The mesoscopic conductance of disordered rings and its random matrix theory

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arXiv reference:

A. Stotland, R. Budoyo, T. Peer, T. Kottos and D. Cohen, (2007)

$DIP, $BSF

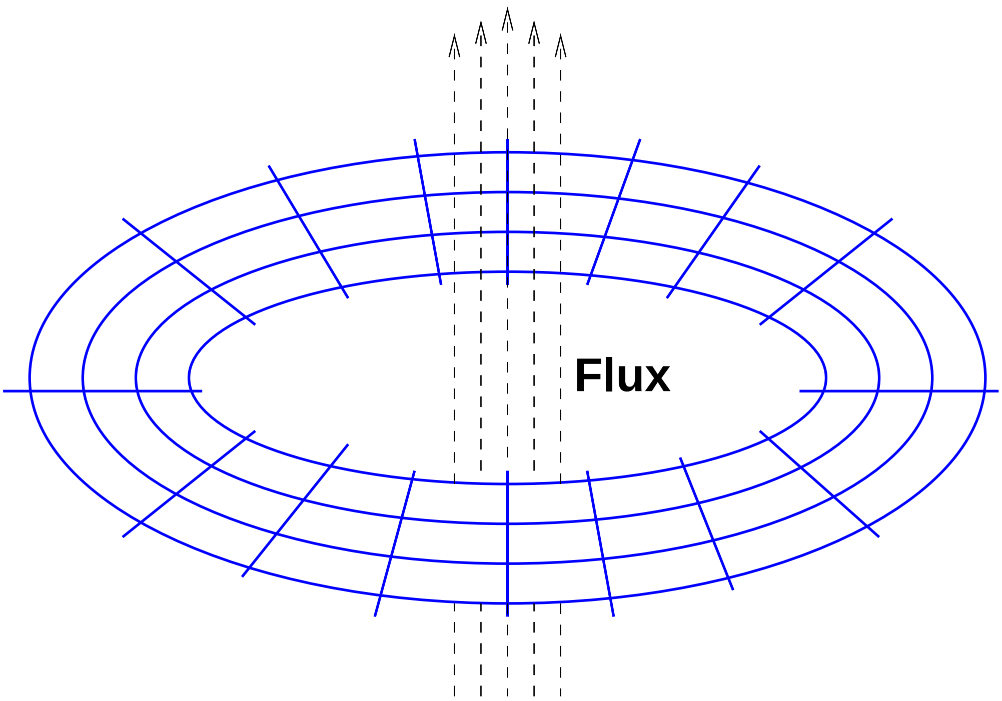
The model

Non interacting "spinless" electrons in a ring.

$$\mathcal{H}(r, p; \Phi(t))$$

$$-\dot{\Phi} \quad = \quad \text{electro motive force (RMS)}$$

$$\boldsymbol{G}\,\dot{\Phi}^2 \quad = \quad \text{rate of energy absorption}$$



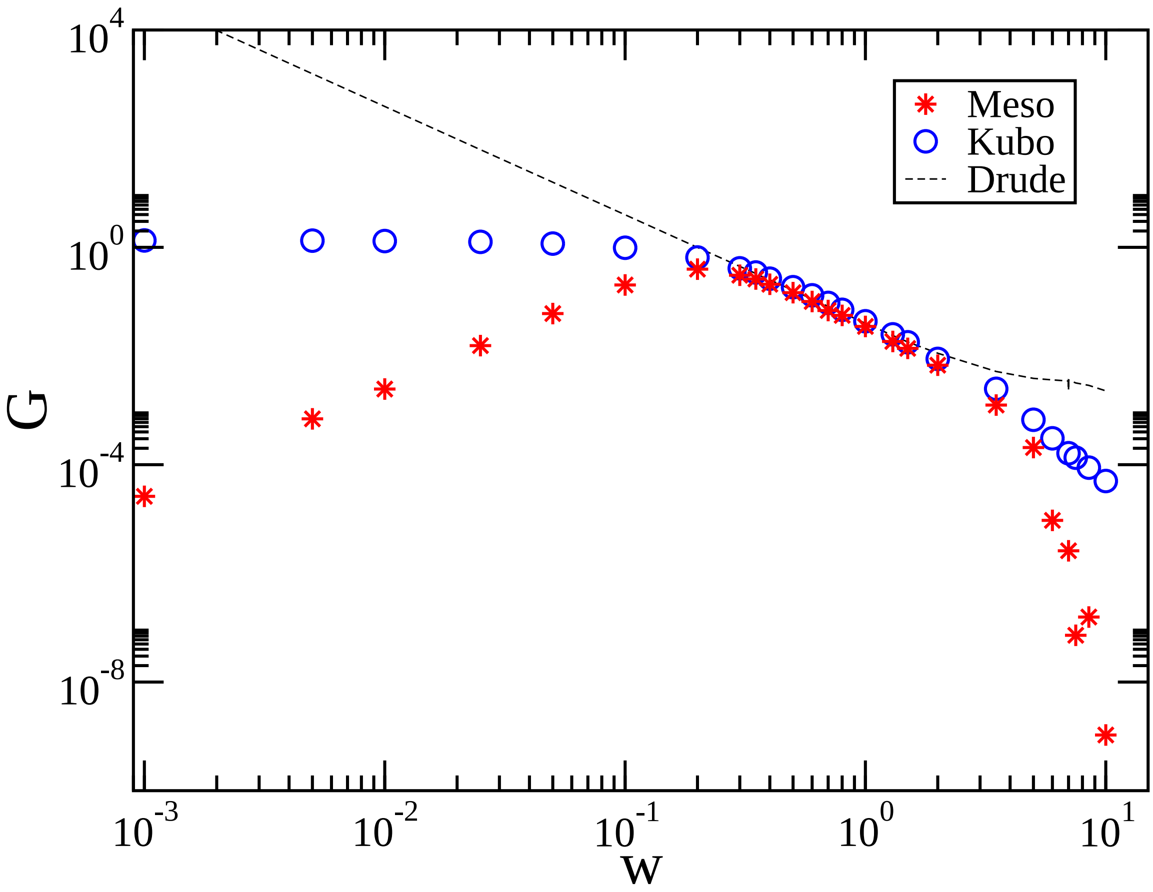
$\mathcal{M}$ mode ring of length L with disorder W.

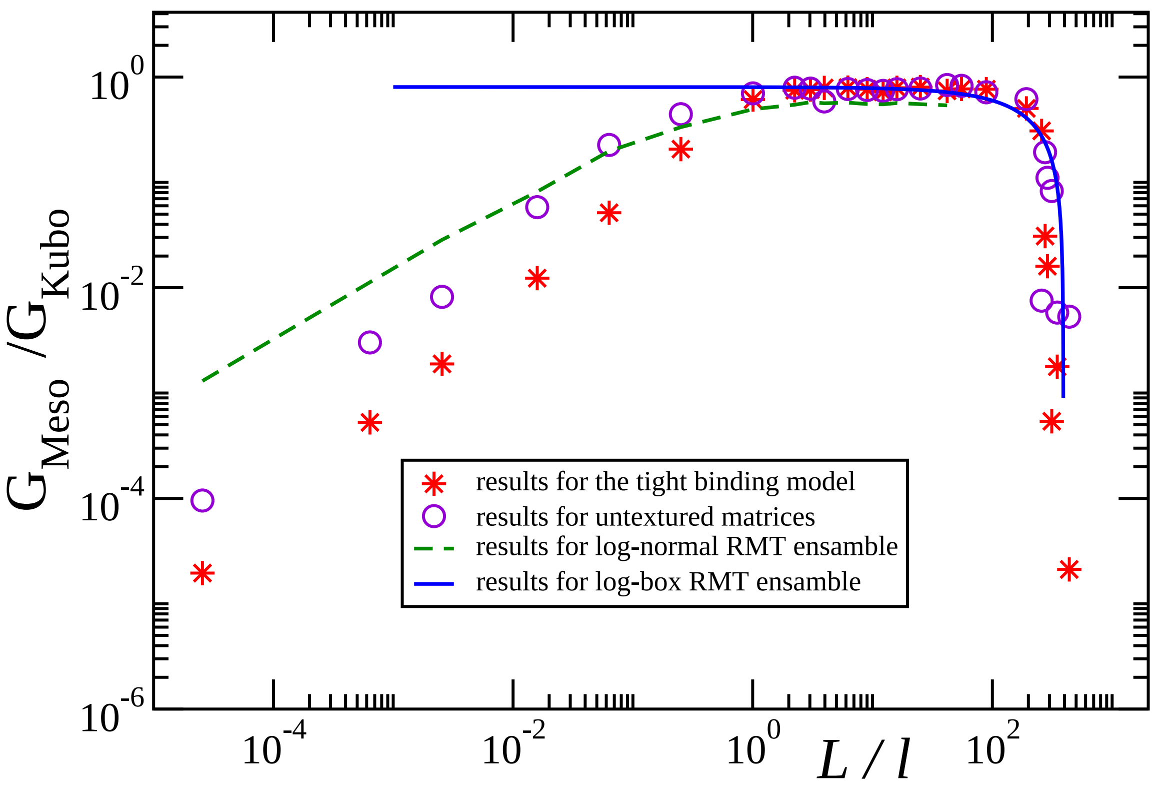
$$\boldsymbol{G} = \pi \left(\frac{e}{L}\right)^2 \mathrm{DOS}^2 \, \langle\langle |v_{mn}|^2 \rangle\rangle$$

$$\langle\langle |v_{mn}|^2 \rangle\rangle_{\text{harmonic}} \ll \langle\langle |v_{mn}|^2 \rangle\rangle_{\text{meso}} \ll \langle\langle |v_{mn}|^2 \rangle\rangle_{\text{algebraic}}$$

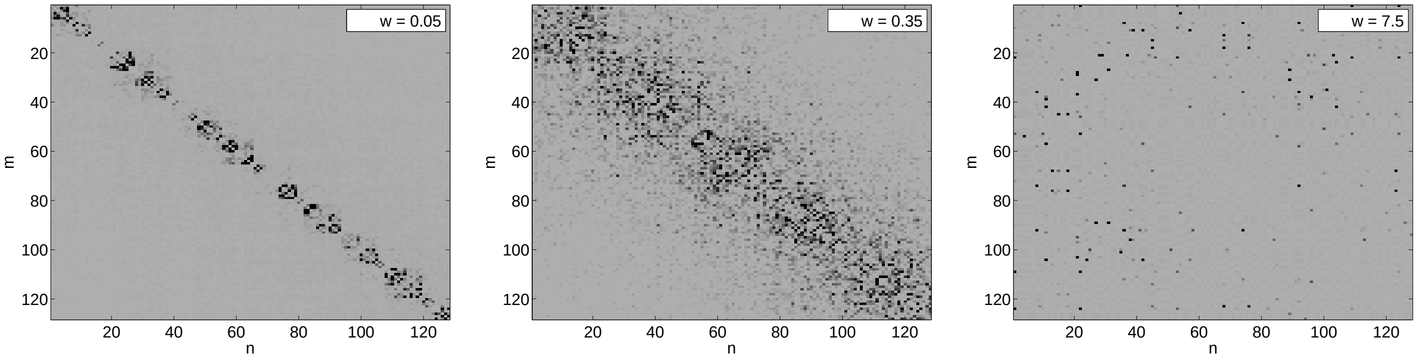
Numerical Results

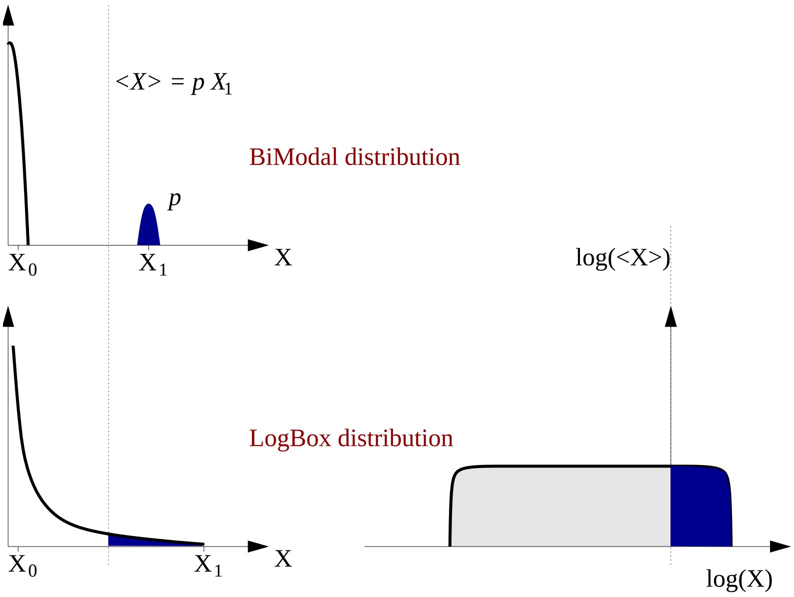
Regimes: ballistic; diffusive; localizaion





$\{|v_{nm}|^2\}$ as a random matrix $\{X\}$





$p \equiv \mathrm{F}(\langle X \rangle)$

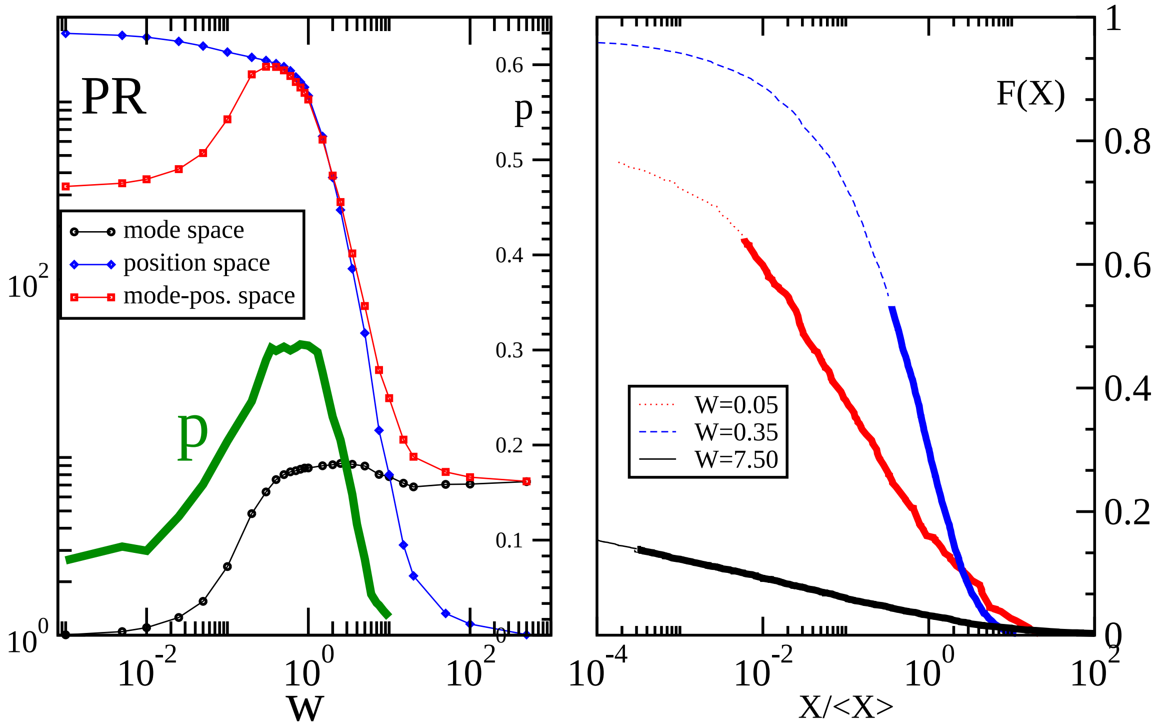
Ballistic:

$X \sim \mathrm{LogNormal}$

Localization:

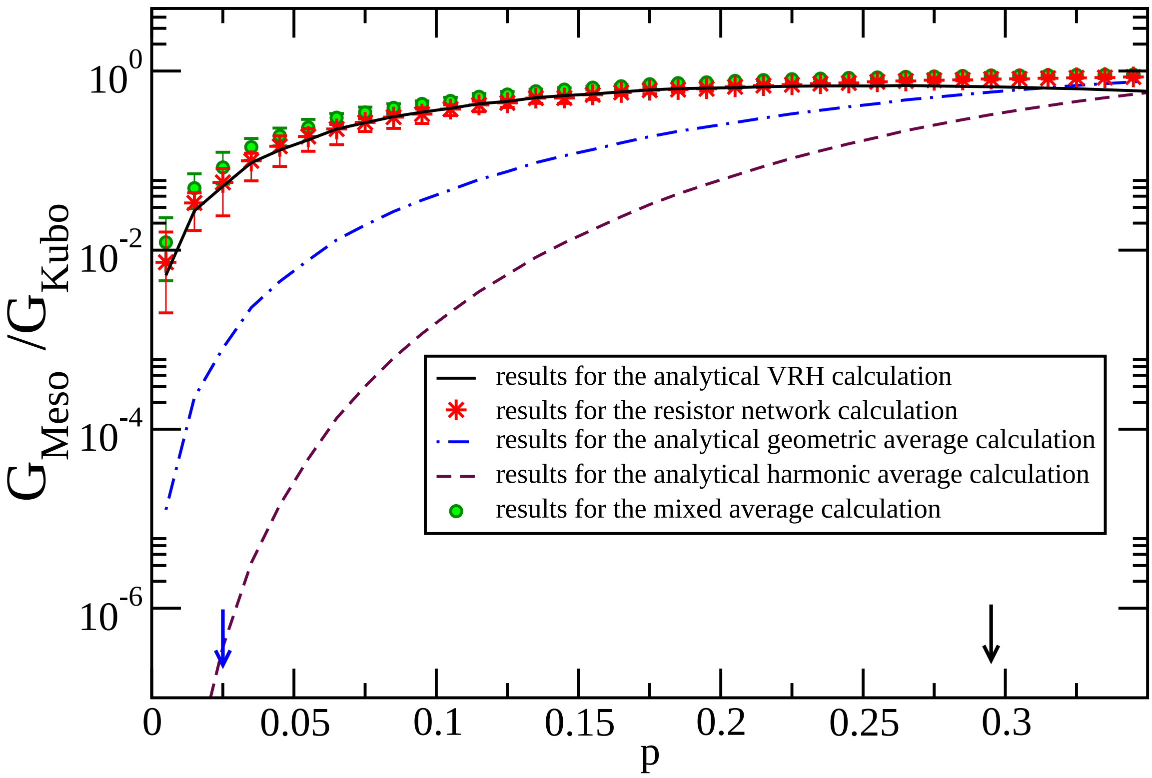
$X \sim \mathrm{LogBox}$

Histograms of X:

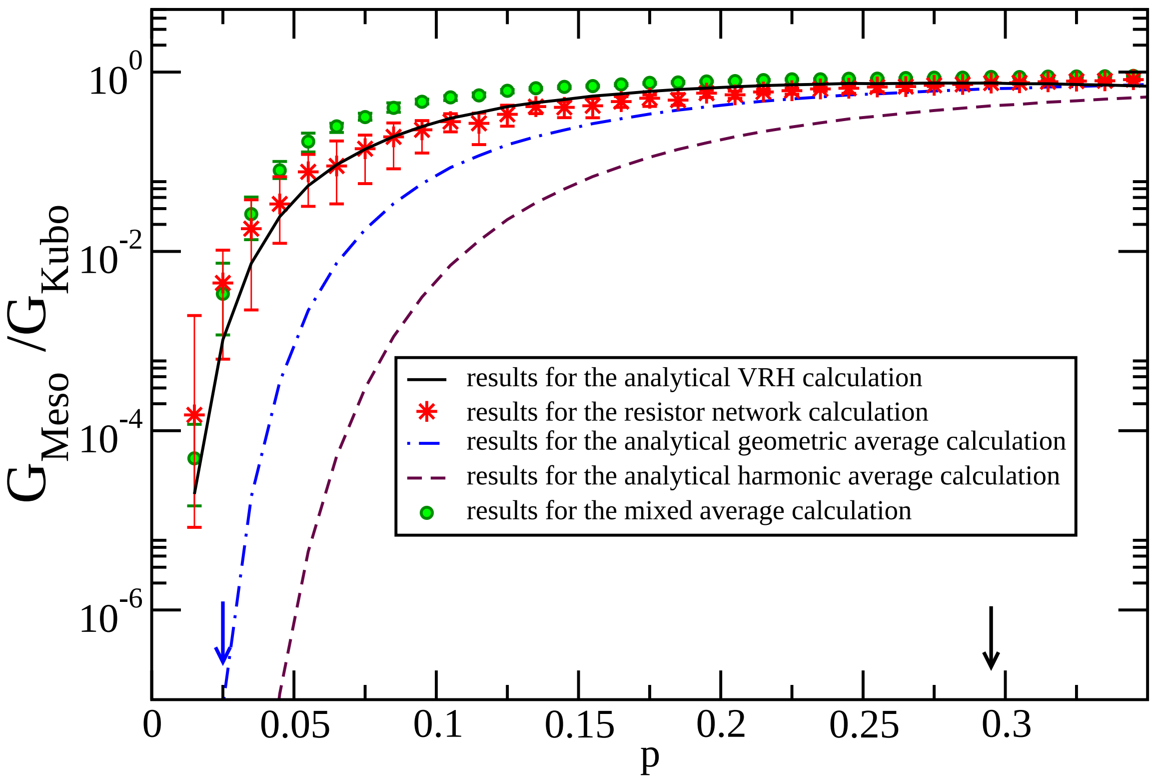


The RMT modeling

Log-normal distribution:



Log-box distribution:



$p \equiv \mathrm{F}(\langle X \rangle)$ - sparsity $\qquad$ b - bandwidth

Linear response theory

$$\boldsymbol{G} \ = \ \pi \left(\frac{e}{L}\right)^2 \sum_{n,m} |v_{mn}|^2 \ \delta_T(E_n - E_F) \ \delta_\Gamma(E_m - E_n)$$

$$\boldsymbol{G} \ = \ \pi \left(\frac{e}{L}\right)^2 \mathrm{DOS}^2 \ \langle\langle |v_{mn}|^2 \rangle\rangle_{\mathrm{algebraic}}$$

applies if

EMF driven transitions $\ll$ relaxation

otherwise

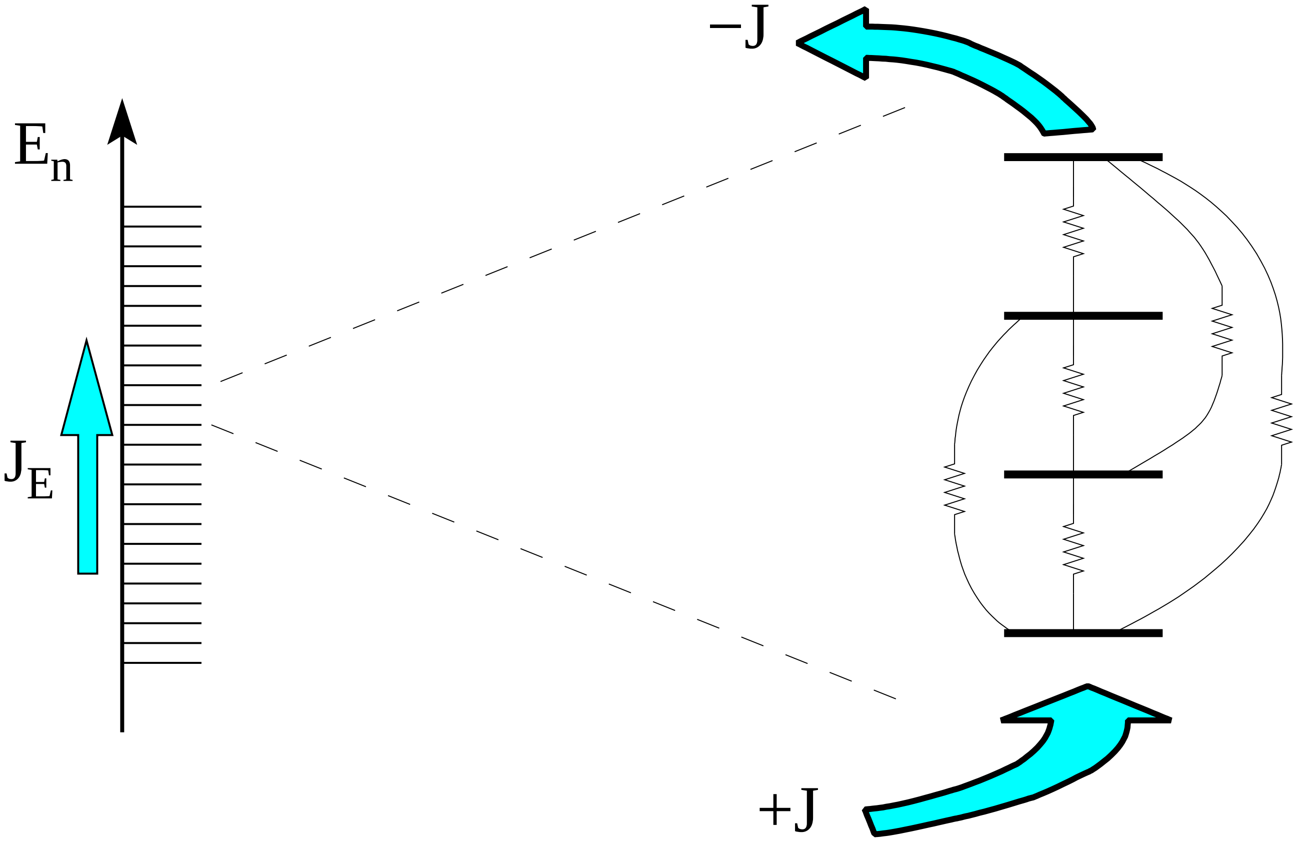
connected sequences of transitions are essential.

leading to

Semi Linear Response Theory (SLRT)

Semi Linear Response Theory

$$\boldsymbol{G} \;=\; \pi\left(\frac{e}{L}\right)^2 \mathrm{DOS}^2\ \langle\langle|v_{mn}|^2\rangle\rangle_{\mathrm{meso}}$$



$$\mathrm{g}_{nm} = 2\varrho_{\mathrm{F}}^{-3}\ \frac{|v_{nm}|^2}{(E_n{-}E_m)^2}\ \delta_{\Gamma}(E_n{-}E_m)$$

$\langle\langle|v_{mn}|^2\rangle\rangle_{\mathrm{meso}} \equiv$ inverse resistivity of the network

$$\langle\langle|v_{mn}|^2\rangle\rangle_{\mathrm{harmonic}} \ll \langle\langle|v_{mn}|^2\rangle\rangle_{\mathrm{meso}} \ll \langle\langle|v_{mn}|^2\rangle\rangle_{\mathrm{algebraic}}$$

Conclusions

(*) Wigner's idea ($\sim$ 1955):

The perturbation is represented by a random matrix whose elements are taken from a Gaussian distribution.

Not always...

1. Ballistic ring $\Longrightarrow$ log-normal distribution.
2. Strong localization $\Longrightarrow$ log-box distribution.
3. Resistors network calculation to get G_{meso}.
4. Generalization of the VRH calculation procedure.
5. SLRT* is essential whenever the distribution of matrix elements is wide ("sparsity") or if there is a "texture".

[1] D. Cohen, T. Kottos and H. Schanz, JPA (2006)

[2] S. Bandopadhyay, Y. Etzioni and D. Cohen, EPL (2006)

[3] M. Wilkinson, B. Mehlig and D. Cohen, EPL (2006)

[4] D. Cohen, PRB (2007)

[5] A. Stotland, R. Budoyo, T. Peer, T. Kottos and D. Cohen, arXiv (2007)

Semi Linear Response Theory

$$H = \{E_n\} - \frac{e}{L}\Phi(t)\{v_{nm}\}$$

$$\frac{dp_n}{dt} = -\sum_m w_{nm}(p_n - p_m)$$

$$w_{nm} \quad = \quad \text{const} \times \mathrm{g}_{nm} \times \text{EMF}^2$$

Scaled transition rates:

$$\mathrm{g}_{nm} = 2\varrho_{\mathrm{F}}^{-3} \; \frac{|\mathcal{I}_{nm}|^2}{(E_n - E_m)^2} \; \delta_{\Gamma}(E_n - E_m)$$