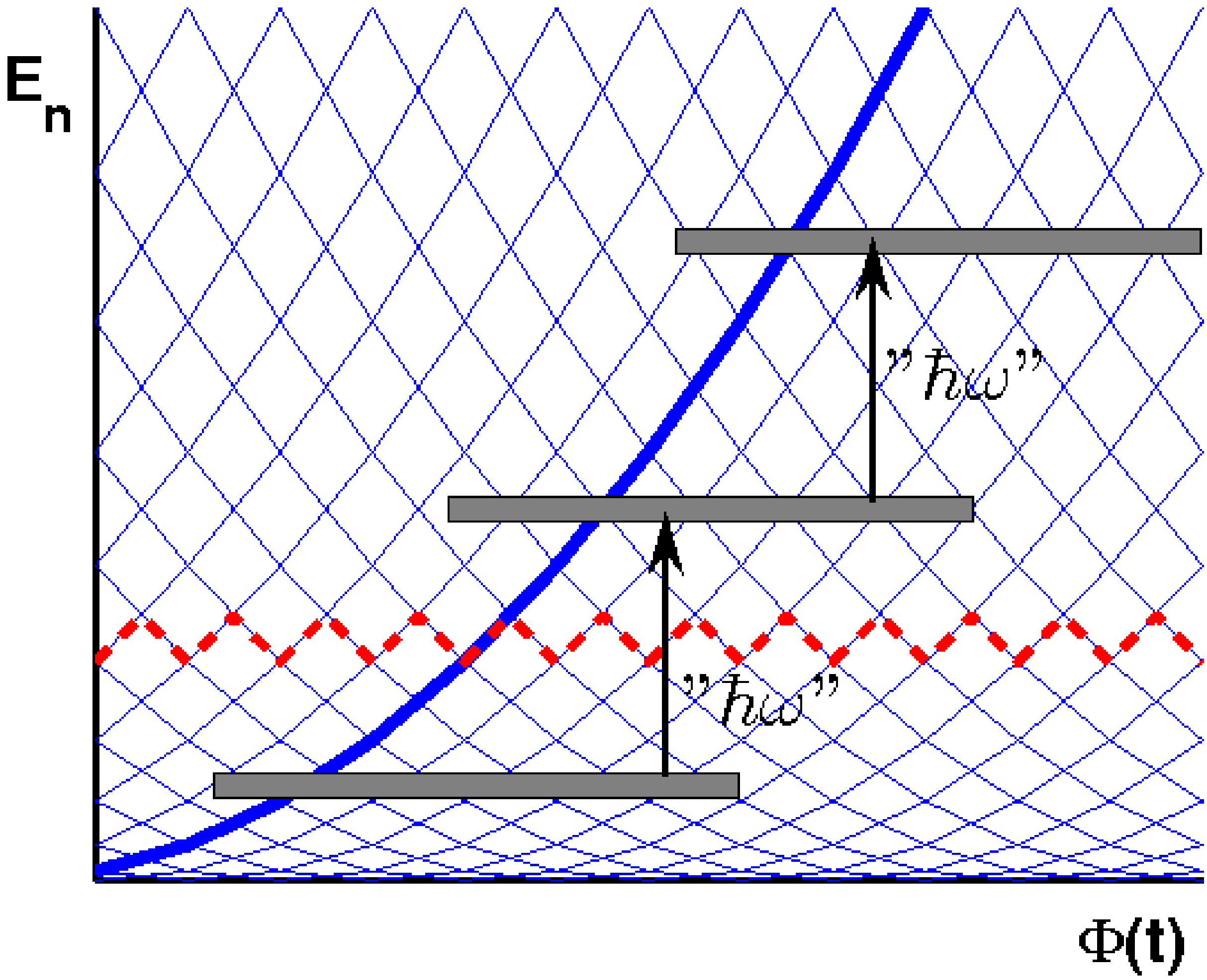
Uni-directional oscillations — Bi-directional oscillations

Ring

Different regimes:



Semiclassical regime: $\delta E_{\rm cl} \gg \Delta \quad \Longleftrightarrow \quad V_{\rm EMF} \gg \frac{\hbar v_{\rm E}}{L}$



The End!