QCD High Energy Evolution: from Basics to NLO

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with ... Misha Lublinsky, Yair Mulian

High Energy Scattering is Interesting

Hadronic cross sections grow with energy

$\sigma_{p-p} \sim s^{.08}$

$\frac{d\sigma^{DIS}}{dQ^2} \sim s^{.2-.3}$

Within perturbative QCD the growth is described by BFKL equation

$\phi(p_T; x_{Bj})$ - TMD, transverse momentum dependent gluon density

$$\frac{d\phi(p_T)}{dY} = \int_{k_T} K_{BFKL}(p_T, k_T)\phi(k_T)$$

$Y = \ln 1/x_{Bj} \propto \ln s$ - rapidity

BFKL - linear homogeneous equation, with solutions

$$\phi(Y) = e^{\lambda_i Y}\phi_0$$

with the maximal eigenvalue $\lambda = \frac{\alpha_s N}{\pi} \ln 2$

Power growth in energy violates Froissart bound.

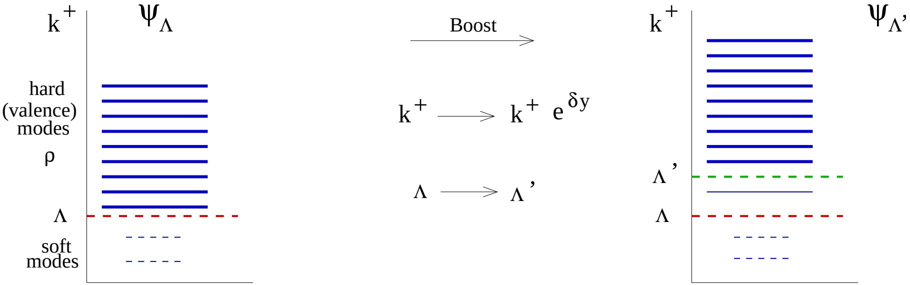
Cross section has to unitarize.

Two distinct effects contribute to the growth of cross section: growth of local gluon density, and growth of transverse size of a hadron.

The maximal BFKL eigenvalue reflects the growth of density.

Growth of transverse size is more subtle - subleading in BFKL, but very stubborn. Cannot be tackled by perturbative methods, and is outside the scope.

Why Does the Gluon Density grow?



Under Boost Longitudinal Momenta grow.

New Gluons Rise From The "Bottomless Pit" which is the **zero mode**.

Color Field becomes strong because of these extra WEIZSACKER-WILLIAMS gluons.

How Does the Gluon Density grow?

The mechanism is very simple.

Hadronic state $|H\rangle$ contains colored partons (mostly guons).

Color charge density j^a comes with non-dynamical longitudinal Coulomb fields.

When boosted **longitudinal** field acquires **transverse** component - LIVE Weiszacker-Williams GLUONS.

$$E^i(r) = \frac{g}{4\pi}\frac{r^i}{|r|^3} \qquad \to E^i = \frac{g}{2\pi}\frac{X^i_\perp}{X^2_\perp}\delta(X^-)$$

How many WW gluons materialize?

$$E^i(k) = i\sqrt{\omega(k)}[a_i(k) - a_i^\dagger(k)] = i\sqrt{k^+}[a_i(k) - a_i^\dagger(k)]$$

Compare:

$$a(k) \sim g\frac{1}{\sqrt{k^+}}\frac{k^i_\perp}{k^2_\perp}; \qquad n(k_\perp) = \int dk^+\langle a_i^\dagger(k)a_i(k)\rangle = \frac{\alpha_s}{k^2_\perp}\int\frac{dk^+}{k^+} = \frac{\alpha_s}{k^2_\perp}Y$$

These gluons also carry color charge density - so the color charge density is increased by the boost.

When boosted again the WW gluons create their own WW field and more gluons...

- AND SO IT GOES...

At very high energies evolution is nonlinear.

At "low" energies evolution is linear: the change in color charge density is proportional to color charge density itself:

$$\delta j(x) \propto j(x)$$

But the gluon density grows - nonlinear effects become important.

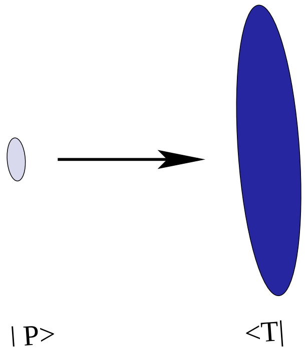
How to describe them and what do they do?

Dense objects - cross sections are not proportional to gluon number. Multiple scatterings are important.

High energy - scattering is eikonal.

The "Paradigm".

Scatter EIKONALLY a "projectile" hadron $|P\rangle$ on a "target" hadron $|T\rangle$ at high energy



$|P\rangle$ - a distribution of color charge density $j^a(x)$.
$|T\rangle$- an ensemble of (possibly strong) color fields $\alpha^a(x)$.

The eikonal S - matrix

Every projectile gluon keeps its transverse position but acquires a color "phase"

$$|x, a\rangle \to S^{ab}(x)|x, b\rangle$$

with

$$S^{ab}(x) = \mathcal{P} \exp\left\{i \int dx^- \; T^a \, \alpha_T^a(x, x^-)\right\}^{ab} .$$

The forward scattering amplitude of $|P\rangle$:

$$\mathcal{S} \; = \langle \mathrm{IN}|\mathrm{OUT}\rangle =< \langle P|\hat{S}|P\rangle >_T$$

$$=< \int dj \; W^P[j] \;\; \exp\left\{i \int \; d^2x \, j_P^a(x) \; \alpha_T^a(x)\right\} >_T$$

$W^P[j]$ is the probability distribution of the projectile color charge density.

The “Hamiltonian” evolution.

Boost the projectile - S-matrix changes, since $W^P[j]$ changes.

The change is due to “materialization” of the soft modes (growth of coherence time of soft fluctuations).

We need to know the “soft gluon” part of the hadronic wave function, to find the change of $S - matrix$.

$$\mathcal{S}_{Y+\Delta Y} = \langle \mathrm{IN}|\mathrm{OUT}\rangle = \langle P_{\mathrm{valence}}|\langle P_{\mathrm{soft}}|\hat{S}|P_{\mathrm{soft}}\rangle|P_{\mathrm{valence}}\rangle$$

Here the “Vacuum” of the soft gluons in the presence of “valence” charge density j:

$$|P_{\mathrm{soft}}\rangle \equiv P_{\mathrm{soft}}[a^\dagger_{soft}; j(x)]|0_{soft}\rangle$$

The phase space of $|P_{\mathrm{soft}}\rangle$ is proportional to ΔY.

Thus we can write

$$\langle P_{\rm soft}|\hat{S}|P_{\rm soft}\rangle = [1 - \mathcal{H}[j, \delta/\delta j]\ \Delta Y + ...]\hat{S}_{valence}$$

Or

$$\mathcal{S}_{Y+\Delta Y} = [1 - \mathcal{H}[j, \delta/\delta j]\ \Delta Y]\mathcal{S}_Y$$

More generally, $\mathcal{H}$ generates a Hamiltonian evolution for any observable which is calculated as average over the probability distribution W:

$$\frac{d}{dY} W^P[j] = -\mathcal{H}[j, \delta/\delta j]\ W^P[j]$$

Hamiltonian Assembly Instructions.

1. Calculate the soft gluon wave function at fixed valence color charge density $P_{soft}[a^{a\dagger}(x), j^a(x)]$.
2. Eikonally propagate $|P_{soft}\rangle$ through the target fields:

$$|IN\rangle = P_{soft}[a^\dagger(x), j(x)] \to |OUT\rangle = P_{soft}[S(x)^{ab} a^{b\dagger}(x), S(x)^{ab} j^a(x)]$$

3. Calculate the soft gluon part of the overlap: $\langle IN|OUT\rangle$
4. Expand to first order in ΔY and extract $\mathcal{H}$.

JIMWLK Hamiltonian: projectile is allowed to be dense $\alpha_s j_P(x) \sim 1$,

but the target is assumed to be dilute $\alpha_T(x) \sim g$.

The eigenfunction P_{soft} is found to all orders in $\alpha_s j$, and to leading order in α_s.

It is a Gaussian in the soft gluon field $a(x)$, $a^\dagger(x)$.

The state $|OUT\rangle$ is expanded to order α_T^2.

This corresponds to expansion of $\mathcal{H}$ to order $(\delta/\delta j)^2$.

Convenient to write in terms of $S_P(x)$

- the eikonal scattering matrix *on the PROJECTILE color field*

- a complicated nonlinear function of $\alpha_s j_P$.

JIMWLK Hamiltonian.

We end up with a 2+1 dimensional Euclidean quantum field theory, with 2 transverse spatial dimensions, and role of time is played by rapidity.

$$H^{JIMWLK} = \frac{\alpha_s}{2\pi^2} \int d^2z \, Q_i^a(z) \, Q_i^a(z)$$

the Hermitian amplitudes $Q_i^a(z)$ are "single inclusive gluon emission amplitude"

$$Q_i^a(z) = \int d^2x \frac{(x-z)_i}{(x-z)^2} [S^{ab}(z) - S^{ab}(x)] J_R^b(x) \, .$$

the generators of color rotation J_R

$$J_R^a(x) = -\mathrm{tr} \left\{ S(x) T^a \frac{\delta}{\delta S^\dagger(x)} \right\}$$

Balitsky hierarchy and JIWMLK evolution.

Like any QFT, Hamiltonian formulation is equivalent to Dysin-Schwinger equations.

E.g. act on a dipole $d(x,y) \equiv \frac{1}{N} tr[S^\dagger(x)S(y)]$

$$\frac{d}{dY} d(x,y) = -H^{JIMWLK} d(x,y)$$

$$= -\frac{\alpha_s N}{\pi} \int d^2z \frac{(x-y)^2}{(x-z)^2 (y-z)^2} \left[d(x,y) - d(x,z)d(z,y)\right]$$

Same for other functions of S

Balitsky hierarchy = Dyson-Schwinger equations of high energy theory.

What's the physics.

Qualitatively one understands dilute and dense limit behavior

Dilute system: (Dipole scattering amplitude)

$$d(x,y) \sim 1 - \frac{1}{2}\left[\int_x^y dx_i b_i(x)\right]^2$$

b_i - WW field is proportional to the color charge density

$$b_i^a \propto \frac{\partial_i}{\partial^2} j^a$$

The number of gluons in the wave function $n \propto \langle j^2 \rangle$.

The number of gluons "emitted" due to boost $\delta n \propto \langle b^2 \rangle \propto \langle j^2 \rangle$.

The Balitsky equation for d linearizes, is equivalent to BFKL - leads to exponential growth of the color charge density.

The (square of the) charge density satisfies linear evolution equation

$$\frac{d\langle j^2 \rangle}{dY} = K \langle j^2 \rangle$$

with exponentially growing BFKL solution

$$\langle j^2(Y) \rangle = e^{KY} \langle j_0^2 \rangle$$

What if the hadron is dense?

There are nonlinear effects in the emission

$$\delta b_i \propto \frac{D_i}{D^2} j, \qquad \text{with} \quad D = \partial - b$$

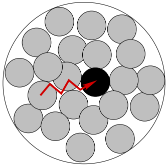
For large j:

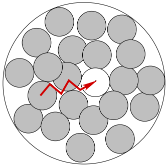
$$\delta b = \frac{b}{b^2} j \quad \sim \quad \text{independent of } j$$

"Bleaching of Color"

Gluons are mostly emitted not into empty space, but on top of other gluons.

The color Casimir at this point will increase or decrease with equal probability.





The two effects together make the color charge density random walk!

$$j^2(Y) \sim j_0^2 + MY$$

Saturation: GROWTH MUCH SLOWER THAN EXPONENTIAL!

Now what does it mean "weak field" or "strong field?

The field is dimensional with dimension of momentum.

In fact the relevant dimensionless scale is precisely gb/Q.

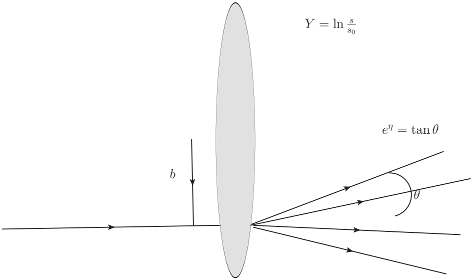
At transverse momentum Q larger than typical value of the field, the field is weak and linear evolution prevails.

At momenta Q smaller than the field gb the random walk rules.

The boundary between the two regimes defines the special transverse momentum scale Q_s - saturation momentum.

Q_s - **typical "gluon density" in the wave function. Also "typical momentum" of gluons in the wave function. Also the only dimensional parameter that determines the bulk of physical properties of the nucleus.**

Q_s is versatile

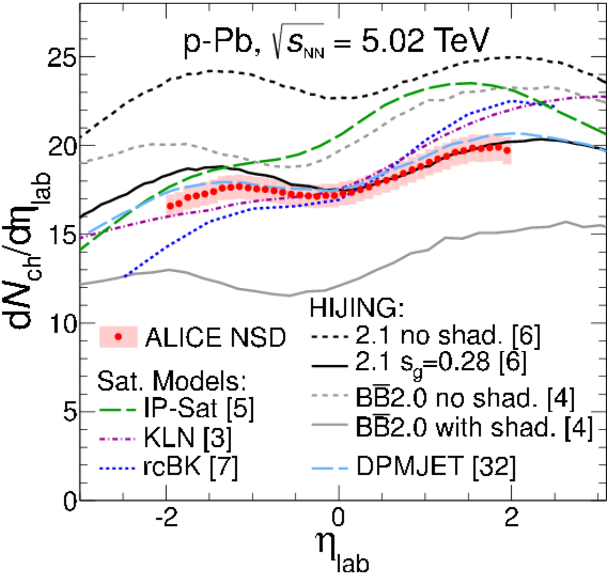


$Q_s^2 = Q_{s0}^2(b)e^{\#\alpha_s(Y+\eta)}$

Y - dependence on the energy of the process

η - dependence on the rapidity in the final state

$Q_{s0}^2(b) \propto S(b)$ - dependence on the centrality of the collision



Kharzeev-Levin-Nardi "KLN" model prediction for particle multiplicity in $p - Pb$ collisions at LHC.

THE "RIDGE" IN p-p AND p-A

First observed by CMS in p-p, followed by ALICE, ATLAS, CMS IN p-Pb- two particle correlations, long range in rapidity and peaked in the same azymuthal direction - "Ridge"

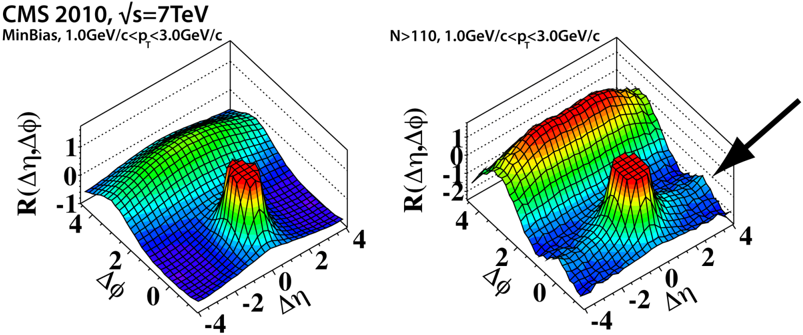
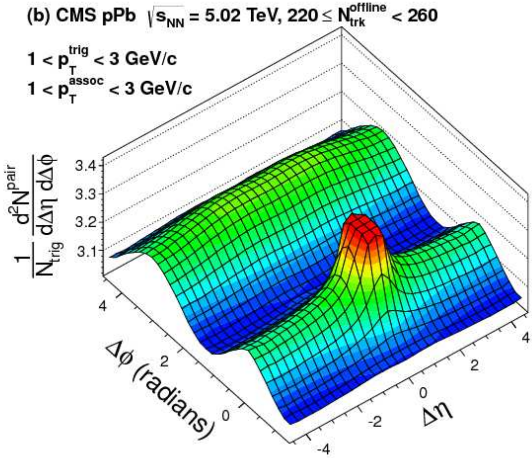


Figure: THE CMS p-p RIDGE-ONLY IN HIGH MULTIPLICITY EVENTS $\sim 10^{-5}$.



The CMS p-Pb ridge - similar at ALICE, ATLAS.

Detailed saturation based fits by Dusling-Venugopalan.

Q_s sets the scale of transverse momenta at which the correlations exist.

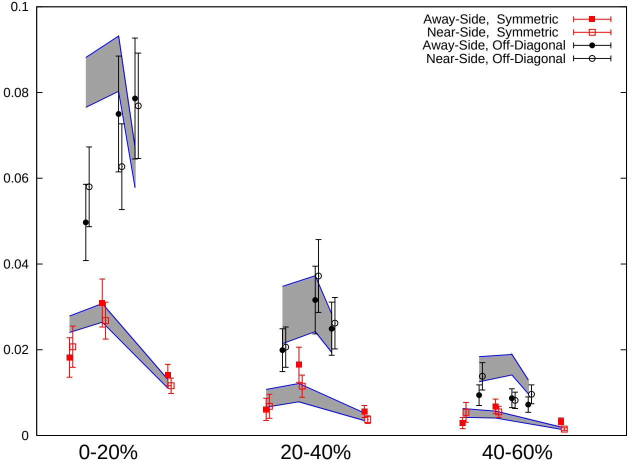


Figure: Dusling-Venugopalan CGC fit to ALICE Ridge data.

Two more interesting applications:

1. Suppression of particle production at forward rapidity in $d-Au$ collisions at RHIC: utilizes atomic number dependence $Q^2_{s0} \propto A^{1/3}$.

2. Decorelation in di-hadron production $d-Au$ collisions at RHIC: outgoing partons get "kicked" when passing through the target with Q_s, and get decorrelated in transverse plane.

Does that mean we have observed QCD saturation?

Unfortunately not.

1. A lot of it is modelling

2. "First principles" leading order calculation gives Q_s^2 growing too fast with energy.

3. NLO is not under control yet. Some corrections are taken into account via running coupling constant (slows down the growth of Q_s). But no fully consistent NLO calculations yet.

We need honest NLO calculations to have reliable QCD saturation predictions.

Next to leading order.

We know that next to leading order corrections are large.

It is technically more challenging to derive NLO JIMWLK, although it can be done and is being done by Misha Lublinsky and Yair Mulian.

But there is a shortcut, called Ian Balitsky.

Balitsky-Chirilli calculated NLO evolution of a dipole. Subsequently Grabovsky calculated some elements of the evolution of a "baryon" in $SU(3)$.

It turns out that these results are (almost) enough to write down the full NLO JIMWLK kernel by inspection!

For $N = 4$ SUSY (like QCD, but a little bit simpler)

$$H^{NLO\ JIMWLK} = \int_{x,y} K_{2,0}(x,y)\ [J_L^a(x)\, J_L^a(y)\ +\ J_R^a(x)\, J_R^a(y)]$$
$$-\,2\int_{x,y,z} K_{2,1}(x,y,z)\, J_L^a(x)\, S_A^{ab}(z)\, J_R^b(y)$$
$$+\int_{x,y,z,z'} K_{2,2}(x,y;z,z')\left[f^{abc}\, f^{def}\, J_L^a(x)\, S_A^{be}(z)\, S_A^{cf}(z')\, J_R^d(y)\ -\ N_c\, J_L^a(x)\, S_A^{ab}(z)\, J_R^b(y)\right]$$
$$+\int_{w,x,y,z,z'} K_{3,2}(w;x,y;z,z') f^{acb}\left[J_L^d(x)\, J_L^e(y)\, S_A^{dc}(z)\, S_A^{eb}(z')\, J_R^a(w)\right.$$
$$\left.-\ J_L^a(w)\, S_A^{cd}(z)\, S_A^{be}(z')\, J_R^d(x)\, J_R^e(y)\right]$$
$$+\int_{w,x,y,z} K_{3,1}(w;x,y;z)\, f^{bde}\left[J_L^d(x)\, J_L^e(y)\, S_A^{ba}(z)\, J_R^a(w)\ -\ J_L^a(w)\, S_A^{ab}(z)\, J_R^d(x)\, J_R^e(y)\right]$$
$$+\int_{w,x,y} K_{3,0}(w,x,y) f^{bde}\left[J_L^d(x)\, J_L^e(y)\, J_L^b(w)\ -\ J_R^d(x)\, J_R^e(y)\, J_R^b(w)\right]\,.$$

$$K_{2,2}(x,y;z,z') = \frac{\alpha_s^2}{16\,\pi^4}\Bigg[\frac{(x-y)^2}{X^2Y'^2(z-z')^2}\Big(1+\frac{(x-y)^2(z-z')^2}{X^2Y'^2-X'^2Y^2}\Big) -$$

$$-\frac{(x-y)^2}{X'^2Y^2(z-z')^2}\Big(1+\frac{(x-y)^2(z-z')^2}{X'^2Y^2-X^2Y'^2}\Big)\Bigg]\ \ln\frac{X^2Y'^2}{X'^2Y^2}$$

$$K_{2,1}(x,y,z) = \frac{\alpha_s^2 N_c}{48\pi}\frac{(x-y)^2}{X^2Y^2}K_{2,1}(x,y;z)$$

$$K_{2,0}(x,y) = \frac{\alpha^2 N_c}{16\pi^3}\int_z \frac{(x-y)^2}{X^2Y^2}\Big[\frac{\pi^2}{3}+2\ln\frac{Y^2}{(x-y)^2}\ \ln\frac{X^2}{(x-y)^2}\Big]$$

$$K_{3,2}(w;x,y;z,z') =$$

$$= \frac{i}{2}\Big[M_{x,y,z}M_{y,z,z'}+M_{x,w,z}M_{y,w,z'}-M_{y,w,z'}M_{x,z',z}-M_{x,w,z}M_{y,z,z'}\Big]\ln\frac{W^2}{W'^2}$$

$$K_{3,1}(w;x,y;z) = \int_{z'}\big[K_{3,2}(y;w,x;z,z')-K_{3,2}(x;w,y;z,z')\big]$$

$$K_{3,0}(w,x,y) = -\frac{1}{3}\Big[\int_{z,z'}K_{3,2}(w,x,y;z,z') + \int_z K_{3,1}(w,x,y;z)\Big]$$

with

$$X\equiv x-z;\ \ X'\equiv x-z';\text{etc.},\quad \text{and}\ M(x,y,z)\equiv\frac{\alpha_s}{2\,\pi^2}\,\frac{(x-y)^2}{X^2\,Y^2}$$

The conformal puzzle.

$N = 4$ SUSY is conformally invariant. So is QCD at the tree level.

So is LO JIMWLK.

But NLO JIMWLK is apparently not!

Not totally suprising: NLO JIMWLK was derived with sharp rapidity cutoff, wich is not conformally invariant.

Balitsky, Chirilli - evolution of a dipole at NLO is not invariant, but can define a "conformal dipole" which does satisfy conformally invariant equation. Is it general? How to redefine other operators?

With operatorial NLO JIMWLK we can resolve these questions.

Is NLO JIMWLK really noninvariant?

Effective theory is obtained by integrating some degrees of freedom:

$$\mathcal{L}(\alpha,\beta) \to \mathcal{L}'(\beta)$$

Suppose $\mathcal{L}$ was symmetric under

$$\alpha \to \alpha + g(\alpha,\beta); \qquad \beta \to \beta + f(\alpha,\beta)$$

Even though α mixes with β in the transformation, integrating out α does not destroy the symmetry in $\mathcal{L}'$, but modifies it

$$f(\alpha,\beta) \to f'(\beta) \neq f(\alpha = 0,\beta)$$

Noninvariant cutoff is similar - the "fast" and "slow" degrees of freedom mix under conformal transformation.

Can we find a modified conformal transformation, which is an exact symmetry of H^{NLO}_{JIMWLK}?

Modified inversion symmetry.

Naive inversion transformation ($x_\pm = x_1 \pm ix_2$):

$$\mathcal{I}_0 : S(x_+, x_-) \to S(1/x_-, 1/x_+);\ J_{L,R}(x_+, x_-) \to \frac{1}{x_+ x_-} J_{L,R}(1/x_-, 1/x_+)$$

$$\mathcal{I}_0 H^{NLO}_{JIMWLK} \mathcal{I}_0 = H^{NLO}_{JIMWLK} + O(\alpha_s)$$

But it is easy to check that to order α_s there is invariance under

$$\mathcal{I} = \mathcal{I}_0 \left[1 - \frac{1}{2} \int M_{xyz} \ln\left(\frac{z^2}{a^2}\right) [J^a_L(x) J^a_L(y) + J^a_R(x) J^a_R(y) \right.$$

$$\left. -2 J^a_L(x) S^{ab}_A(z) J^b_R(y)] \right]$$

So there is no conformal puzzle as such: H^{NLO}_{JIMWLK} is indeed conformally invariant.

MORE TO DO AND HOPEFULLY MORE TO COME...