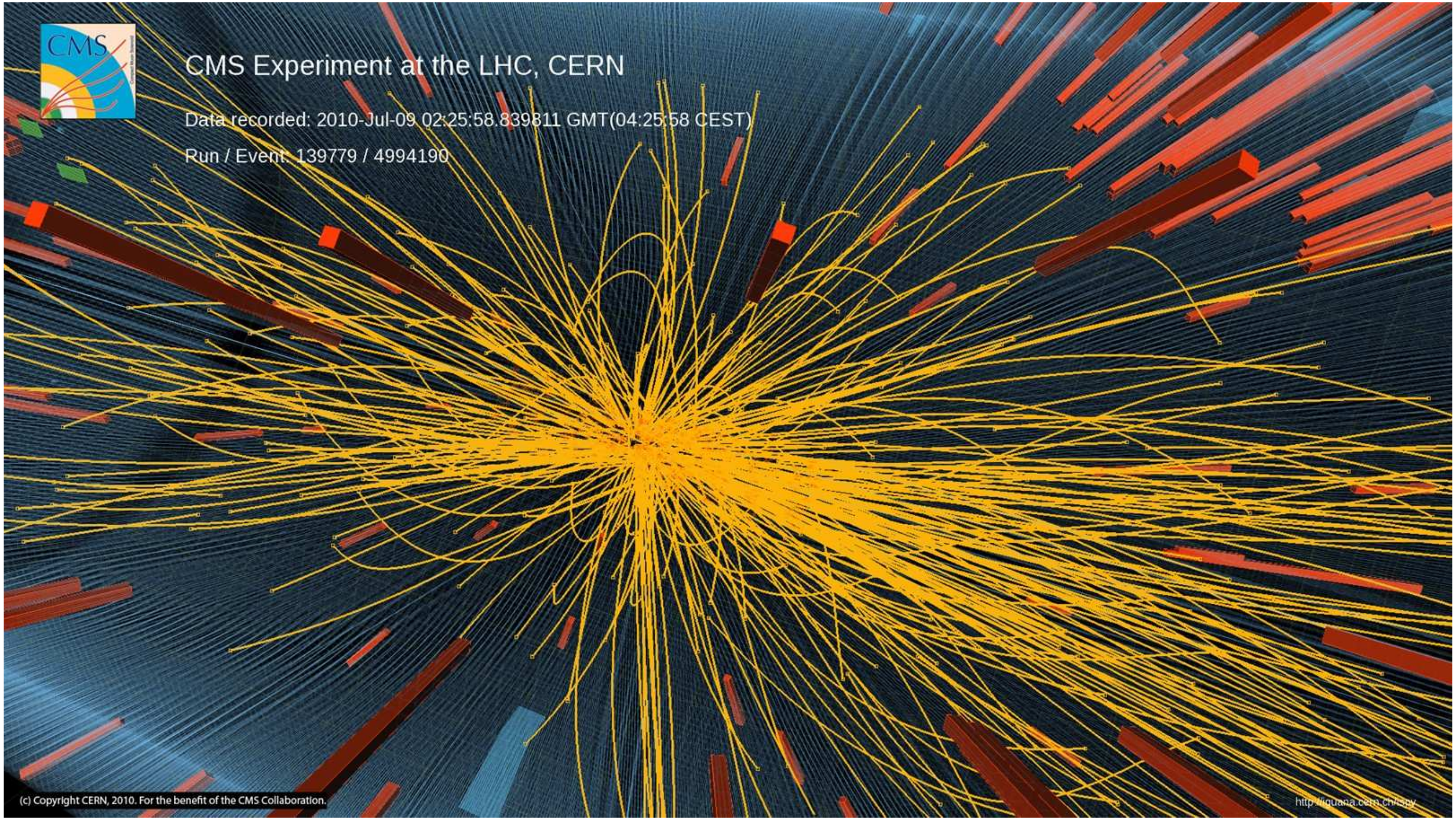
Angular correlations in gluon emission ("ridge") from high energy QCD.

Michael Lublinsky

Ben-Gurion University of the Negev

Beer Sheva, Israel

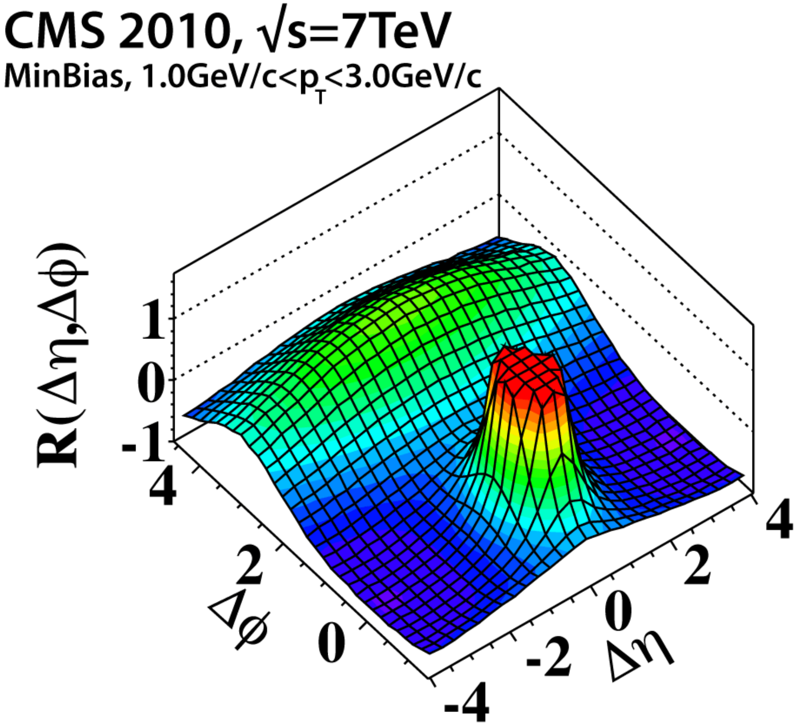
with Alex Kovner; Phys.Rev. D84 (2011) 094011, Phys.Rev. D83 (2011) 034017;
Int.J.Mod.Phys. E22 (2013) 1330001.

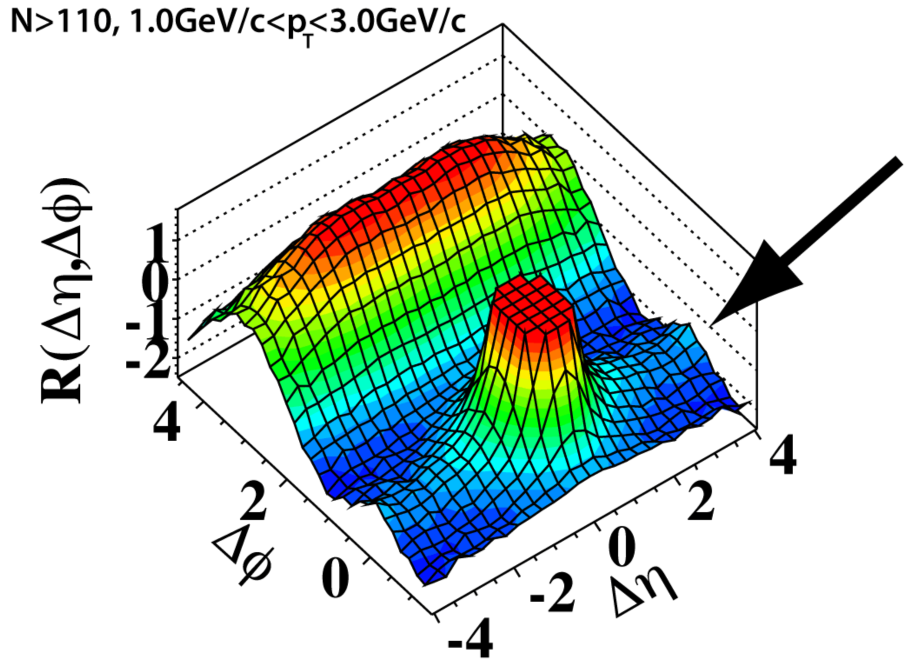


CMS Collaboration: arXiv:1009.4122 [hep-ex]

"RIDGE" - ANGULAR CORRELATIONS

Two particle correlations in $p-p$: long range in rapidity, near-side angular correlations





"High multiplicity" collisions with over a hundred charged particles produced

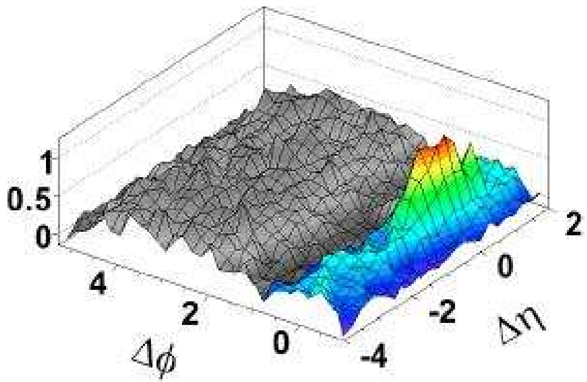
Forward pick. Backward ridge at the angle π – back-to-back correlation.

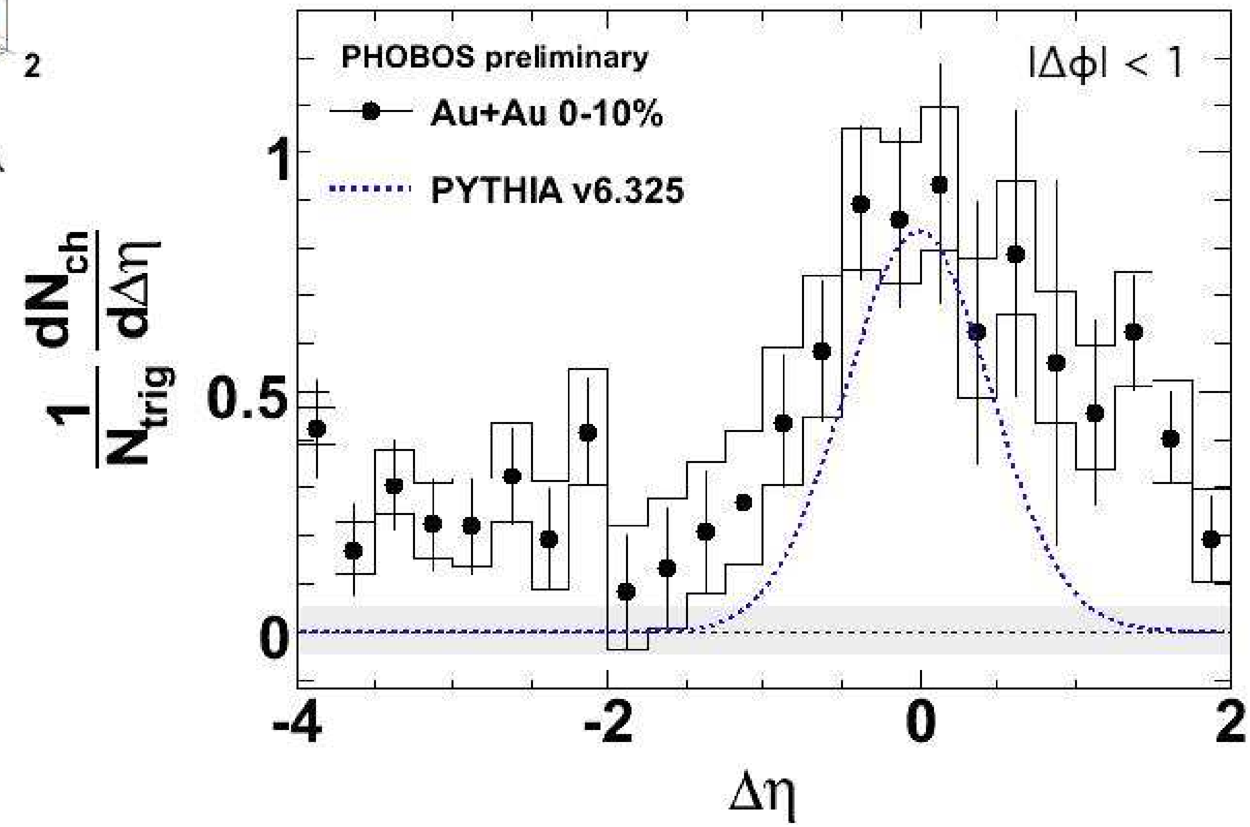
Same-side ridge is a new "correlation" effect PYTHIA and friends fail

a very similar phenomenon in heavy ion collisions at RHIC

PHOBOS: high p_t triggered ridge

Correlated yield on near-side (|Δϕ| < 1):



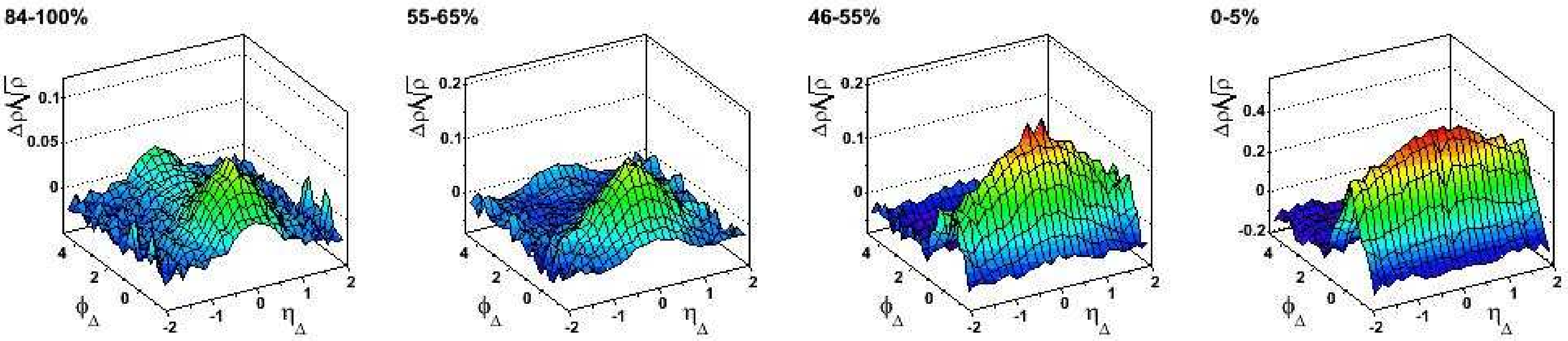


$p_t^{trigger} > 2.5$ GeV

$p_t^{associate} > 20$ MeV

STAR: soft ridge

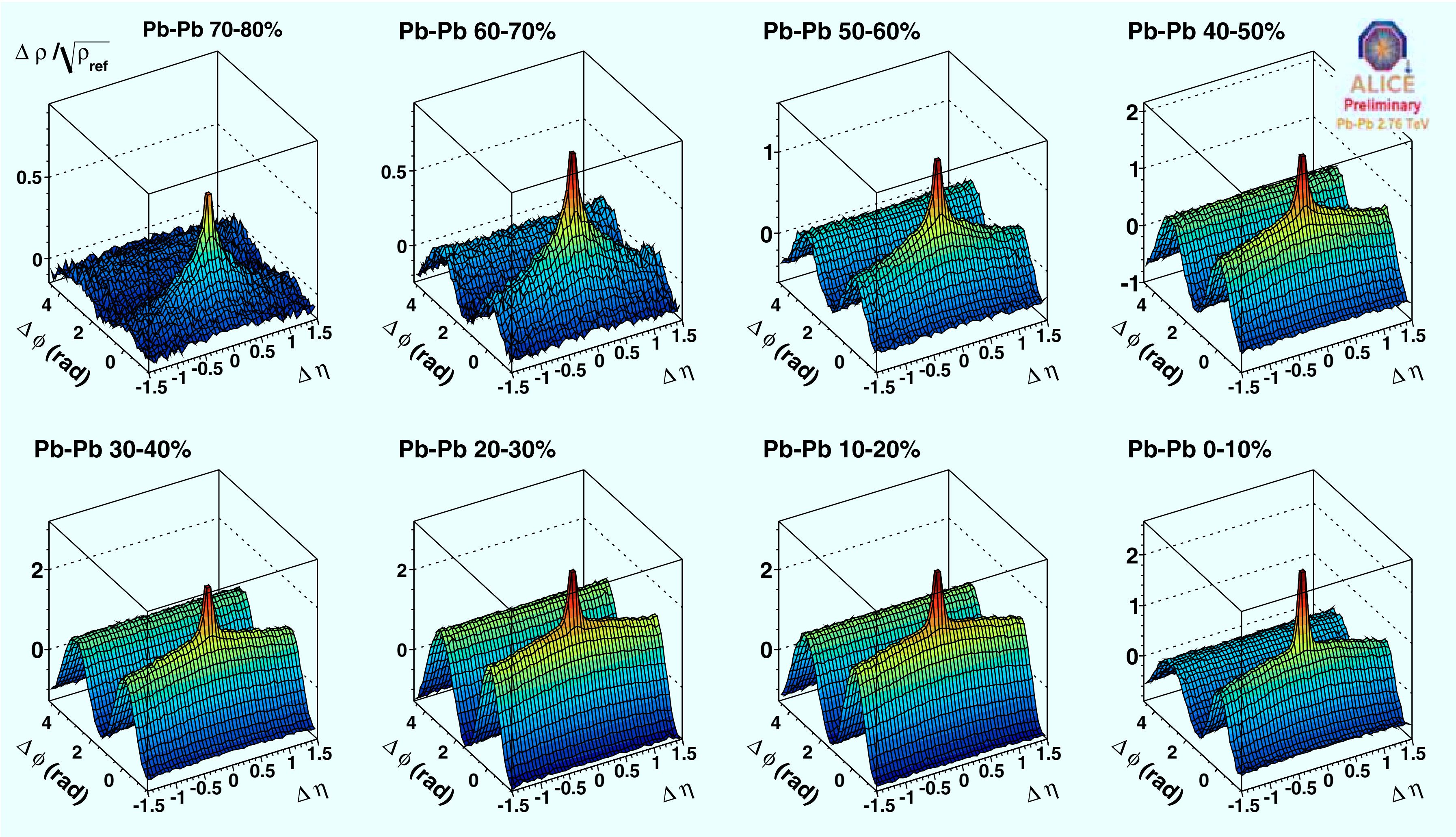
Centrality dependence of the ridge



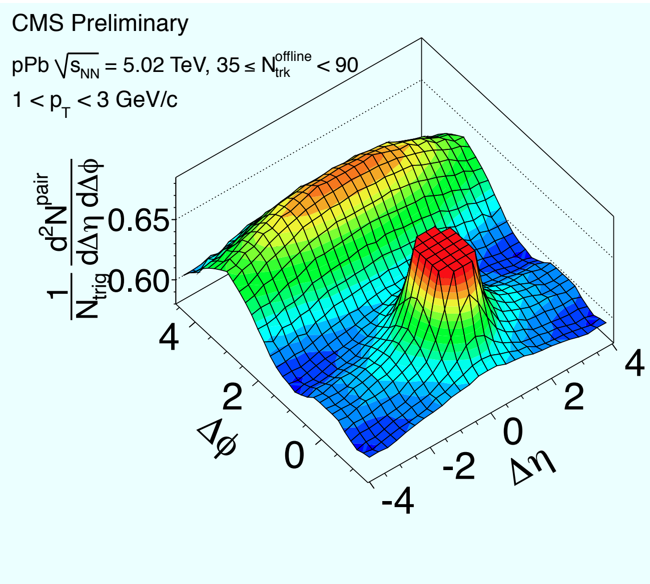
No ridge structure in peripheral collisions

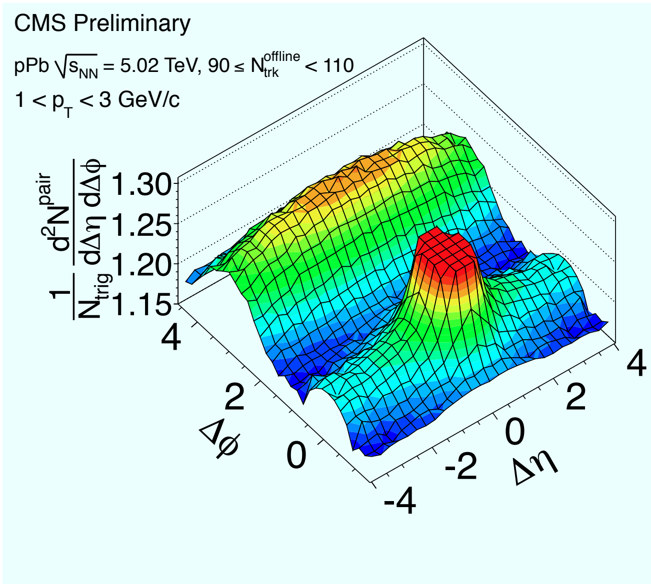
In heavy ion collisions at RHIC, the ridge has rather simple explanation related to explosion of high density matter

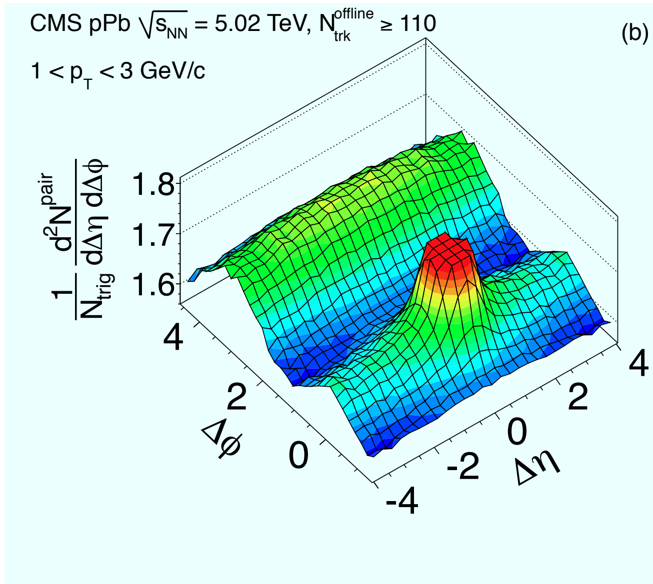
Soft ridge from ALICE



Multiplicity dependence of the ridge in pPb







By no means a complete summary of experimental results:

- **Ridge is seen in $A - A$, $p - A$ and $p - p$ collisions at more or less the same window in p_t**
- $p - A$ **and** $A - A$ **are very similar when compared at the same multiplicities**
- $p - p$ **is definitely a rare fluctuation**
- **while no ridge at all in peripheral** $A - A$
- $\mathbf{p_t}$ **and** $\mathbf{\Delta}\eta$ **dependencies are unclear**

BIg Questions

- **Origin of angular collimation?**
  Could be many. For sure explosive "wind" from hydro would lead to some.

- **Origin of long range rapidity correlations?**
  Causality: correlations exist in early stage of the collision (like in cosmology)

- **Do we see a sign of universality between $p-p$ and $p-A$ and $A-A$?**
  Hopefully Yes! High energy QCD implies this universality. In both experiments the effect emerges only when high densities are involved (color glass condensate (CGC))

- **Do we see a collective phenomenon (QGP?) in $p-p$ or $p-A$?**
  We don't know yet ...

Our Goal

To discuss some general features of gluon production at high energy.

We need to compute correlations in two-gluon inclusive production rate

$$\left[\frac{\mathbf{d^2N}}{\mathbf{d^2p\,d\eta\,d^2k\,d\xi}} - \frac{\mathbf{dN}}{\mathbf{d^2k\,d\xi}}\frac{\mathbf{dN}}{\mathbf{d^2p\,d\eta}}\right]\Big/\frac{\mathbf{dN}}{\mathbf{d^2k\,d\xi}}\frac{\mathbf{dN}}{\mathbf{d^2p\,d\eta}}$$

We don't know how to compute dense on dense (F Gelis, T Lappi, R Venugopalan do know)

We do know quite a lot about dilute on dense (DIS)
For DIS, we do have QCD-derived formulae for multi-gluon production, including high energy evolution between produced gluons.

Here I talk about only one source for the observed phenomena, INITIAL CONDITIONS, as follows from quite general QCD-based considerations, but I have no quantitative results.

A Dumitru, K Dusling, F Gelis, J Jalilian-Marian, T Lappi, R Venugopalan

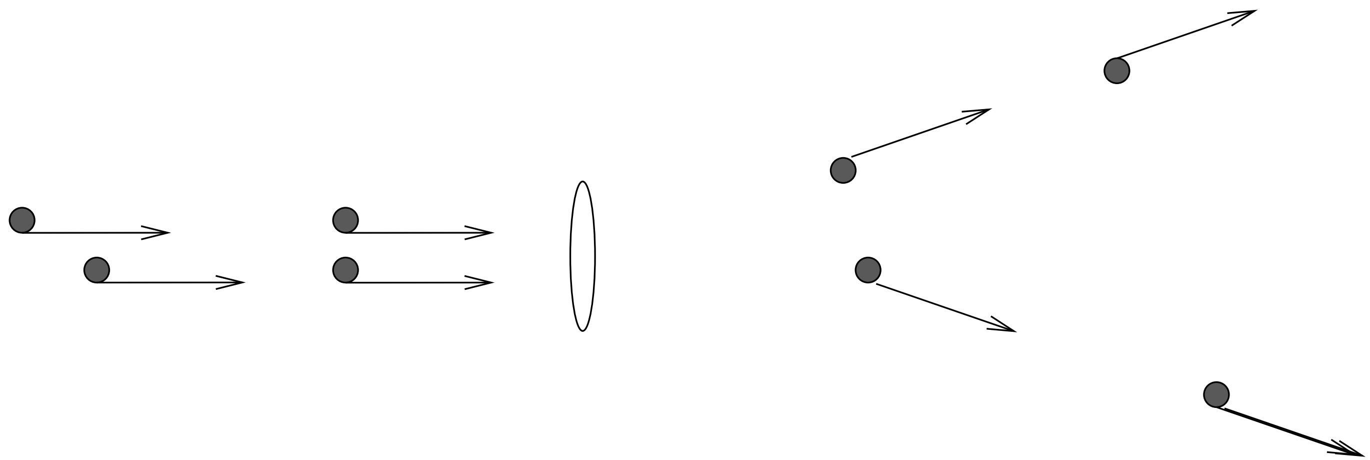
NAIVE PICTURE OF EIKONAL GLUON PRODUCTION

Long range rapidity correlations come for free with boost invariance

Incoming $|P\rangle$ is approximately boost invariant: exactly the same gluon distribution at Y_1 and Y_2.
What happens at Y_1, happens also at Y_2: If it is probable to produce a gluon at Y_1, it is also probable to produce a gluon at Y_2.

But exactly by the same logic there must be angular correlations:
Gluons scatter on exactly the same target
If the first gluon is most likely to be scattered to the right, the second gluon at the same impact parameter will be also scattered to the right

Eikonal scattering is rapidity independent!



High Energy Scattering: CGC-type approach

Target **Projectile**

$$\langle T| \quad \rightarrow \qquad\qquad \leftarrow \quad |P\rangle$$

S**-matrix:**

$$\mathbf{S(Y)} \;=\; \langle \mathbf{T}\, \langle\, \mathbf{P}| \;\; \hat{\mathbf{S}}(\rho^{\mathrm{t}},\, \rho^{\mathrm{p}}) \;\; |\mathbf{P}\rangle\, \mathbf{T}\rangle$$

CGC-type averaging

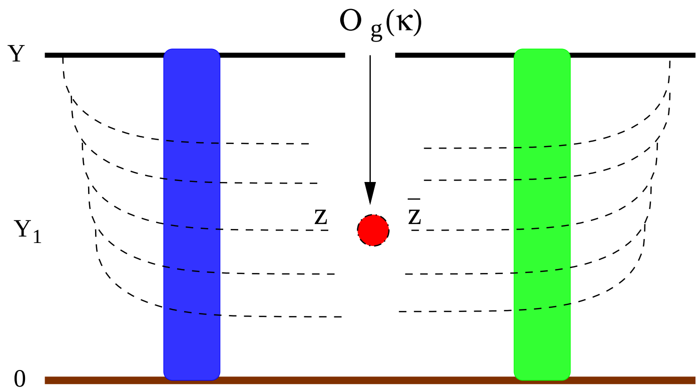
$$\mathbf{S(Y)} \;=\; \int \mathbf{D}\rho^{\mathrm{p}}\, \mathbf{D}\rho^{\mathrm{t}} \;\; \mathbf{S}[\rho^{\mathrm{p}},\, \rho^{\mathrm{t}}] \;\; \mathbf{W}^{\mathrm{p}}_{\mathbf{Y}-\mathbf{Y_0}}[\rho^{\mathrm{p}}] \;\; \mathbf{W}^{\mathrm{t}}_{\mathbf{Y_0}}[\rho^{\mathrm{t}}]$$

$\mathbf{W}^{\mathrm{p,t}}$ **are probability distributions, subject to high energy evolution equations**

For any other observable $\mathcal{O}$

$$\langle \mathcal{O} \rangle_{\mathbf{P,T}} \;=\; \int \mathbf{D}\rho^{\mathrm{p}}\, \mathbf{D}\rho^{\mathrm{t}} \;\; \mathcal{O}_{\mathbf{Y_0}}[\rho^{\mathrm{p}},\, \rho^{\mathrm{t}}] \;\; \mathbf{W}^{\mathrm{p}}_{\mathbf{Y}-\mathbf{Y_0}}[\rho^{\mathrm{p}}] \;\; \mathbf{W}^{\mathrm{t}}_{\mathbf{Y_0}}[\rho^{\mathrm{t}}]$$

Single inclusive gluon production



The observable

$$\hat{\mathcal{O}}_g \sim \mathbf{a}_i^{\dagger\, a}(\mathbf{k})\, \mathbf{a}_i^{a}(\mathbf{k})$$

$$\frac{dN}{d^2k\,dy} = \langle\, \sigma(\mathbf{k})\, \rangle_{P,T}$$

$$\sigma(\mathbf{k}) = \int_{z,\bar{z},x_1,\bar{x}_1} e^{ik(z-\bar{z})}\vec{f}(\bar{z}-\bar{x}_1)\cdot\vec{f}(x_1-z)\ \Big\{\rho(x_1)[S^\dagger(x_1) - S^\dagger(z)][S(\bar{x}_1) - S(z)]\rho(\bar{x}_1)\Big\}$$

Here

$$f_i(x-y) = \frac{(x-y)_i}{(x-y)^2} \qquad S(x) = \mathcal{P}\exp\left\{i\int dx^-\, T^a\, \alpha_t^a(x,x^-)\right\}. \qquad \text{"}\Delta\text{"}\,\alpha_t = \rho_t \quad (YM)$$

TWO GLUON INCLUSIVE PRODUCTION

Using dilute projectile formulae, but thinking of it as being dense

$$\mathcal{O} = \mathbf{a}^\dagger(\mathbf{k})\,\mathbf{a}(\mathbf{k})\;\mathbf{a}^\dagger(\mathbf{p})\,\mathbf{a}(\mathbf{p})$$

$$\frac{\mathbf{dN}}{\mathbf{d^2pd^2kd}\eta\mathbf{d}\xi} = \sigma_4 = \langle\sigma(\mathbf{k})\;\sigma(\mathbf{p})\rangle_{\mathbf{P,T}}$$

Configuration by configuration
(for fixed configuration of projectile charges ρ and fixed target fields S)

$\sigma(\mathbf{k})$ is a real function of k, which has a maximum at some value $\mathbf{k} = \mathbf{q_0}$. Then the two gluon production probability configuration by configuration has a maximum at

$$\mathbf{k} = \mathbf{p} = \mathbf{q_0} \simeq \mathbf{Q_s}$$

The value of $\mathbf{q_0}$ depends on configuration, but the fact that $\mathbf{k} \simeq \mathbf{p}$ does not.

This is the near side correlation!

Is the maximum of σ_1 unique?

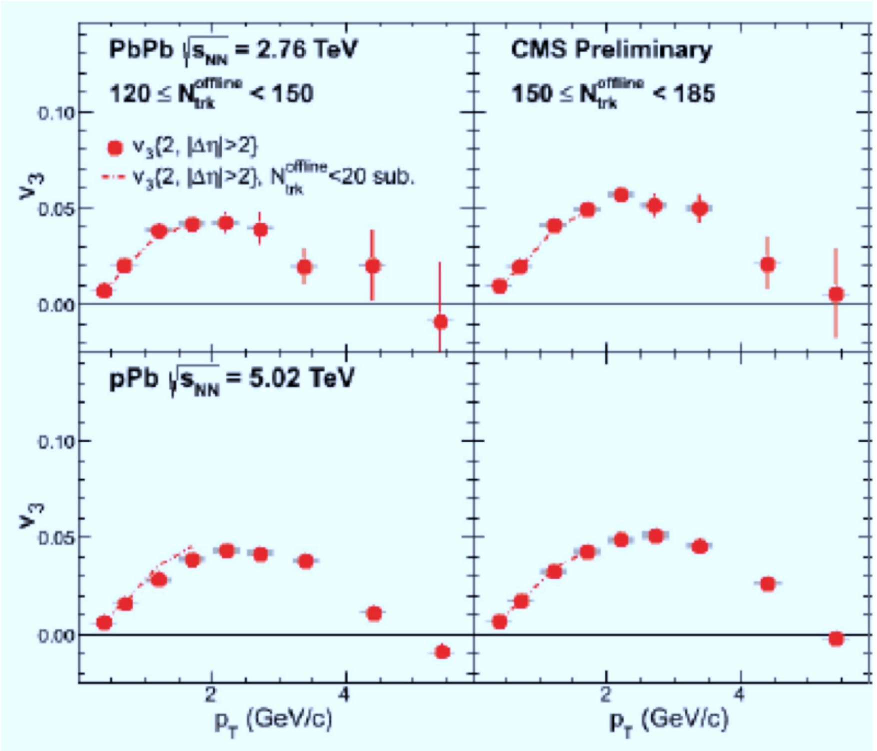
No, σ_1 is symmetric under $\mathbf{k} \to -\mathbf{k}$ and thus has two maxima at $\mathbf{q_0}$ and $-\mathbf{q_0}$

This means that σ^4 has a symmetry $\mathbf{k}, \mathbf{p} \to -\mathbf{k}, \mathbf{p}$ and therefore has maxima at relative angles $\phi = 0$ and $\phi = \pi$

The maximum at $\phi = \pi$ is very difficult to distinguish experimentally.

After all there seems to be some asymmetry between 0 and π angles

The v_3 story:



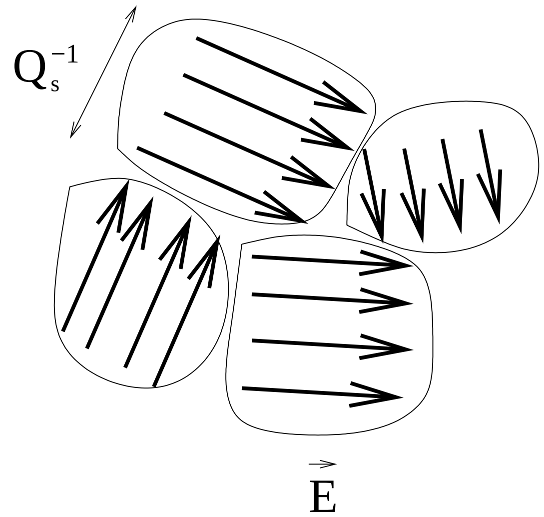
How big is the effect?

To be correlated two gluons have to be in the same incoming color state and have to scatter of the same target field

Transverse correlation length in the hadron $\mathbf{L = 1/Q_s}$ ("mean density")

The correlated production $\propto 1/(Q_s^{max})^2$,

while the total multiplicity $\propto S_A^{min}$



$$\left[\frac{d^2N}{d^2p\,d^2k} - \frac{dN}{d^2k}\frac{dN}{d^2p}\right] \Big/ \frac{dN}{d^2k}\frac{dN}{d^2p} \propto \frac{1}{(Q_s^{max})^2\, S_A^{min}}\,.$$

Q_s grows with energy. Hence correlations should disappear with increasing energy. Less correlations at the LHC than at RHIC? Not obvious, because we fully ignored the flow.

CONCLUSIONS 1

- Gluon production at high energy leads naturally to rapidity correlations and angular correlations. There just have to be many gluons so that more than one is produced at fixed impact parameter (within $\Delta b \sim 1/Q_s$)

- "Classical" term leads to the strongest correlations – thus the correlations should be largest for nucleus projectile where it dominates. On the other hand effect becomes weaker with increasing Q_s. So, maybe actually the other way around – it is strongest for $p - p$ in a limited range in energy?

- None of these qualitative features depends on what averaging procedure we use to average over the projectile and target fields, but quantitative of course it will.

Too Many sources of uncertainty:

– large N_c

– target/projectile averaging

– target/projectile evolution

– rapidity evolution between produced gluons

– the role of high multiplicity trigger

– QGP hydro explosion

High Energy Evolution

Hadron wave function in the gluon Fock space

$$|\Psi\rangle_{Y_0} = \Psi[a_i^{\dagger a}(x)]|0\rangle_{Y_0} \qquad |\Psi\rangle = |v\rangle$$

Increase of energy = boosting one of the hadrons

High energy limit = soft gluon emission approximation

The evolved wave function

$$|\Psi\rangle_Y = \Omega_Y(\rho,\, a)\,|v\rangle_{Y_0}\,; \qquad |v\rangle_{Y_0} = |v\rangle \otimes |0_a\rangle$$

$$\frac{dW^t}{dY} = H^{HE}\;W^t \qquad \frac{dW^p}{dY} = H^{HE}\;W^p$$

Dilute limit:

$$\Omega_Y(\rho \to 0) \equiv \mathcal{C}_Y = \mathrm{Exp}\left\{ i \int d^2z\, b_i^a(z) \int_{e^{Y_0}\Lambda}^{e^Y \Lambda} \frac{dk^+}{\pi^{1/2}|k^+|^{1/2}} \left[a_i^a(k^+, z) + a_i^{\dagger a}(k^+, z)) \right] \right\} .$$

The classical WW field $\quad \mathbf{b_i^a(z) = \frac{g}{2\pi} \int d^2x \frac{(z-x)_i}{(z-x)^2} \;\; \rho^a(x)}$

$\mathbf{H^{KLWMIJ} = H^{HE}(\rho \to 0)}$ - A. Kovner and M.L., Phys.Rev.D71:085004, 2005

Dense limit: $\quad \mathbf{\Omega(\rho \sim 1/\alpha_s) = \mathcal{C}\, B}$ A. Kovner, M.L, and U. Wiedemann (2007)

$\mathbf{H^{JIMWLK} = H^{HE}(\rho \to \infty)}$ - Jalilian Marian, Iancu, McLerran, Leonidov, Kovner (1997-2002)

Baltitsky-Kovchegov (BK) is the large $\mathrm{N_c}$ version of JIMWLK

Evolution with Pomeron Loops (model):

$$\mathbf{H^{HE} \simeq H^{JIMWLK}(\rho \to \infty) \;'' + ''\; H^{KLWMIJ}(\rho \to 0)}$$

Target correlations $\langle tr[S^\dagger S]\, tr[S^\dagger S]\rangle_T$ from the BK equation

BKe for imaginary part of the dipole scattering amplitude $\mathbf{N(\vec{r}) = 1 - tr[S^\dagger_x\, S_y]/N_c}$

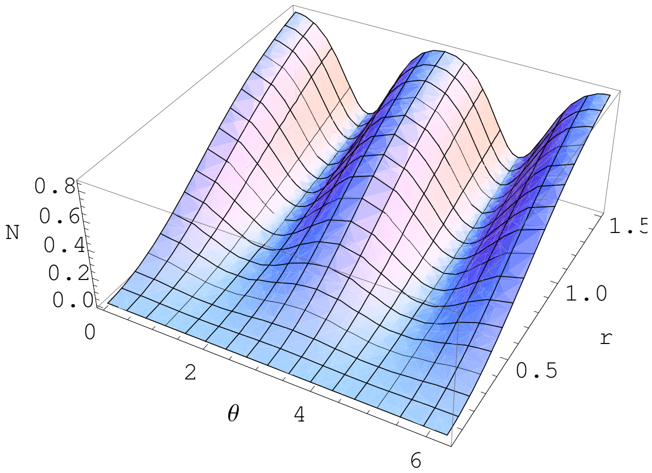
$$\partial_Y\, \mathbf{N(\vec{r})} = \frac{\mathbf{C_F}\,\alpha_s}{\mathbf{2}\,\pi} \int \mathbf{d^2\vec{r}'} \frac{\mathbf{\vec{r}^2}}{\mathbf{\vec{r}'^{\,2}\,(\vec{r}-\vec{r}')^2}} \left[\mathbf{N(\vec{r}') + N(\vec{r}-\vec{r}') - N(\vec{r}) - N(\vec{r}')\,N(\vec{r}-\vec{r}')}\right]$$

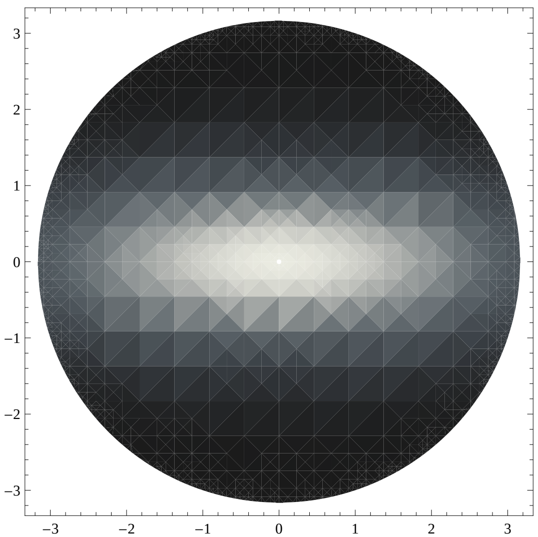
$\mathbf{\vec{r} = \vec{x} - \vec{y}}$ is a vector of the dipole moment.

Anisotropic initial conditions at some initial rapidity $Y_0 = \ln 10^2$.

$$\mathbf{N(Y_0, \vec{r}) = 1 - Exp[-a\, r^2\, xg^{LOCTEQ6}(x_0, 4/r^2)\, F(\theta)]}; \qquad \mathbf{a} = \frac{\alpha_s(\mathbf{r^2})\,\pi}{\mathbf{2\,N_c\,R^2}}$$

$$\mathbf{F(\theta) = 1 - A + 2\,A\cos^2(\theta)} \qquad \mathbf{A = 3/4}$$



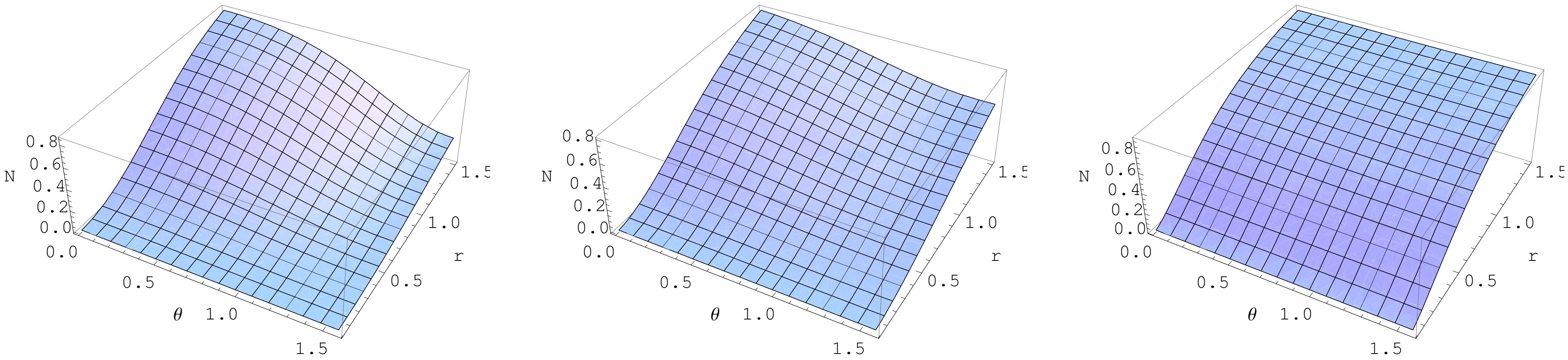


$\mathbf{W}[\delta] = \mathbf{1/2}\pi$, constant for any δ ranging from 0 to 2π.

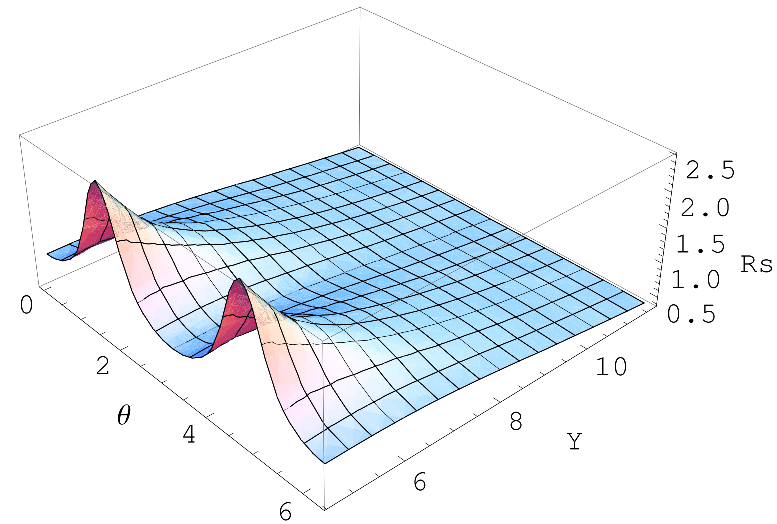
$$\langle \mathbf{F} \rangle_\delta = \int_0^{2\pi} \mathbf{d}\delta \, \mathbf{F}(\theta + \delta) \, \mathbf{W}[\delta] = \mathbf{1}$$

We are interested in the two-dipole correlator $\langle \mathbf{N}(\mathbf{Y}, \mathbf{r_1}, \theta_1, \delta) \, \mathbf{N}(\mathbf{Y}, \mathbf{r_2}, \theta_2, \delta) \rangle_\delta$.

Single configuration solution



the saturation scale $\quad N(Y, R_S, \theta) \;=\; 1/2$

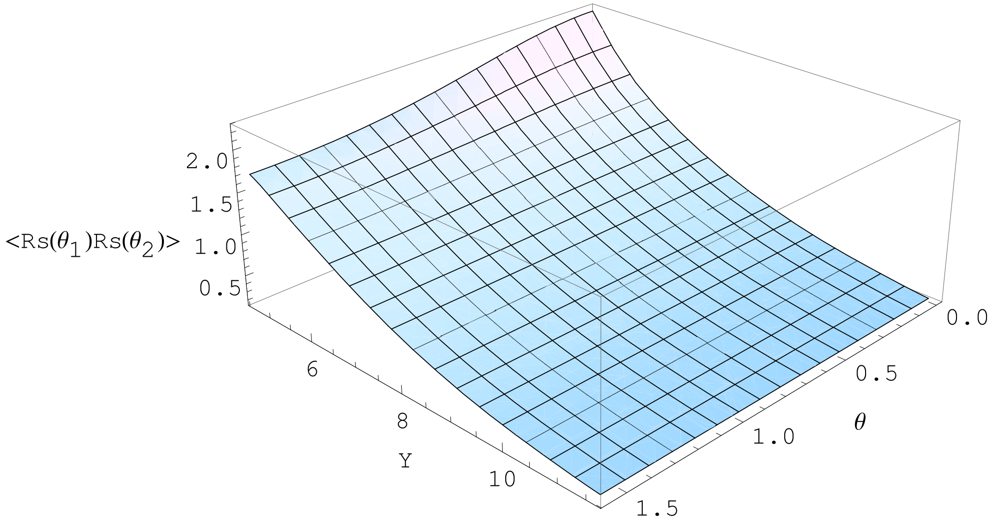


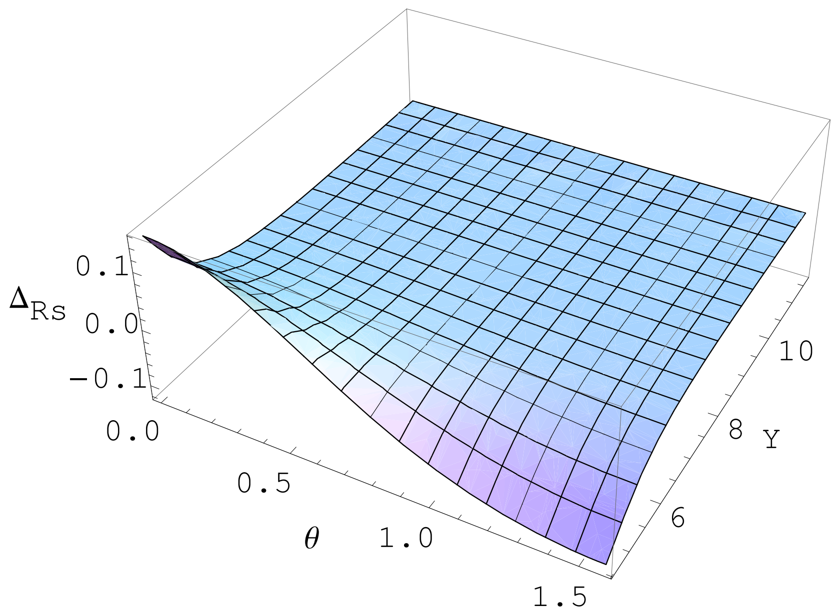
Very fast isotropization!

Angular correlations of the saturation radius

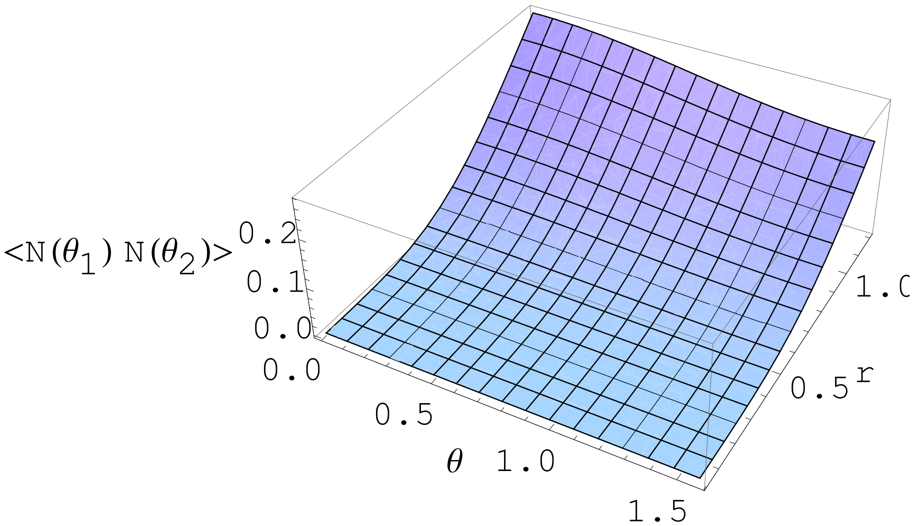
Two quantities of interest: correlator of two saturation scales $\langle \mathbf{R_s}(\theta_1)\mathbf{R_s}(\theta_2)\rangle_\delta$ and

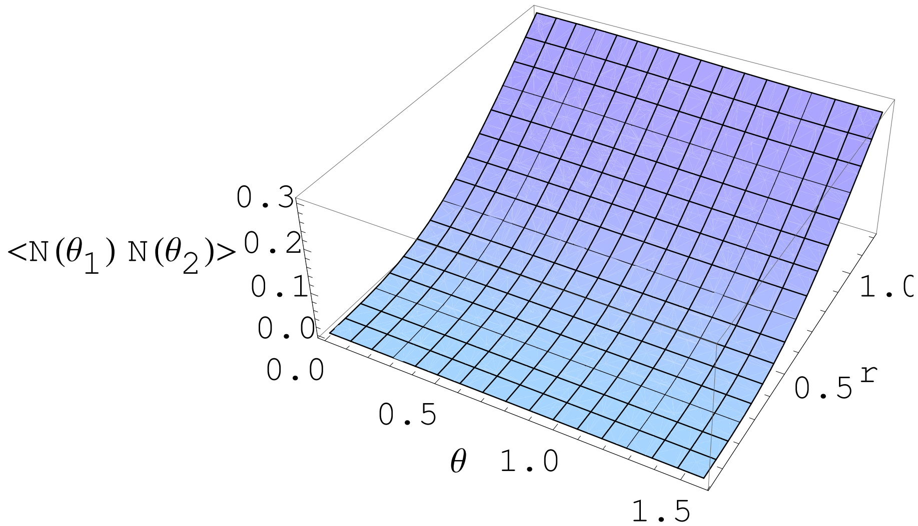
$$\mathbf{\Delta_{R_s}}(\mathbf{Y},\mathbf{r},\theta) \equiv \frac{\langle \mathbf{R_s}(\mathbf{Y},\theta_1,\delta)\,\mathbf{R_s}(\mathbf{Y},\theta_2,\delta)\rangle_\delta - \langle \mathbf{R_s}(\mathbf{Y},\theta_1,\delta)\rangle_\delta\,\langle \mathbf{R_s}(\mathbf{Y},\theta_2,\delta)\rangle_\delta}{\langle \mathbf{R_s}(\mathbf{Y},\theta_1,\delta)\rangle_\delta^{\mathbf{2}}}, \qquad \theta = \theta_1 - \theta_2$$





Angular correlations $\langle \mathbf{N(Y,r,\theta_1)N(Y,r,\theta_2)} \rangle_\delta$





Again fast anizotropization

$$\textbf{Ang. correlations} \sim \mathbf{e^{-\lambda Y}}, \qquad \lambda \simeq \mathbf{0.6}$$

Presumably related to the second BFKL eigenvalue

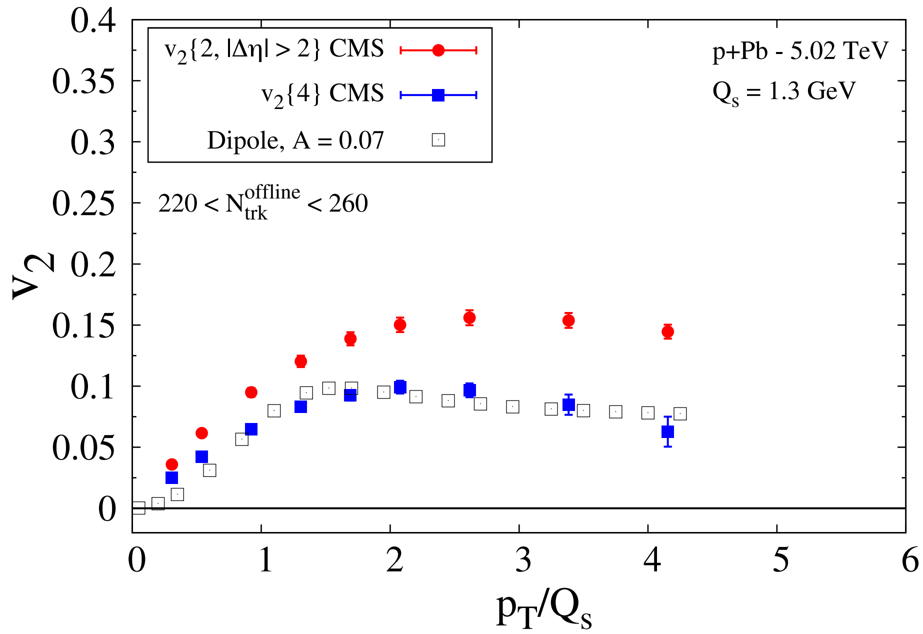
$$\omega_{\mathbf{n=0}} = \mathbf{4} \ln \mathbf{2}\, \bar{\alpha}_{\mathbf{s}}; \qquad \omega_{\mathbf{n=2}} = \mathbf{4}\,(\ln \mathbf{2} - \mathbf{1})\, \bar{\alpha}_{\mathbf{s}}$$

CONCLUSIONS 2

- **Within the "projectile" dipole model, we find an exponentially fast isotropization with the exponent $\lambda_A \simeq 0.6$.**

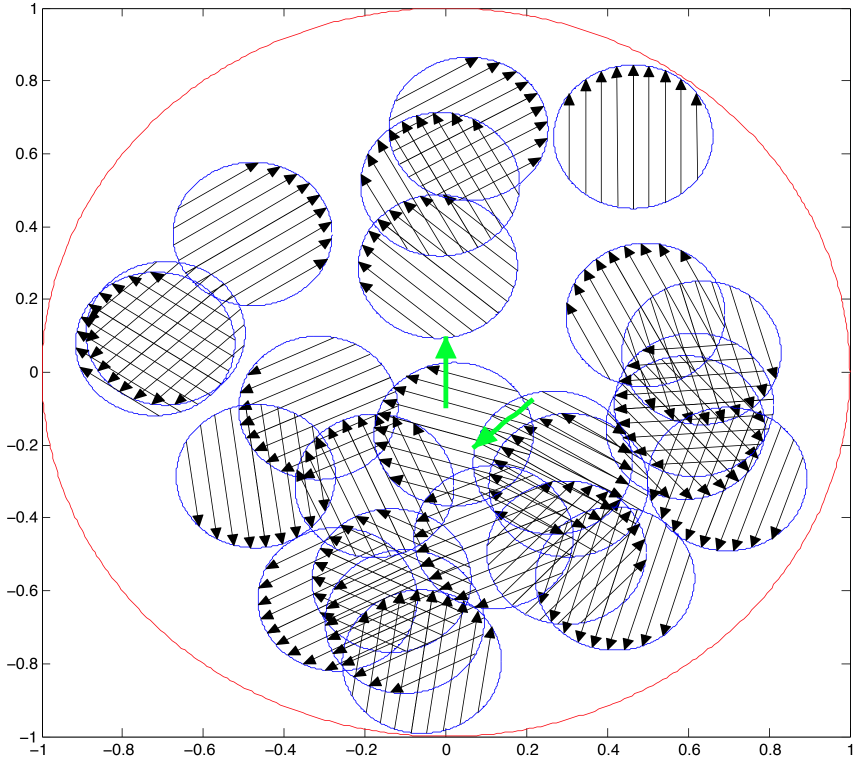
- **Observed correlations must arise dynamically. Those we find in the "target" dipole model. Pomeron loops are needed**

A. Dumitru, A. Giannini, arXiv:1406.5781 [hep-ph]



Work in progress

with Andrej Kormilitzin



$$\mathbf{N}(\mathbf{Y_0}, \vec{\mathbf{r}}, \vec{\mathbf{b}}) \; = \; \mathbf{1} \; - \; \mathbf{Exp}[\, -\, (\vec{\mathbf{r}}\, \vec{\mathbf{E}}(\vec{\mathbf{b}}))^{2}] \, ;$$

$$\vec{\mathbf{E}}(\vec{\mathbf{b}}) \; = \; \sum \; \vec{\mathbf{E}}_0(\vec{\mathbf{b}}) \, \mathbf{e}^{-\, \mathbf{d}^2 \, \mathbf{Q_s^2}} \, ; \qquad \mathbf{E_0} \; = \; \mathbf{Q_s}$$

Towards correlations in symmetric collisions.

T. Altinoluk, A. Kovner, E. Levin, ML, JHEP 1404 (2014) 075

$$\frac{\mathbf{dN}}{\mathbf{d^2pd^2kd}\eta\mathbf{d}\xi} = \langle\sigma(\mathbf{k})\ \sigma(\mathbf{p})\rangle_{\mathbf{P,T}}$$

$$\sigma(\mathbf{k}) = \int_{\mathbf{z},\bar{\mathbf{z}},\mathbf{x_1},\bar{\mathbf{x}}_1} \mathbf{e}^{\mathbf{ik(z-\bar{z})}}\vec{\mathbf{f}}(\bar{\mathbf{z}}-\bar{\mathbf{x}}_1)\cdot\vec{\mathbf{f}}(\mathbf{x_1}-\mathbf{z})\ \left\{\rho(\mathbf{x_1})[\mathbf{S}^\dagger(\mathbf{x_1})-\mathbf{S}^\dagger(\mathbf{z})][\mathbf{S}(\bar{\mathbf{x}}_1)-\mathbf{S}(\mathbf{z})]\rho(\bar{\mathbf{x}}_1)\right\}$$

Identify target Pomeron $P_A^T(x,y) \equiv 1 - \langle S(x)S^\dagger(y)\rangle_T/N_c$

and projectile Pomeron as $P_A^P(x,y) \sim \frac{1}{\nabla^2}(x-\bar{x})\frac{1}{\nabla^2}(y-\bar{y})\ \langle\rho(\bar{x})\rho(\bar{y})\rangle_P$

after color projection algebra and some little massage

$$\frac{d\sigma}{d\eta\, dk^2\, d\xi\, dp^2} \sim \frac{1}{k^2}\frac{1}{p^2}\int_{x,y,u,v} \cos k(x-y)\cos p(u-v)$$
$$\times\left\{\frac{1}{4}\frac{\partial}{\partial(ij\bar{i}\bar{j})}[\bar{P}^T_A(x,y)\bar{P}^T_A(u,v)]\Delta^{ijkl}\Delta^{\bar{i}\bar{j}\bar{k}\bar{l}}\frac{\partial}{\partial(kl\bar{k}\bar{l})}[\bar{P}^P_A(x,y)\bar{P}^P_A(u,v)]\right.$$
$$\left.-\frac{8}{N_c^2}\frac{\partial}{\partial(ij\bar{i}\bar{j})}[\bar{N}^T_{xy}\bar{N}^T_{uv}\bar{Q}^T_{yuvx}]\Delta^{ijkl}\Delta^{\bar{i}\bar{j}\bar{k}\bar{l}}\frac{\partial}{\partial(kl\bar{k}\bar{l})}[\bar{N}^P_{yx}\bar{N}^P_{vu}\bar{Q}^P_{xvuy}]\right\}$$

where we have defined

$$\frac{\partial}{\partial(ijkl)} \equiv \frac{\partial}{\partial x_i}\frac{\partial}{\partial y_j}\frac{\partial}{\partial u_k}\frac{\partial}{\partial v_l}$$

$$\Delta^{ijkl} \equiv \delta^{ij}\delta^{kl} + \delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk}$$

Here $Q^T(yuvx) = tr[S(y)S^\dagger(u)S(v)S^\dagger(y)]$ **(quadrupole/B-Reggeon)**

The expression is manifestly symmetric with respect to target/projectile.