Semilinear response and RMT for the heating rate of cold atoms in vibrating traps

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References:

A. Stotland, R. Budoyo, T. Peer, T. Kottos and D. Cohen, JPA (FTC) (2008)

A. Stotland, D. Cohen and N. Davidson, EPL (2009)

A. Stotland, and D. Cohen, in preparation

$DIP, $BSF

Diffusion and Energy absorption

Driven chaotic system with Hamiltonian $\mathcal{H}(X(t))$

$$X \quad = \quad \text{some control parameter}$$

$$\dot{X} \quad = \quad \text{rate of the (noisy) driving}$$

$\rightsquigarrow$ diffusion in energy space:

$$\boldsymbol{D} \quad = \quad \boldsymbol{G}_{\text{diffusion}} \ \overline{\dot{X}^2}$$

$\rightsquigarrow$ energy absorption:

$$\dot{\boldsymbol{E}} \quad = \quad \boldsymbol{G}_{\text{absorption}} \ \overline{\dot{X}^2}$$

[Ott, Brown, Grebogi, Wilkinson, Jarzynski, D.C.]

There is a dissipation-diffusion relation.

In the canonical case $\dot{E} = D/T$.

Below we use for G scaled units.

LRT - Kubo

Hamiltonian:

$$\mathcal{H}(X(t)) \;\approx\; \mathcal{H}_0 + f(t)V$$

$$\mathcal{H}_0 = \mathcal{H}(X_0)$$

$$f(t) = X(t) - X_0$$

$$V = \frac{\partial \mathcal{H}}{\partial X}$$

Kubo formula:

$$G = \int \tilde{C}(\omega)\tilde{S}(\omega)d\omega$$

$$\tilde{C}(\omega) = \text{FT}\,\langle V(t)V(0)\rangle$$

$$\tilde{S}(\omega) = \text{FT}\,\langle \dot{f}(t)\dot{f}(0)\rangle$$

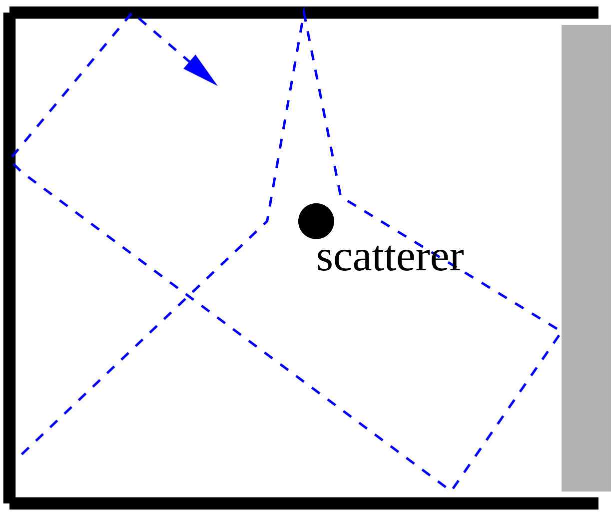
Diffusion and heating:

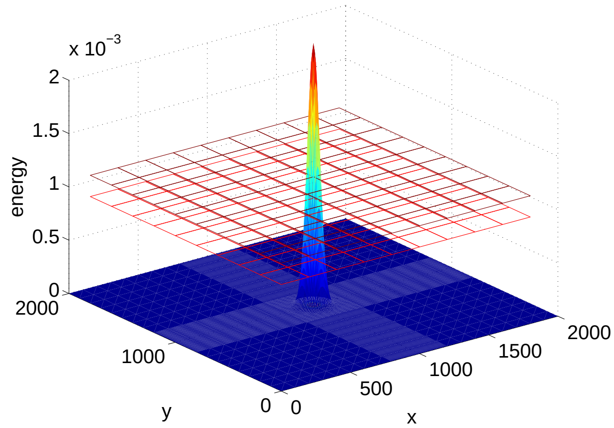
$$\boldsymbol{D} = G\;\overline{\dot{f}^2}$$

$$\dot{\boldsymbol{E}} = \boldsymbol{D}/T$$

The model - classical description

A particle in a 2-D box with a vibrating wall.





Deforming potential: smooth Gaussian / s-scatterer

$$\mathcal{H} = \left.\frac{p^2}{2\mathsf{m}}\right|_{\text{box}} + U(x,y) + W(x - X(t))$$

$$\approx \mathcal{H}_0 + U(x,y) + f(t)V$$

$$f(t) = X(t) - X_0 \quad ; \quad V = -\frac{\partial W}{\partial x} = \text{force}$$

Equations of motion:

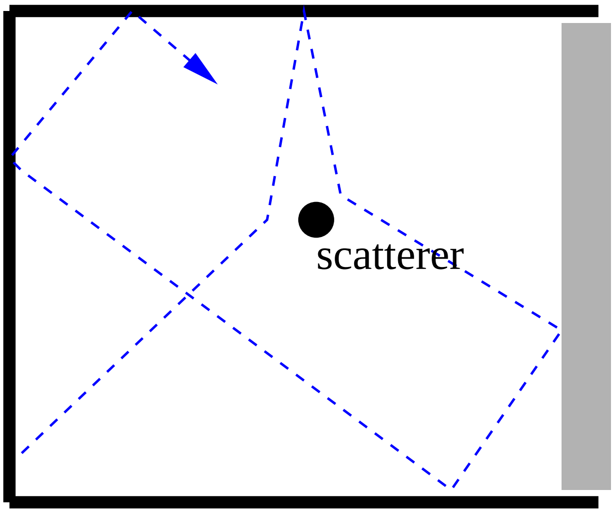
$$\dot{x} = \frac{p_x}{\mathsf{m}} \qquad \dot{p}_x = -\frac{\partial U}{\partial x}$$

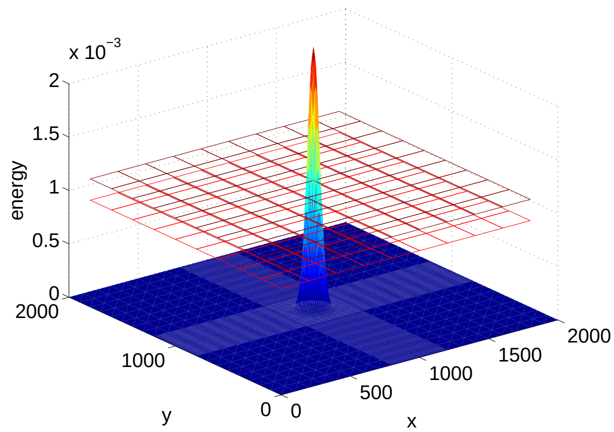
$$\dot{y} = \frac{p_y}{\mathsf{m}} \qquad \dot{p}_y = -\frac{\partial U}{\partial y}$$

$$\tilde{C}(\omega) = \text{FT } \langle V(t)V(0)\rangle$$

The model - quantum description

A particle in a 2-D box with a vibrating wall.





Deforming potential: smooth Gaussian / s-scatterer

The Hamiltonian in the $\boldsymbol{n} = (n_x, n_y)$ basis:

$$\mathcal{H} = \mathrm{diag}\{E_{\boldsymbol{n}}\} + u\{U_{\boldsymbol{nm}}\} + f(t)\{V_{\boldsymbol{nm}}\}$$

The matrix elements for the wall displacement:

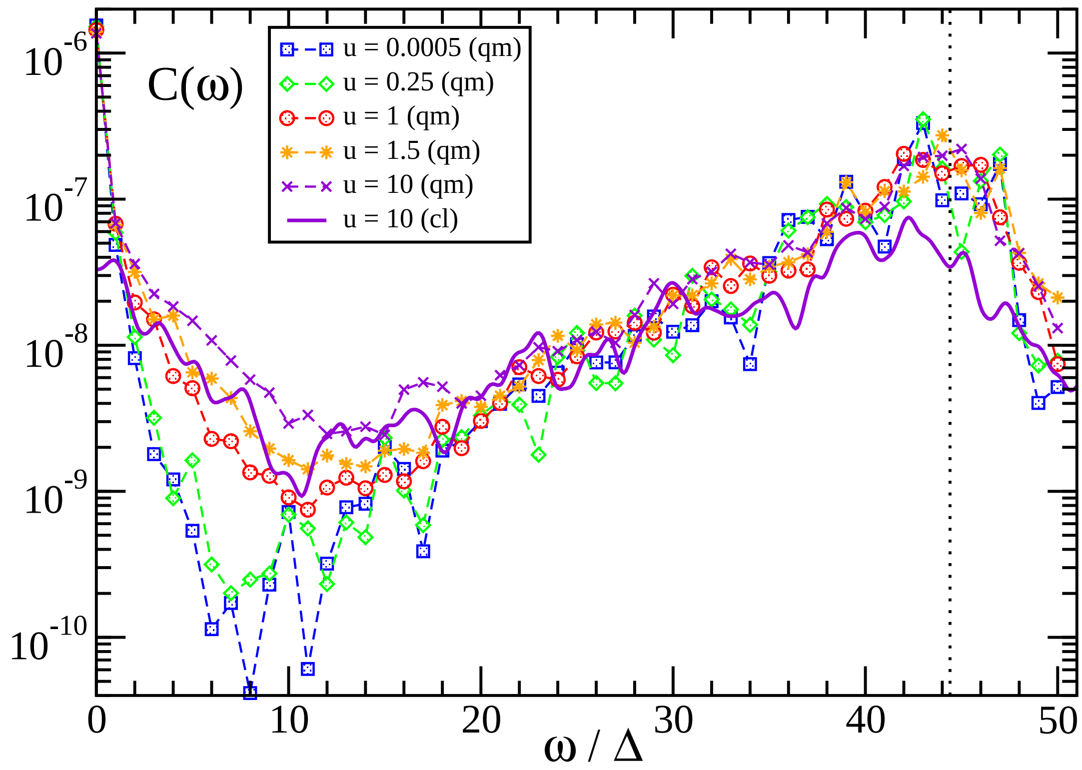
$$V_{\boldsymbol{nm}} = -\delta_{n_y,m_y} \times \frac{\pi^2}{\mathsf{m} L_x^3} n_x m_x$$

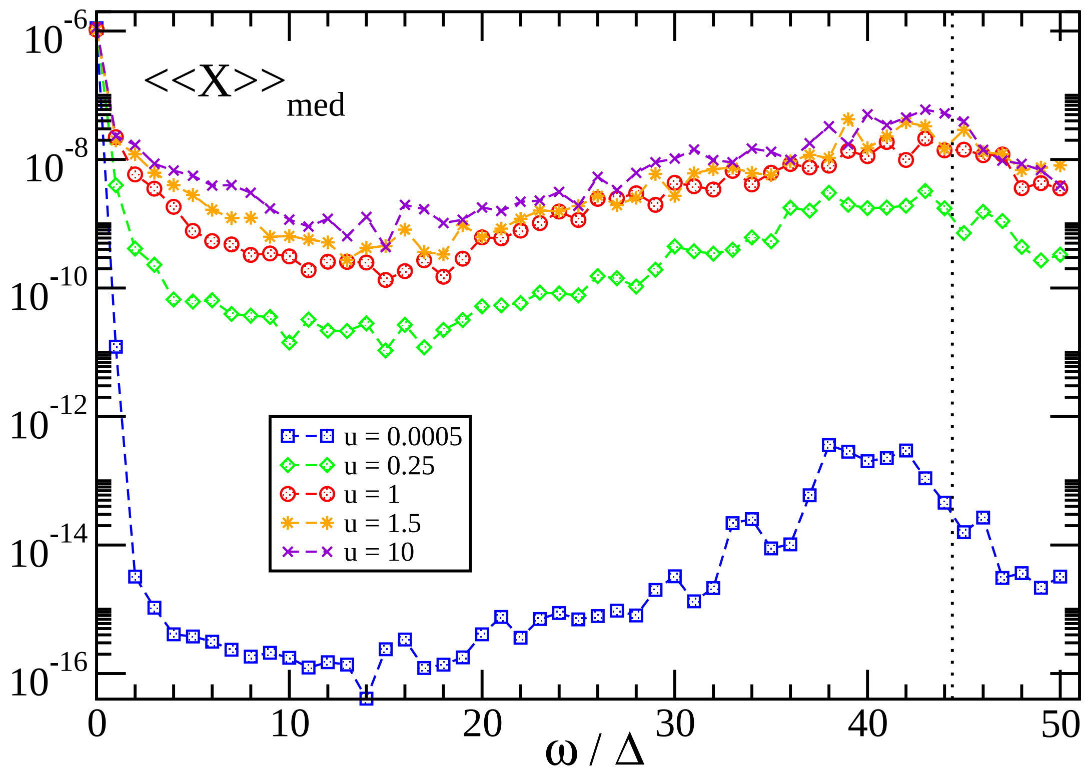
The Hamiltonian in the E_n basis:

$$\mathcal{H} = \mathrm{diag}\{E_n\} + f(t)\{V_{nm}\}$$

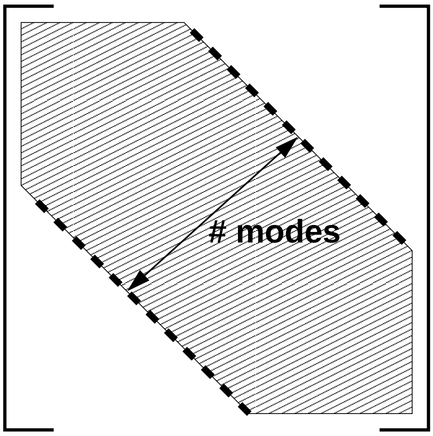
$$\tilde{C}(\omega) = \left\langle \left\langle \sum_n |V_{nn_0}|^2 \, \delta(\omega - (E_n - E_{n_0})) \right\rangle \right\rangle_{\text{algebraic}}$$

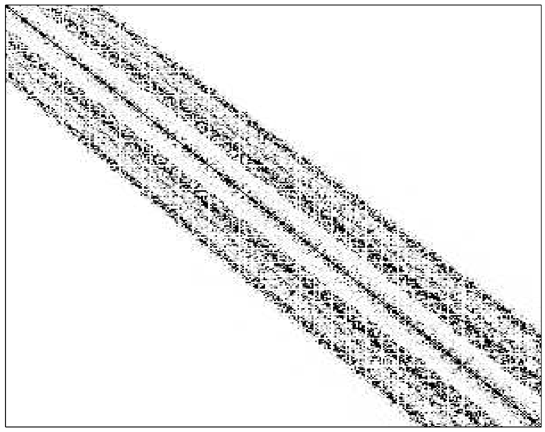
Correlation function

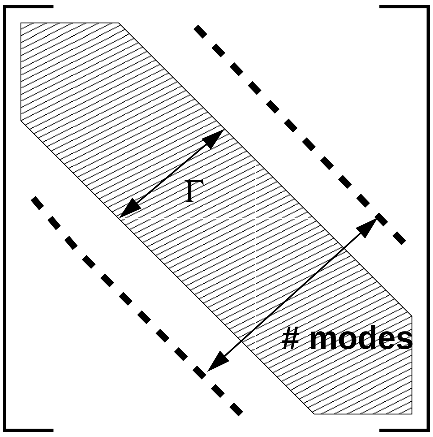




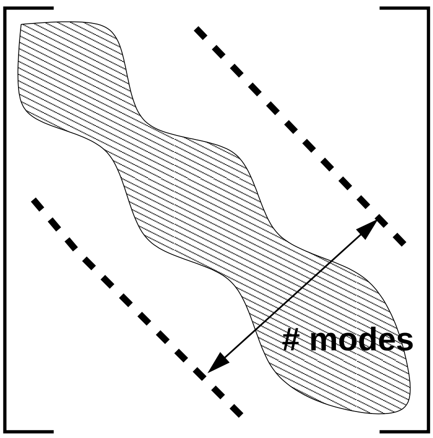
Matrices - Gallery

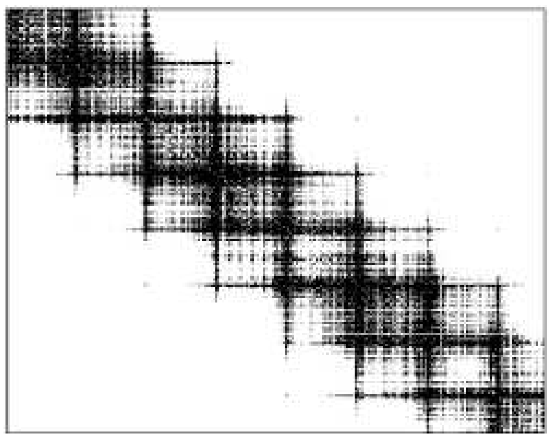


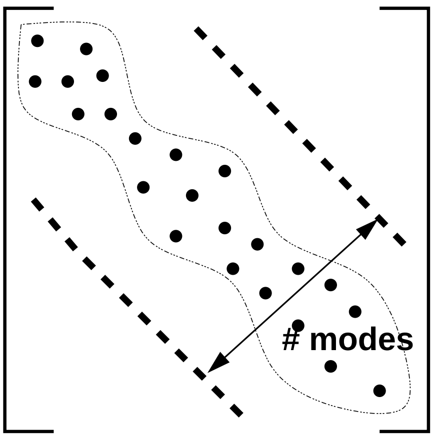


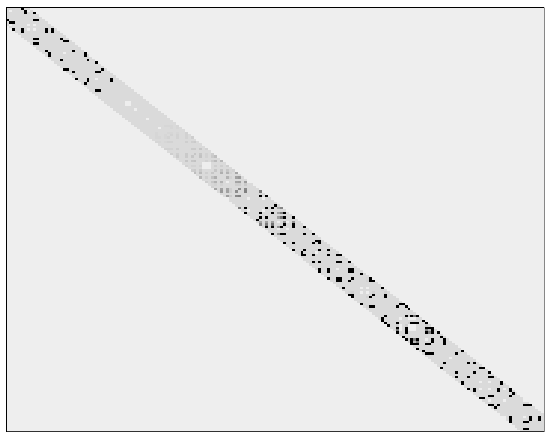


Sparsity and texture are not reflected by $\tilde{C}(\omega)$



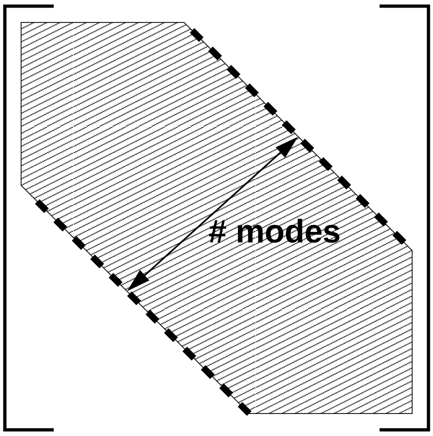






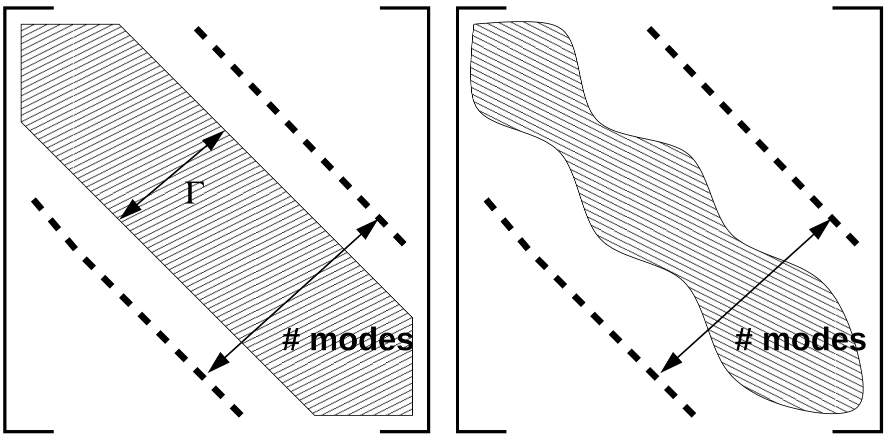
Regimes

- Strong Quantum Chaos



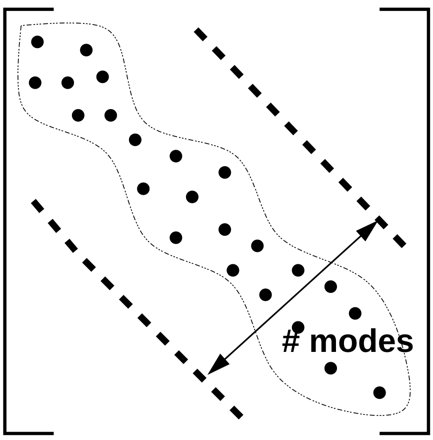
$$u > u_{\text{prt}}$$
$$u_{\text{prt}} = \sqrt{\mathcal{M}} \times u_c$$
$$\mathcal{M} = \#\text{modes}$$

- Weak Quantum Chaos



$$u_c < u < u_{\text{prt}}$$

- First Order Perturbation Theory



$$u < u_c$$
$$u_c \sim \frac{1}{\mathsf{m} \times A_{\text{sctr}}}$$

Derivation - some formulas

$$\mathcal{H} = \mathrm{diag}\{E_{\boldsymbol{n}}\} + u\{U_{\boldsymbol{nm}}\} + f(t)\{V_{\boldsymbol{nm}}\}$$

Sum rule:

$$\sum_{|r|\lesssim b} |V_{n+r,n}|^2 \quad \approx \quad V_0^2$$

$$V_0 \quad = \quad \frac{\mathsf{m} v_{\mathrm{E}}^2}{L}$$

$$\Rightarrow \quad |V_{nm}|^2 \quad \sim \quad \frac{1}{b}\, V_0^2$$

- Strong Quantum Chaos

$$b = \mathcal{M}$$

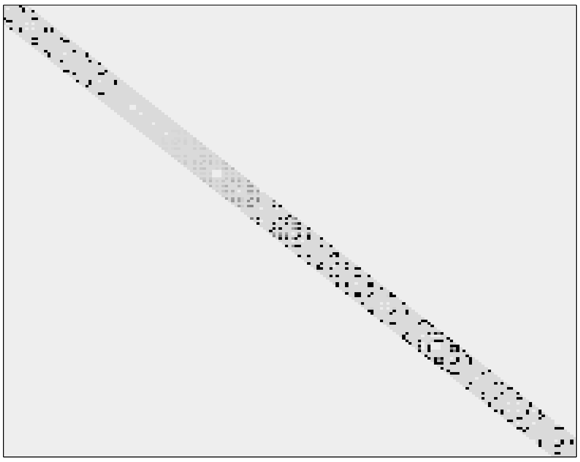
- Weak Quantum Chaos

$$b \quad = \quad \frac{\Gamma}{\Delta} \quad \sim \quad \left(\frac{u}{u_c}\right)^2$$

- First Order Perturbation Theory

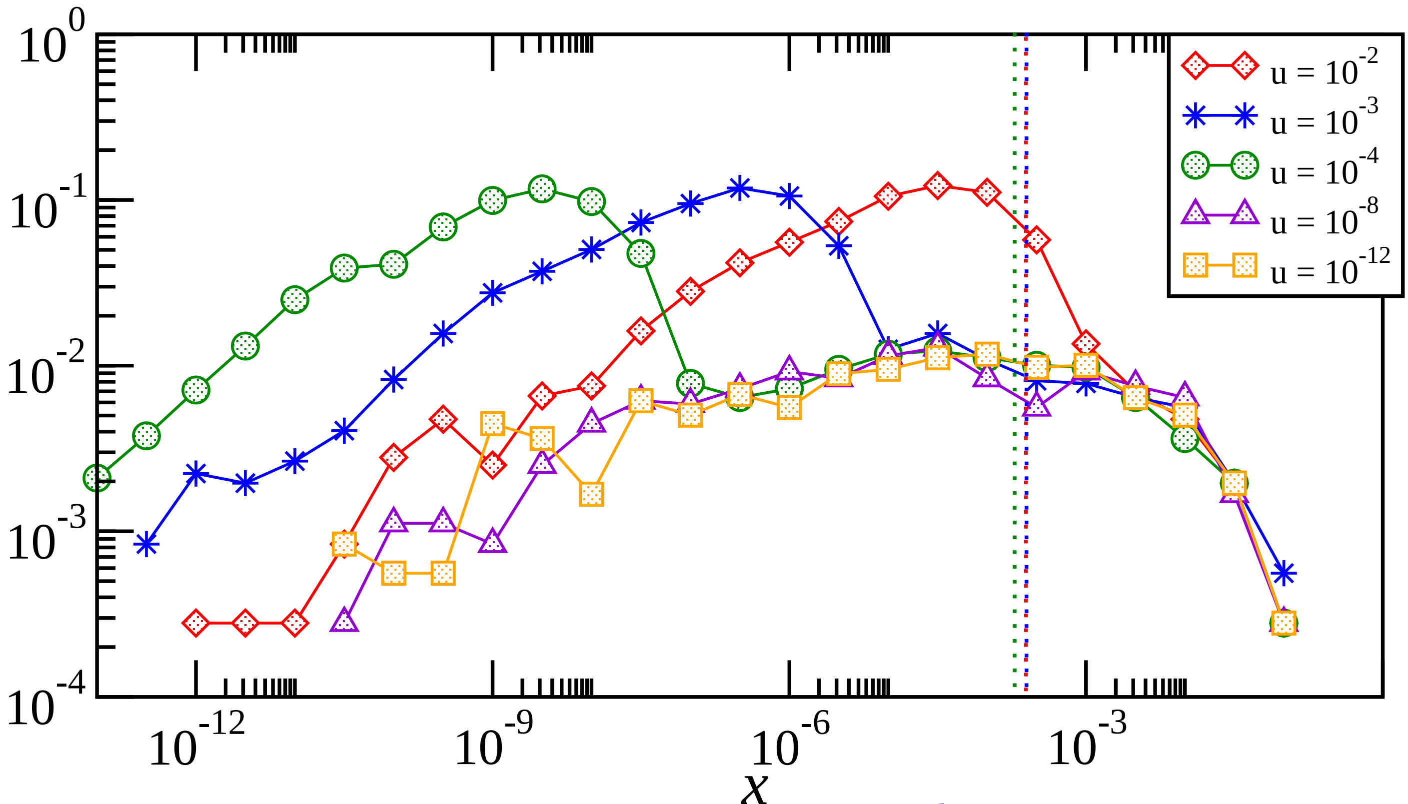
$$|V_{nm}|^2 \approx |\langle \boldsymbol{m}|n\rangle|\; V_0^2 \approx \left|\frac{uU_{\mathbf{nm}}}{E_{\boldsymbol{n}}-E_{\boldsymbol{m}}}\right| V_0^2 = \dots$$

$\{|V_{nm}|^2\}$ as a random matrix $\{X\}$



- bandwidth
- texture
- sparsity

Histogram of X :



$X \sim$ LogNormal

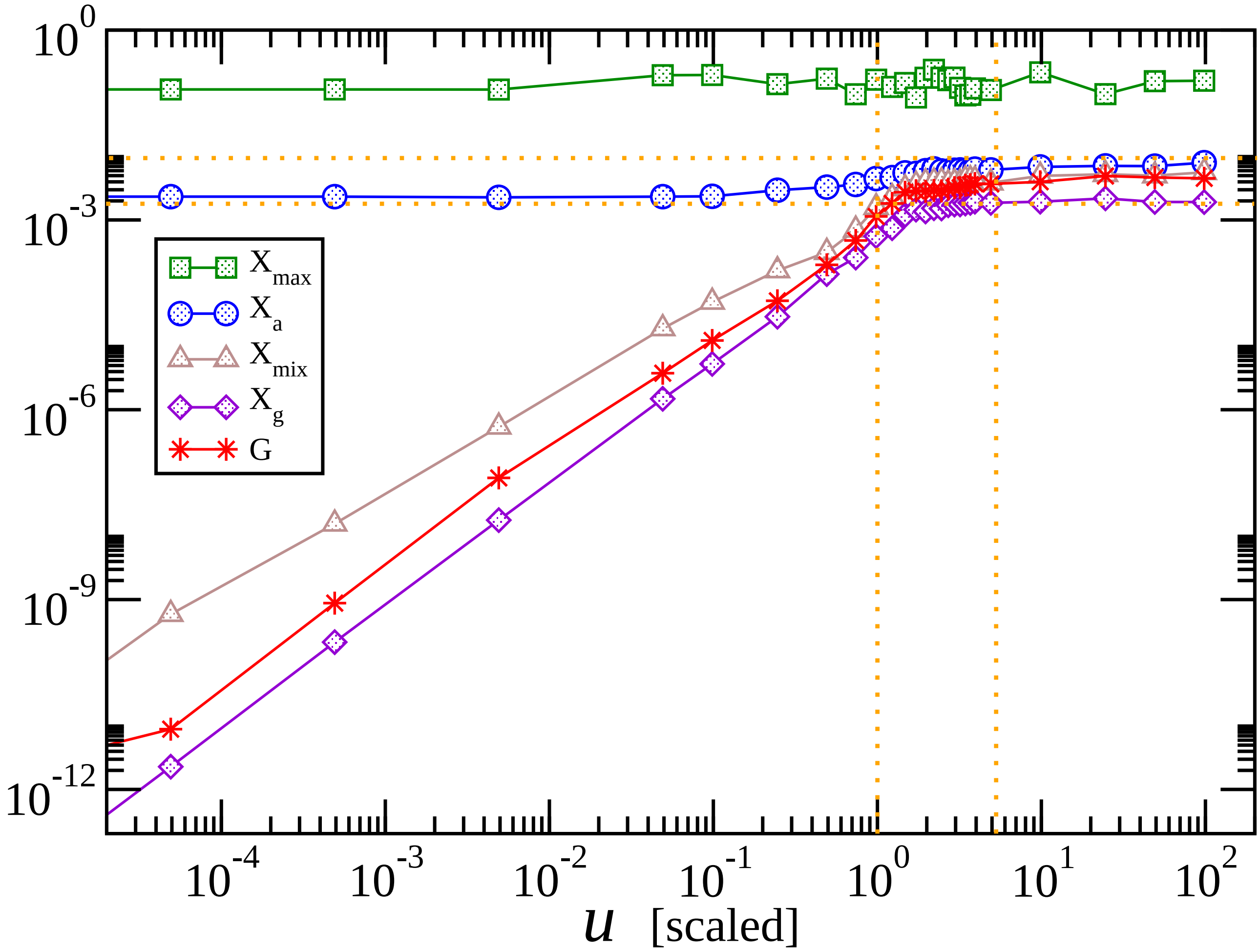
Algebraic average: $\langle\langle x\rangle\rangle_a = \langle x\rangle$

Harmonic average: $\langle\langle x\rangle\rangle_h = [\langle 1/x\rangle]^{-1}$

Geometric average: $\langle\langle x\rangle\rangle_g = \exp[\langle \log x\rangle]$

$$\langle\langle x\rangle\rangle_h \quad \ll \quad \langle\langle x\rangle\rangle_g \quad \ll \quad \langle\langle x\rangle\rangle_a$$

$\{|V_{nm}|^2\}$ as a random matrix $\{X\}$ - cont.



$$q \equiv \frac{\langle\langle x\rangle\rangle_g}{\langle\langle x\rangle\rangle_a}$$

$$p \equiv \text{Prob}\left(X > \langle X\rangle\right)$$

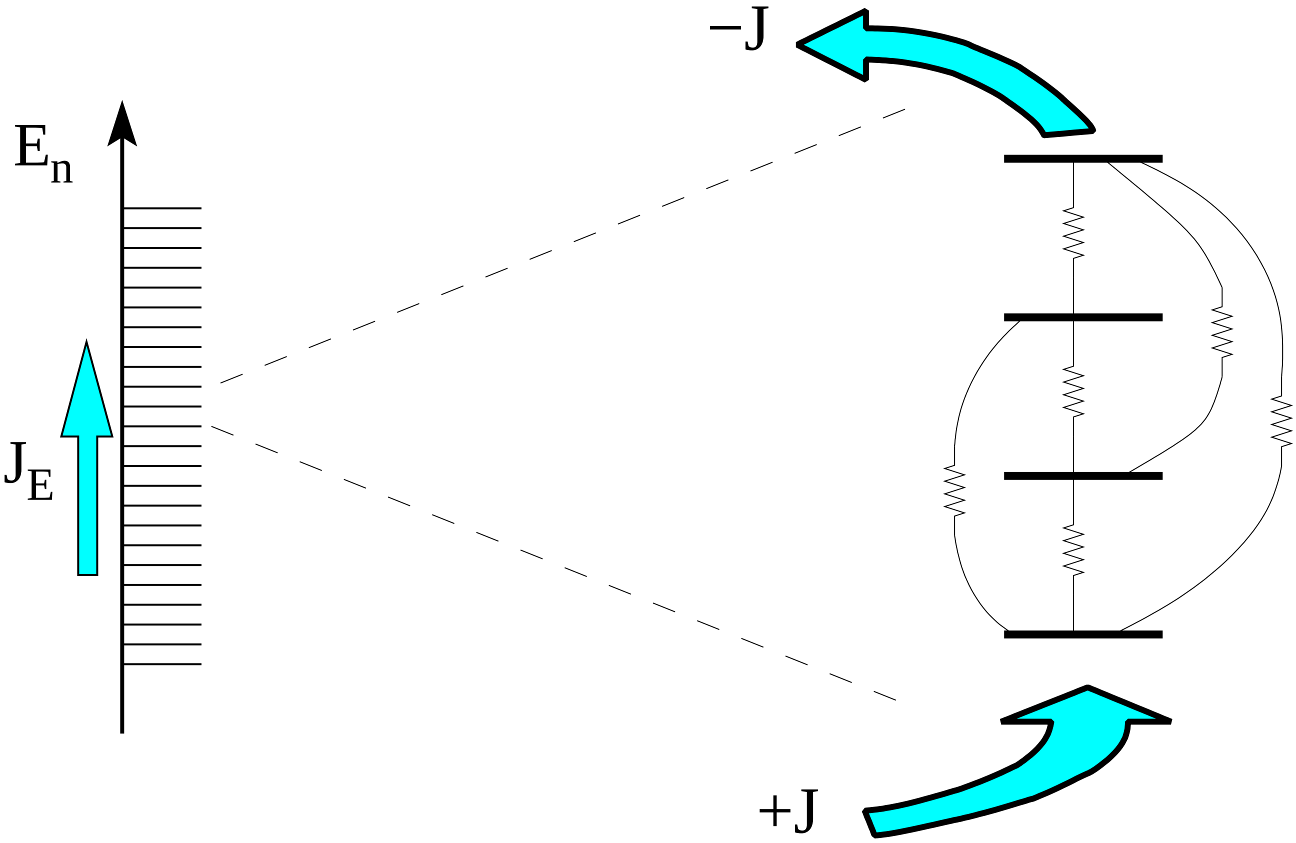
$$\langle\langle x\rangle\rangle_a = \frac{1}{2\pi^2\mathcal{M}}\left(\frac{\mathsf{m}v_{\mathrm{E}}^2}{L_x}\right)^2$$

$$\langle\langle x\rangle\rangle_g \approx \frac{4}{\pi^2}\frac{1}{\mathcal{M}^2}\left(\frac{u}{u_c}\right)^2\left(\frac{\mathsf{m}v_{\mathrm{E}}^2}{L_x}\right)^2$$

Semi Linear Response Theory

$$\mathcal{H} = \{E_n\} - X(t)\{V_{nm}\}$$

$$\boldsymbol{G} \;=\; \pi\varrho_{\mathrm{E}}\ \langle\langle|V_{mn}|^2\rangle\rangle_{\mathrm{SLRT}}$$



$$\mathrm{g}_{nm} = 2\varrho_{\mathrm{F}}^{-3}\ \frac{|V_{nm}|^2}{(E_n - E_m)^2}\ \tilde{S}(E_n - E_m)$$

$\langle\langle|V_{mn}|^2\rangle\rangle_{\mathrm{SLRT}} \equiv$ inverse resistivity of the network

$$\langle\langle|V_{mn}|^2\rangle\rangle_{\mathrm{harmonic}} \ll \langle\langle|V_{mn}|^2\rangle\rangle_{\mathrm{SLRT}} \ll \langle\langle|V_{mn}|^2\rangle\rangle_{\mathrm{algebraic}}$$

The RMT modeling and VRH

- Log-normal RMT modeling

For the rectangular $\tilde{S}(\omega)$ of width ω_c

$$G_{\mathrm{SLRT}} \quad = \quad q \;\exp\left[2\sqrt{-\ln q \;\ln(\omega_c/\Delta)}\right] \times G_{\mathrm{LRT}}$$

Different distributions give different results!

- Log-box RMT modeling

For the rectangular $\tilde{S}(\omega)$ of width ω_c

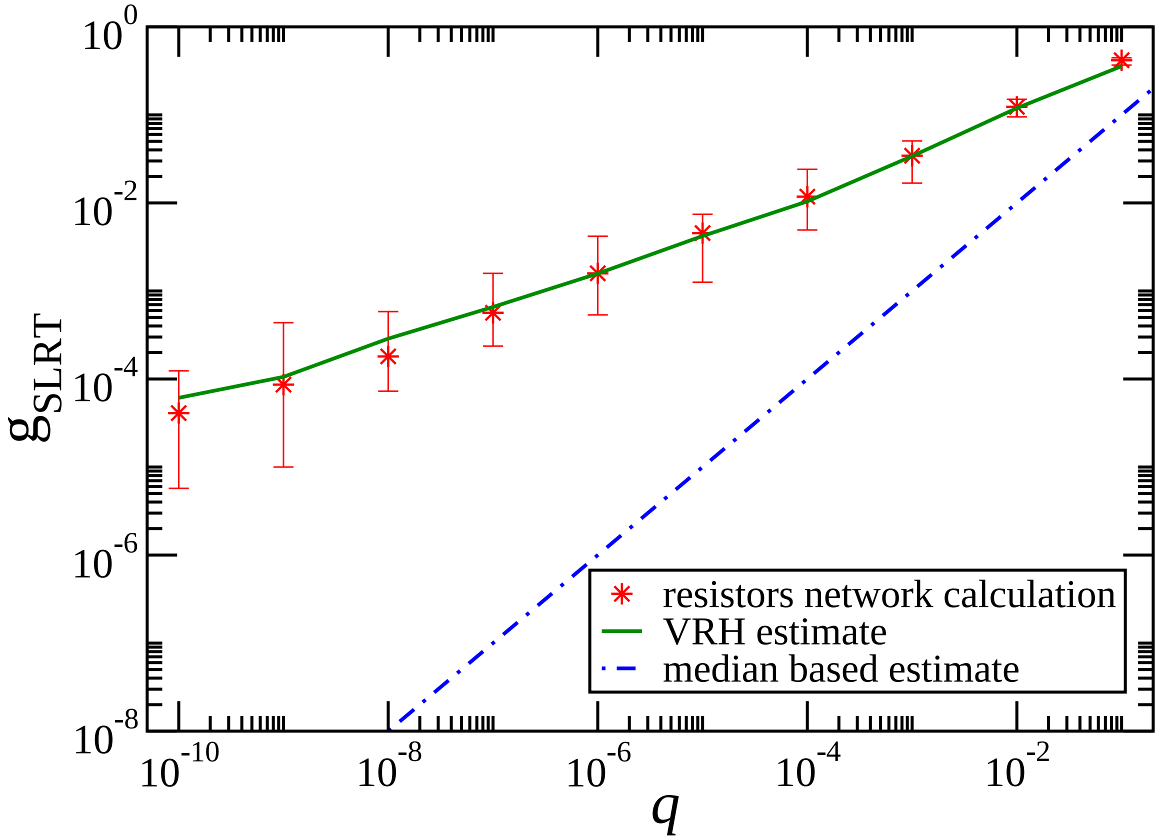
$$G_{\mathrm{SLRT}} \;=\; \frac{1}{p}\;\exp\left[-\frac{\Delta}{p\,\omega_c}\right] \times G_{\mathrm{LRT}}$$

For the exponential $\tilde{S}(\omega)$ of width ω_c

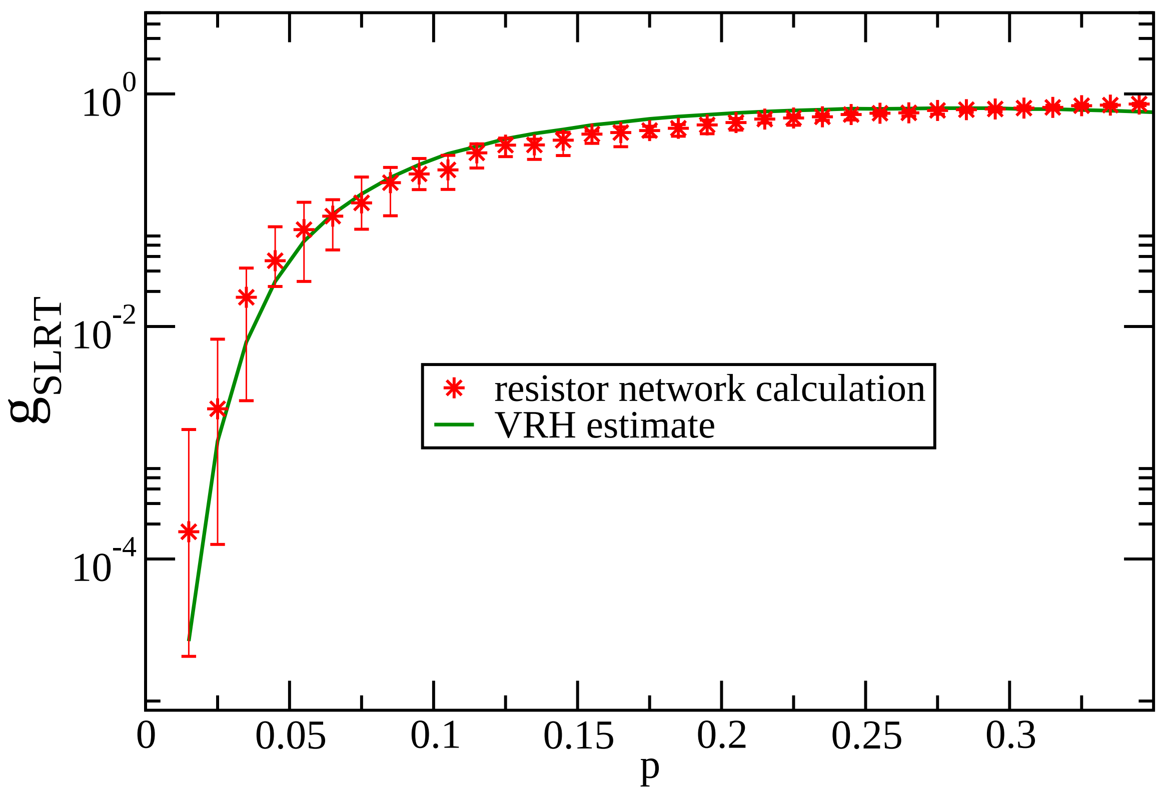
$$G_{\mathrm{SLRT}} \;=\; \frac{1}{p}\exp\left[-2\sqrt{\frac{\Delta}{p\,\omega_c}}\right] \times G_{\mathrm{LRT}}$$

The RMT modeling - Numerical results

- Log-normal RMT modeling



- Log-box RMT modeling



The RMT modeling and VRH

A typical matrix element for connected transitions:

$$\left(\frac{\omega}{\Delta}\right) \text{Prob}\Big(x > x_\omega\Big) \;\sim\; 1$$

Generalized VRH estimate:

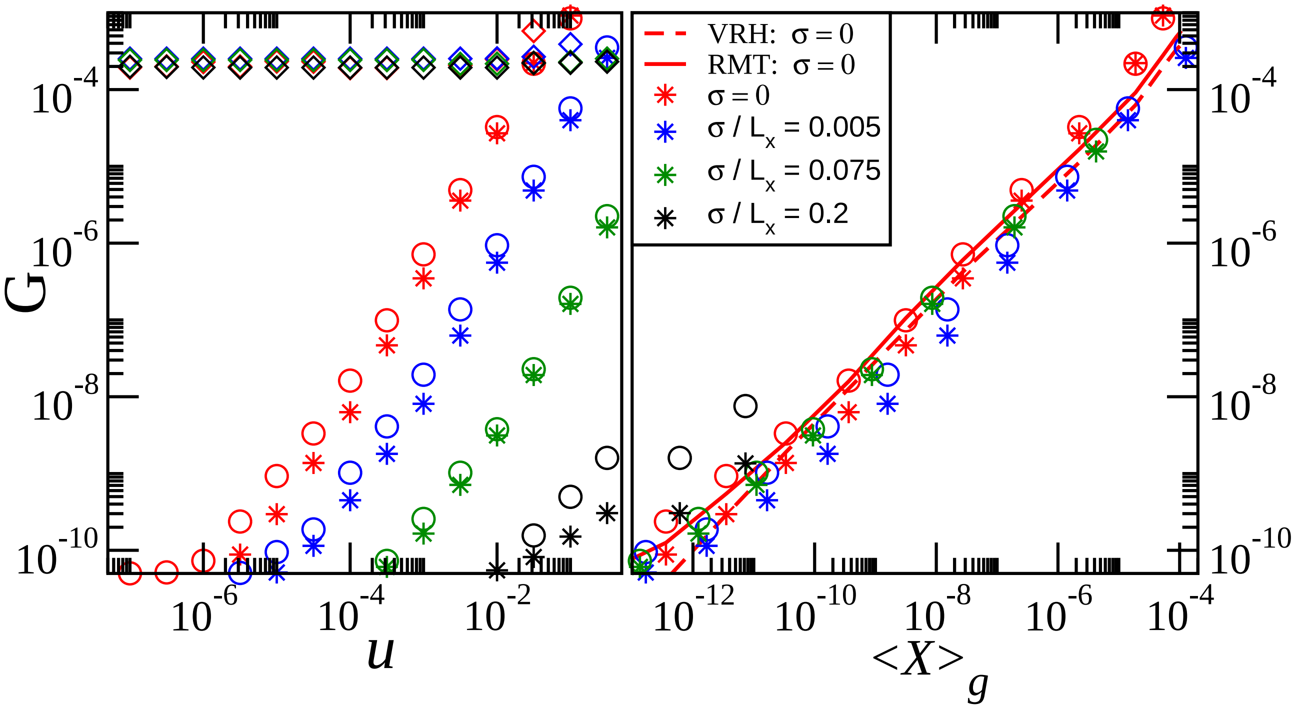
$$G_{\text{SLRT}} \quad = \quad \int x_\omega \tilde{S}(\omega) d\omega$$

In the standard case (strong disorder):

$$\begin{aligned} \text{Prob}(x) \;&=\; \text{log-box} \\ x_\omega \;&\approx\; v_{\text{F}}^2 \;\exp\left(\frac{\Delta_l}{|\omega|}\right) \\ \tilde{S}(\omega) \;&\propto\; \exp\left(-\frac{|\omega|}{T}\right) \\ G_{\text{SLRT}} \;&\propto\; \int \exp\left(-\frac{|\omega|}{T}\right) \exp\left(\frac{\Delta_l}{|\omega|}\right) d\omega \end{aligned}$$

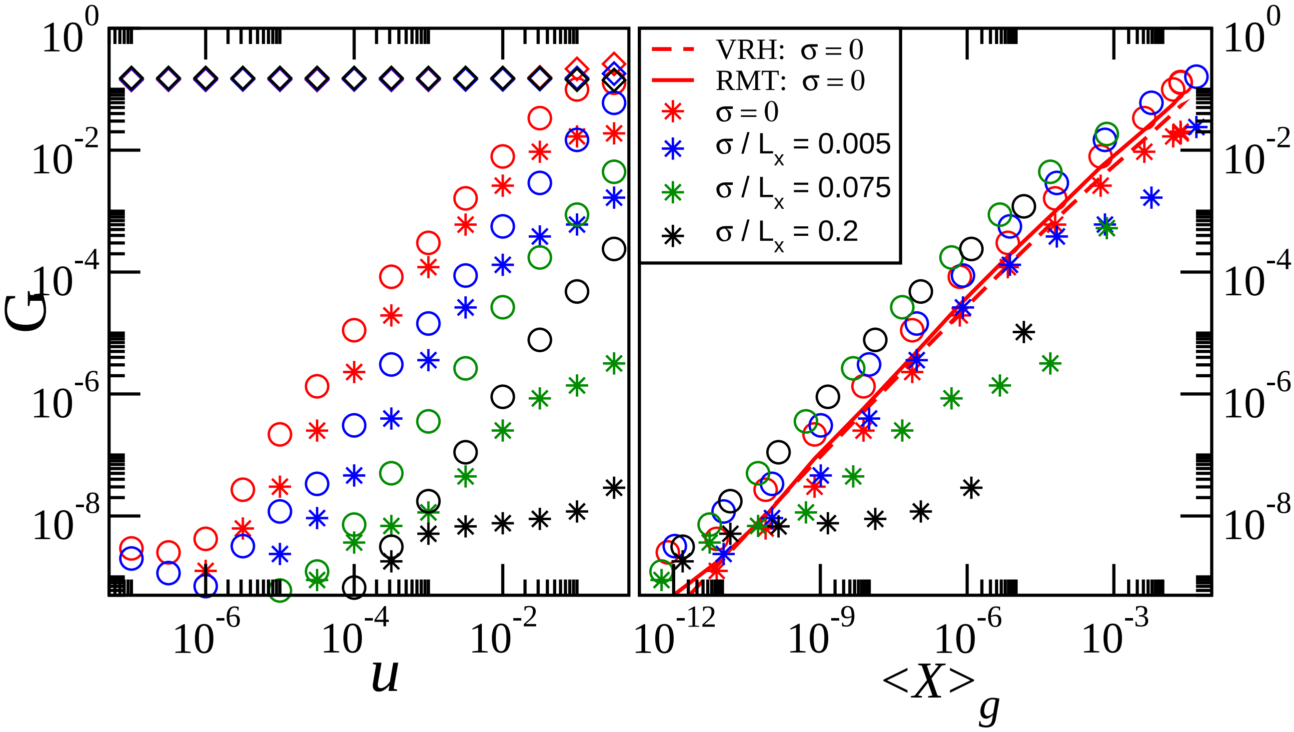
Numerical Results

$AS = 1$



What about texture?

$AS = 20$



Conclusions

(*) Wigner ($\sim$ 1955):
The perturbation is represented by a random matrix whose elements are taken from a Gaussian distribution.

Not always...

1. No "strong quantum chaos" $\Longrightarrow$ log-normal distribution.
2. The heating process $\sim$ a percolation problem.
3. Resistors network calculation to get G_{SLRT}.
4. Generalization of the VRH estimate
5. SLRT is essential whenever the distribution of matrix elements is wide ("sparsity") or if the matrix has "texture".

[1] D. Cohen, T. Kottos and H. Schanz, JPA (2006)

[2] S. Bandopadhyay, Y. Etzioni and D. Cohen, EPL (2006)

[3] M. Wilkinson, B. Mehlig and D. Cohen, EPL (2006)

[4] D. Cohen, PRB (2007)

[5] A. Stotland, R. Budoyo, T. Peer, T. Kottos and D. Cohen, JPA(FTC) (2008)

[6] A. Stotland, D. Cohen and N. Davidson, EPL (2009)