Low-Frequency Nonlinear Stationary Waves and Fast Shocks: Hydrodynamical Description

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Stationary one-dimensional nonlinear waves in two-fluid hydrodynamics are studied analytically in the assumption of polytropic pressure and massless electrons. Particular attention is paid to the presence of soliton solutions, which exist when the asymptotic plasma velocity is in the range $v_+^2 < v^2 < v_F^2$ or $v_{SL}^2 < v^2 < v_-^2$, where v_{SL}, v_I, v_F, and v_S are the slow, intermediate, fast and sound speeds, respectively, and $v_- = \min(v_S, v_I)$, $v_+ = \max(v_S, v_I)$. General nonlinear solution is derived in the parametric representation. Inclusion of weak dissipation changes qualitatively the behavior of solutions allowing for fast shock-like solutions. Generalized expression for the whistler precursor wavelength is derived.

I. INTRODUCTION

Nonlinear low-frequency hydrodynamical waves are of substantial interest, in particular, due to their role in the collisionless shock formation and generic relation to the structure of these shocks. "Low-frequency" in the present context means that the typical frequencies and wavelengths correspond to the range $\omega \ll \Omega_e$, where $\Omega_e = eB/m_e c$ is the electron gyrofrequency. This range includes both magnetosonic and whistler modes Nonlinear fast magnetosonic waves steepen into shocks (see, e.g., Ref. 1 and references therein), and are coupled to whistlers, which are part of the shock structure [2–4] or result from nonlinear dynamics of the shock front [5, 6]. Alternatively, shocks can be considered as formed from weakly damped solitons [7]. It should be noted that throughout the paper the term "soliton" is used in its non-rigorous usage, as a synonym for a stationary one-dimensional localized structure which tends to the same asymptotic state at both spatial infinities (solitary wave), in the spirit of Ref. 7. Since no temporal evolution is considered, the integrability issues are not relevant here.

One-dimensional nonlinear hydrodynamical waves have been thoroughly investigated in various limits (see, e.g., [8–13]), mainly within the mode separation approach (for review see [8, 13]). Particular attention was paid to the existence of soliton solutions. However, mode separation does not seem appropriate for the description of the one-dimensional stationary nonlinear wave of shock structure, which covers spatial scale range from typical MHD scales $\gg c/\omega_{pi}$ (where $\omega_{pi} = (4\pi n e^2/m_i)^{1/2}$) down to much smaller scales $\ll c/\omega_{pi}$, typical for the whistler [3, 4]. Therefore, a shock front itself should be considered as a large amplitude nonlinear wave, incorporating in it properties of several modes.

Two-fluid hydrodynamics allows to extend the description onto scales in the typical whistler range, without extra complications due to the kinetic effects, although in the same time it leaves aside many features typical for high-Mach number shocks, like anisotropy and non-gyrotropy of ion distributions (see a review in Ref. 14). Still two-fluid hydrodynamics is useful in the case of relatively low Mach numbers and may provide some guidelines even for higher-Mach number regimes. Previous studies (see [12, 13] and references therein) treated nonlinear stationary one-dimensional waves (NS1D) semi-quantitatively, using qualitative analysis of the corresponding pseudopotential. In the present paper we complete the analytical study of NS1D in the framework of two-fluid hydrodynamics for a hot plasma, assuming the polytropic form for the state equation. The objective of the present short paper is to present the results of the analysis of the features of these low-frequency stationary nonlinear waves, including the range of parameters when solitons exist. The paper is organized as follows. In section II we derive the basic equations in the dimensionless form. In section III we analyze the stationary points of the above equations, paying particular attention to the conditions for the existence of soliton solutions. In section IV we analytically derive the general nonlinear wave solution in the parametric form. In section V we consider the effects of weak dissipation and make some conclusions for the shock-like solutions, including the generalization for the whistler precursor wavelength.

II. BASIC EQUATIONS

We start with the one-dimensional stationary two-fluid hydrodynamical equations, with $\partial/\partial t = \partial/\partial y = \partial/\partial z = 0$, assuming quasineutrality $n_e = n_i = n$ and neglecting the electron mass $m_e = 0$. We also assume that the electron and ion pressures are isotropic and depend only on the density n. With these assumptions the equations take the following form [15]:

$$m_i v\frac{dv}{dx} = eE_x + \frac{e}{c}\hat{\mathbf{n}}\cdot(\mathbf{U}_i\times\mathbf{B}_\perp) - \frac{1}{n}\frac{dp_i}{dx}, \tag{1}$$

$$0 = -eE_x - \frac{e}{c}\hat{\mathbf{n}}\cdot(\mathbf{U}_e\times\mathbf{B}_\perp) - \frac{1}{n}\frac{dp_e}{dx}, \tag{2}$$

$$m_i v\frac{d\mathbf{U}_i}{dx} = e\mathbf{E}_\perp + \frac{e}{c}v\hat{\mathbf{n}}\times\mathbf{B}_\perp + \frac{e}{c}B_x\mathbf{U}_i\times\hat{\mathbf{n}}, \tag{3}$$

$$0 = -e\mathbf{E}_\perp - \frac{e}{c}v\hat{\mathbf{n}}\times\mathbf{B}_\perp - \frac{e}{c}B_x\mathbf{U}_e\times\hat{\mathbf{n}}, \tag{4}$$

$$\hat{\mathbf{n}}\times\frac{d\mathbf{B}_\perp}{dx} = \frac{4\pi}{c}ne(\mathbf{U}_i-\mathbf{U}_e), \tag{5}$$

$$nv = J = \text{const}, \tag{6}$$

where $\hat{\mathbf{n}}$ is the unity vector along x axis, $\mathbf{B}_\perp \perp \hat{\mathbf{n}}$, $\mathbf{U} \perp \hat{\mathbf{n}}$, $\mathbf{E}_\perp \perp \hat{\mathbf{n}}$, and $\mathbf{E}_\perp = \text{const}$.

Straightforward calculations give

$$\mathbf{U}_i = \frac{B_x}{4\pi n v m_i}(\mathbf{B}_\perp - \mathbf{B}_{\perp 0}), \tag{7}$$

$$\mathbf{U}_e = \frac{B_x}{4\pi n v m_i}(\mathbf{B}_\perp - \mathbf{B}_{\perp 0}) - \frac{c}{4\pi e n}\hat{\mathbf{n}}\times\frac{d\mathbf{B}_\perp}{dx}, \tag{8}$$

where $\mathbf{B}_{\perp 0}$ is the integration constant. After substitution of Eq. (8) into Eq. (4) one has

$$\begin{aligned}&\frac{B_x}{4\pi n}\frac{d\mathbf{B}_\perp}{dx} - \left(\frac{e}{c}v - \frac{eB_x^2}{4\pi n v m_i}\right)\hat{\mathbf{n}}\times\mathbf{B}_\perp \\ &= e\mathbf{E}_\perp + \frac{eB_x^2}{4\pi n v m_i}\hat{\mathbf{n}}\times\mathbf{B}_{\perp 0} \equiv \mathbf{G} = \text{const}.\end{aligned} \tag{9}$$

These equations are accompanied by the pressure balance equation

$$nm_i v^2 + p_e + p_i + \frac{\mathbf{B}_\perp^2}{8\pi} = P = \text{const}, \tag{10}$$

which is obtained by summing up Eq. (1) and (2) and nothing but the momentum conservation (constancy of the momentum flux).

It is convenient and instructive to write the equations in the dimensionless form. To do so we choose a reference point, where $n = n_0$, $v = v_0$, and $\mathbf{B} = \mathbf{B}_0$. Using the freedom in the choice of a reference frame, we require that $\mathbf{U}_i = 0$ in the reference point, so that $\mathbf{B}_0 = (B_x, \mathbf{B}_{\perp 0})$, $B_x = B_0\cos\theta$, $|\mathbf{B}_{\perp 0}| = B_0\sin\theta$. We shall also use the freedom in the coordinate choice to choose z axis along $\mathbf{G}$. Now, normalizing the variables as follows:

$$\frac{n}{n_0} = N, \quad \frac{\mathbf{B}_\perp}{B_0} = (b_y, b_z), \tag{11}$$

and introducing the local Alfven velocity in the reference point $v_A^2 = B_0^2/4\pi n_0 m_i$ one arrives at the following equations:

$$l_W\frac{db_z}{dx} = (1-N\chi)b_y, \tag{12}$$

$$l_W\frac{db_y}{dx} = \mu N\sin\theta(1-\chi) - (1-N\chi)b_z, \tag{13}$$

where we normalized the reference point plasma velocity with the Alfven velocity $V = v_0/v_A$ (this is in fact the local flow Alfven Mach number in the reference point), and $\mu = \text{const}$ and $l_W = c\cos\theta v_0/V^2\Omega_0$, $\Omega_0 = eB_0/m_i c$. We also denoted $\chi = \cos^2\theta/V^2$ for convenience and compactness of writing.

The pressure balance equation takes the following dimensionless form:

$$\frac{2V^2}{N} + \beta p(N) + \mathbf{b}^2 = 2V^2 + \beta + \sin^2\theta, \tag{14}$$

where $\beta = 8\pi(p_e(n_0) + p_i(n_0))/B_0^2$ and $p(N) = (p_e + p_i)/(p_e(n_0) + p_i(n_0))$. In what follows we shall make the simplifying assumption $p(N) = N^\Gamma$.

It is worth noting that if the solution tends to the asymptotically homogeneous state, one can choose the reference point there, and $\mu = 1$.

III. STATIONARY POINT ANALYSIS

The global behavior of the solutions of Eqs. (12)-(14) is determined by the location and character of the stationary points, in which $db_y/dx = db_z/dx = 0$. In such point $b_y = 0$ and

$$b_z = \mu N \sin\theta \frac{1-\chi}{1-N\chi}. \tag{15}$$

If there is no stationary point on a solution, the solution describes a periodic nonlinear wave (or diverges, in which case we say that there is no solution). Any soliton solution starts and ends in the same stationary point, while a shock-like solution goes from one stationary point to another.

Let as assume that a stationary point *does* belong to the solution. In this case we *always* may choose our reference point in this stationary point, that is, $N = 1$, $\mathbf{b} = (0, \sin\theta)$ should satisfy both (15) and (14), so that $\mu = 1$. This point will be hereforth referred to as *upstream point*. Analyzing perturbations near the upstream point $b_y = \delta b_y$, $b_z = \sin\theta + \delta b_z$, and $N = 1 + \delta N$, and assuming $\delta b_y, \delta b_z, \delta N \propto \exp(\kappa x)$, one easily finds

$$\kappa^2 l_W^2 = -\frac{(V^2 - v_{SL}^2)(V^2 - v_I^2)(V^2 - v_F^2)}{V^4(V^2 - v_S^2)}, \tag{16}$$

where $v_S = (\beta/2)(dp/dN)^{1/2}_{|N=1} = (\Gamma\beta/2)^{1/2}$ is the normalized local sound velocity, and v_{SL}, v_I, and v_F are normalized local slow, intermediate, and fast velocities, respectively, in the upstream point:

$$v_I^2 = \cos^2\theta, \tag{17}$$

$$v_{SL}^2 = \tfrac{1}{2}[(v_S^2 + 1) - \sqrt{(v_S^2+1)^2 - 4v_S^2 v_I^2}], \tag{18}$$

$$v_F^2 = \tfrac{1}{2}[(v_S^2 + 1) + \sqrt{(v_S^2+1)^2 - 4v_S^2 v_I^2}]. \tag{19}$$

The above expression (16) shows that the upstream point is a center if $V > v_F$ or $v_- < V < v_+$ or $V < v_{SL}$, and it is a saddle if $v_+ < V < v_F$ or $v_{SL} < V < v_-$, where $v_- = \min(v_I, v_S)$ and $v_+ = \max(v_I, v_S)$.

If a stationary point is a center, it is an isolated point, that is, it itself represents the trivial constant solution and does not belong to any other solution. On the other hand, if a stationary point is a saddle point, there exists a solution which tends to the stationary point at either $x \to \infty$ or $x \to -\infty$, that is, the solution is asymptotically homogeneous. Absence of the asymptotically homogeneous state automatically implies absence of a soliton solution. From the above relations one can immediately conclude that fast ($V > v_F$) magnetosonic soliton does not exist. Since the pressure is expected to be almost isotropic near the asymptotically homogeneous state, the above conclusion that there are no fast magnetosonic solitons in the non-dissipative two-fluid hydrodynamics, is quite general. Therefore, the suggested in 7 "shock from a soliton" scenario does not work in the oblique case.

It is easy to see, that, with the substitution $\kappa = ik$, $V = v_{ph} = \omega/k$, Eq. (16) is nothing but the dispersion relation for small-amplitude fast, intermediate, and slow waves [13], and the ranges of soliton existence coincide with the ranges where the linear mode dispersion is negative. This suggests simple qualitative explanation for the soliton absence when $\kappa^2 < 0$. Indeed, let us assume, for example, that a fast soliton forms in a plasma with the asymptotic velocity $V > v_F$. From the point of view of the initial value problem this soliton should be considered as a spontaneous emitter of small-amplitude fast waves, propagating in both directions (upstream and downstream). The wave, propagating downstream, is convected by the plasma flow and leaves eventually the system, leaving the stationary soliton alone. The wave, propagating upstream, will be convected by the plasma flow, if its velocity is less than the plasma velocity, but will stand in the flow, thus breaking the upstream soliton asymptotics, if its velocity can be equal to the flow velocity. If the fast wave dispersion were negative, all small-amplitude waves would have phase velocities $v_{ph} < v_F < V$ and would be convected downstream and further out of the system, leaving again the stationary soliton alone. However, Eq. (16) shows that the fast wave is positively dispersive, and its phase velocity is not limited from above. Thus, for each given $V > v_F$ a small-amplitude wave exists, whose phase velocity $v_{ph} = V$. In this case the soliton would have an option to break spontaneously into two stationary structures, one of which is a phasestanding monochromatic wave. We will not go into further detail of this qualitative argument, mentioning only that it is similar to the well known evolutionarity consideration for MHD discontinuities [16].

Thus, solitons in the non-dissipative two-fluid hydrodynamics with polytropic pressure exist if their asymptotic velocity is in the range $v_{SL}, V < v_-$ or $v_+ < V < v_F$.

However, nothing forbids for a solution with $V > v_F$ to pass infinitesimally close to the upstream point. Let us analyze whether such solution can start (end) in another stationary point, which we hereafter call *downstream point*. If the downstream point exists, the following relations should be satisfied: $b_y = 0$,

$$b_z = \frac{N \sin\theta(1-\chi)}{1-N\chi}, \tag{20}$$

$$P(N) = \frac{2V^2}{N} + \beta N^\Gamma + [\frac{N\sin\theta(1-\chi)}{1-N\chi}]^2 \\ = 2V^2 + \beta + \sin^2\theta, \tag{21}$$

where we assume $N > 1$, without loss of generality. Since $d^2P(N)/dN^2 > 0$ and $P(N) \to +\infty$ when $N = 1/\chi$, it is obvious that the necessary and sufficient condition that the second stationary point exists is

$$\frac{dP}{dN}_{|N=1} = -2\frac{(V^2 - v_F^2)(V^2 - v_{SL}^2)}{V^2 - v_I^2} < 0. \tag{22}$$

Thus, a solution with $V > v_F$ can pass infinitesimally close to the upstream point and end in the downstream point. If the upstream point becomes unstable because of other factors, not taken into account in the current approximation (see below), the solution becomes a shock-like solution.

It is obvious that the downstream point should be a saddle, and the locally defined normalized (with the values of the magnetic field and density in the downstream point) velocity should satisfy the same inequalities as above.

To illustrate the last statement let us consider the simplest limit $\beta \to 0$, $\cos\theta \to 0$. Let us denoting the density in the downstream point by N_m and the magnetic field $b_y = 0$, $b_z = b_m = N_m$. Analyzing small perturbations $\delta N, \delta\mathbf{b} \propto \exp(\kappa_m x)$, one finds the general expression for corresponding κ_m in the following form:

$$\kappa_m^2 l_W^2 = -\left(\frac{V^2 - N_m v_I^2}{V^2}\right)\left[\frac{V^2 - N_m v_I^2}{V^2} \right. \\ \left. - \frac{\sin^2\theta N_m^3 (V^2 - v_I^2)^2}{(V^2 - N_m v_I^2)^2 (V^2 - v_S^2 N_m^{\Gamma+1})}\right]. \tag{23}$$

In the above defined limit $b_m = N_m$,

$$N_m = \frac{1}{2}[(8V^2 + 1)^{1/2} - 1], \tag{24}$$

and (23) takes the form

$$\kappa_m^2 l_W^2 = \frac{N_m^3}{V^2} - 1. \tag{25}$$

Taking into account the mass conservation $NV = const$ and $v_A^2 \propto b^2/N$, one finds that $V^2/N_m^2 = V_m^2/v_{Am}^2$, as was expected. One can see also from (24) and (25) that the downstream point is a saddle when $V > 1$, that is in the fast range.

IV. GENERAL SOLUTION

The equations (12)-(14) were usually treated qualitatively (see, for example, [12, 13]), using graphical examination of the corresponding pseudopotential. However, it is not too difficult to find the general solution. From Eqs. (12)-(13) one immediately has

$$\frac{db_y}{db_z} = \frac{\mu N \sin\theta(1-\chi) - (1 - N\chi)b_z}{(1-N\chi)b_y}, \tag{26}$$

which can be easily transformed further in the following equation:

$$\frac{db_z}{db^2} = \frac{1 - N\chi}{2\mu N\sin\theta(1-\chi)}. \tag{27}$$

Using Eq. (14) with the polytropic pressure $P(N) = \beta N^\Gamma$ for the relation between b^2 and N , one finally obtains

$$\frac{db_z}{dN} = \frac{1 - N\chi}{2\mu N \sin\theta(1-\chi)}(\frac{2V^2}{N^2} - \Gamma\beta N^{\Gamma-1}). \tag{28}$$

Eq. (28) is easily integrated to

$$b_z = \sin\theta + \frac{V^2}{2\mu\sin\theta(1-\chi)}(1 - \frac{1}{N^2}) - \frac{\Gamma\beta}{2\mu\sin\theta(1-\chi)(\Gamma-1)}(N^{\Gamma-1} - 1) \\ + \frac{\cos^2\theta}{\mu\sin\theta(1-\chi)}(\frac{1}{N} - 1) + \frac{\beta\cos^2\theta}{2\mu\sin\theta(1-\chi)V^2}(N^\Gamma - 1), \tag{29}$$

where the boundary conditions $b_z = \sin\theta$, $b_y = 0$, at $N = 1$, are already taken into account. Eq. (29) provides the parametric representation for $b_z(N)$. It should be completed with the equation for b_y:

$$b_y = \pm(2V^2(1-1/N) + \beta(1-N^\Gamma) + \sin^2\theta - b_z^2)^{1/2}. \tag{30}$$

Obviously, the solution is limited by the conditions $N > 0$ and $b_y^2 > 0$, which gives

$$|b_z| \leq \sqrt{2V^2(1-1/N) + \beta(1-N^\Gamma)}. \tag{31}$$

The points, where $b_y = 0$ are "reflection points", since there db_z/dx changes its sign. The solution is a periodic nonlinear wave or a soliton depending on the choice of the parameters V and μ. The spatial dependence can be found from

$$x = l_W \int b_y^{-1} \frac{db_z}{dN} dN, \tag{32}$$

although the integration cannot be carried out explicitly in general case. This general solution in the parametric form completes the studies of NS1D in polytropic two-fluid hydrodynamics with massless electrons.

In the following figures we show the phase portraits and spatial profiles of the nonlinear waves, which are obtained using the above derived expressions. For this graphical presentation the parameters were chosen as follows: $\beta = 0.3$, $\Gamma = 5/3$, and $\theta = 75°$, which corresponds to the following set of characteristic velocities:$v_{SL} = 0.1164$, $v_I = 0.2588$, $v_S = 0.5$, and $v_F = 1.112$. In Fig. 1 we present the phase portraits (b_z, b_y) for a number of soliton solutions, when $\mu = 1$ and $v_S < V < v_F$. Only the $b_y > 0$ half of the phase portrait is shown. The solitons are rarefactive, both density and magnetic fields are minimum in the central part of the profile. The amplitude of the soliton increases with the decrease of its velocity (Mach number) V. It is worth mentioning that b_z changes sign for sufficiently low V. For the velocities near v_F the noncoplanar magnetic field b_y always remains substantially less than the main component. For lower velocities the two components become comparable.

Fig. 2 presents the spatial profiles $b_z(x)$ for these soliton solutions. Again only half of the profile is shown. As could be expected the solitons become wider with the increase of their velocity and decrease of the amplitude. However, nearly in all of them b_z decreases from its maximum value to the asymptotic value on the scale of about $(4-5)l_W$.

In Fig. 3 we show the phase portraits for nonlinear periodic fast waves. In this case $\mu \neq 1$, so that a specific $V = 2 > v_F$ is chosen, while μ was varied. The part of the figure to the left from $b_z = \sin\theta$ corresponds to the same solutions that were shown earlier, only the parameters are different. The part to the right corresponds to the periodic nonlinear waves. Portraits with cusps are not artificial. They appear because of the double-valued character of the function $N(b^2)$ as defined by Eq. 14. The cusps correspond to the transition from one branch to another (cf. 13). In the waves with relatively small amplitudes b_y remains small, while with the increase of the amplitude the two magnetic field components become comparable.

Fig. 4 shows the profiles of these periodic nonlinear waves (only half-wavelength is shown). For the relative amplitude of about 0.5 the wave half-wavelength is about $4l_W$. For higher amplitude waves this parameter is slightly less, but the dependence on the amplitude does not seem to be strong.

The cusped solutions are singular, since $db_z/dx \to 0$. They are hydrodynamical idealization of the same kind as MHD discontinuities or cusped perpendicular magnetosonic soliton [7]. In fact, the massless electron approximation breaks down where magnetic field gradients become large and the typical scale of the spatial variation approaches the electron inertial length c/ω_{pe}. The magnetic field behavior near the cusps should be significantly affected by the electron inertia effects (which are not considered here), so that at small scales in the cusp vicinity the wave profile will be smoothed out. At scales substantially larger than the electron inertial length the cusped profile is still as a reasonable approximation.

V. WEAKLY DISSIPATIVE REGIME

Real plasmas are dissipative, often because of turbulent collisions. We shall model phenomenologically this dissipation by introducing effective friction $\propto (\mathbf{U}_i - \mathbf{U}_e)$ in the equations of motion, as follows:

$$m_i v \frac{d\mathbf{U}_i}{dx} = e\mathbf{E}_\perp + \frac{e}{c} v\hat{\mathbf{n}} \times \mathbf{B}_\perp + \frac{e}{c} B_x \mathbf{U}_i \times \hat{\mathbf{n}} - \nu(\mathbf{U}_i - \mathbf{U}_e), \tag{33}$$

$$0 = -e\mathbf{E}_\perp - \frac{e}{c} v\hat{\mathbf{n}} \times \mathbf{B}_\perp - \frac{e}{c} B_x \mathbf{U}_e \times \hat{\mathbf{n}} - \nu(\mathbf{U}_e - \mathbf{U}_i). \tag{34}$$

This friction results in the resistivity $\eta = \nu/ne^2$.

Taking into account that the dissipation is assumed to be small, Eqs. (33)-(34) give eventually

$$l_W \frac{db_z}{dx} + \epsilon l_W \frac{db_y}{dx} = (1 - N\chi) b_y, \tag{35}$$

$$l_W \frac{db_y}{dx} - \epsilon l_W \frac{db_z}{dx} = \mu N \sin\theta (1-\chi) - (1 - N\chi) b_z, \tag{36}$$

where $\epsilon = l_d/l_W = \nu/m_i\Omega_i \cos\theta \ll 1$.

Let us analyze again the upstream point $N = 1$, $b_z = \sin\theta$, $b_y = 0$, which exists when $\mu = 1$. It is easy to find that in this case:

$$\kappa = \kappa_{ND} + \epsilon(2l_W)^{-1}[(V^2 - v_{SL}^2)(V^2 - v_F^2) + (V^2 - v_I^2)(V^2 - v_S^2)][V^2(V^2 - v_S^2)]^{-1}, \tag{37}$$

where κ_{ND} is the nondissipative exponent (see section III) and $\kappa_{ND}^2 < 0$. One can easily see that in the fast range $V > v_F$ the character of the upstream point changes to the unstable focus. In this case a solution with $V > v_F$ can start from this point at $x \to -\infty$. As we have seen above, there is another stationary point (saddle downstream point) in the fast range, which means that a shock-like solution is possible in the weakly dissipative regime.

Once the existence of a shock-like solution is established, it is natural to identify the parameter V with the shock Mach number M, so that $\chi = \cos^2\theta/M^2$. Then Eqs. (16) and (37) give the wavelength λ_W and the damping length Δ_W, respectively, of the whistler precursor in the following form:

$$\lambda_W = 2\pi l_W \left[\frac{M^4(M^2 - v_S^2)}{(M^2 - v_{SL}^2)(M^2 - v_I^2)(M^2 - v_F^2)}\right]^{1/2}, \tag{38}$$

$$\Delta_W = \frac{2l_W}{\epsilon}\frac{M^2(M^2 - v_S^2)}{(M^2 - v_{SL}^2)(M^2 - v_F^2) + (M^2 - v_I^2)(M^2 - v_S^2)}, \tag{39}$$

which generalizes the expression for the upstream whistler precursor found in [2].

VI. CONCLUSIONS

Using the two-hydrodynamics with polytropic pressure and massless electrons for the description of one-dimensional stationary nonlinear low-frequency waves, we derived the most general equations for the structure of these waves. Soliton solutions exist only for rather restrictive constraints on the asymptotic plasma velocity. In particular, fast magnetosonic solitons do not exist in such plasmas without dissipation. Studies of one-dimensional stationary nonlinear waves are completed with the derivation of the general solution for a nonlinear wave in the parametric form. Inclusion of weak dissipation changes qualitatively the character of solutions, allowing existence of fast shocks. The expression for the whistler precursor wavelength is generalized for the hot plasma case.

Acknowledgments

The research was supported in part by the Binational Science Foundation under grant No. 94-00047.

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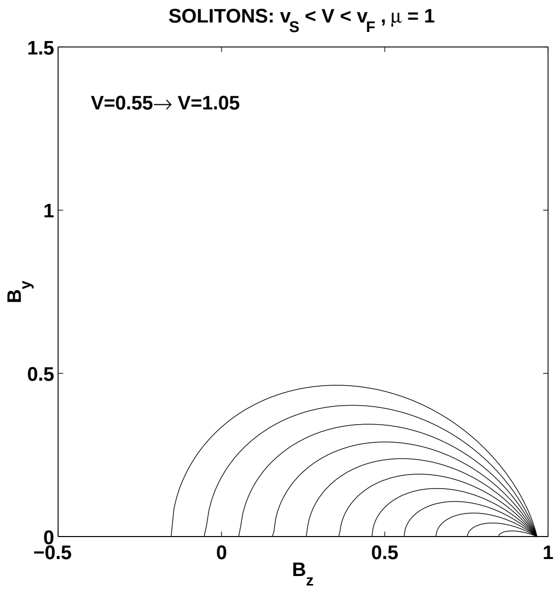

FIG. 1: Phase portrait for rarefactive soliton solutions.

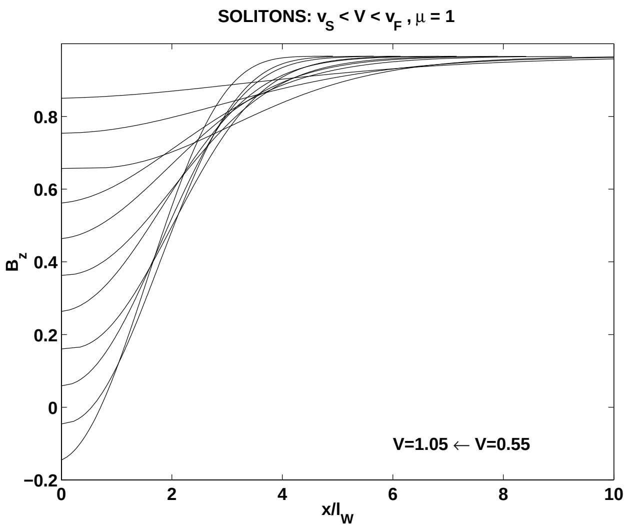

FIG. 2: Spatial profiles for the soliton solutions shown in Fig. 1. Only half of the soliton is shown.

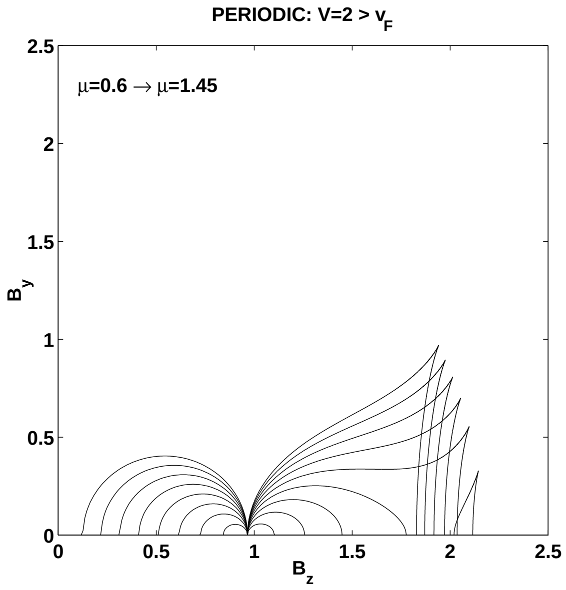

FIG. 3: Phase portraits of the periodic nonlinear fast waves.

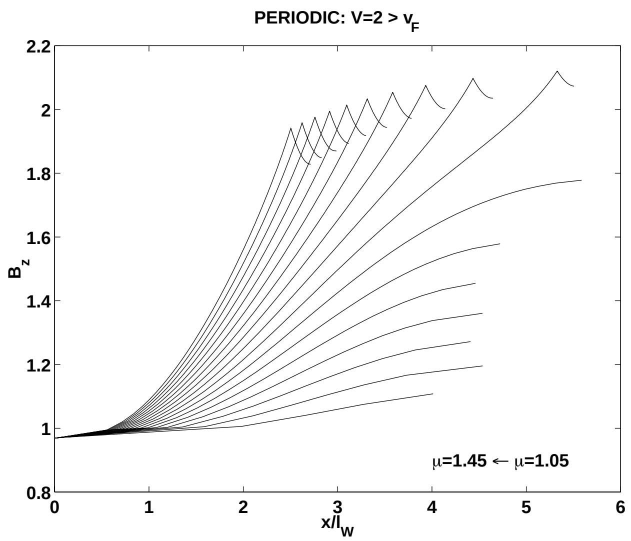

FIG. 4: Profiles of the periodic nonlinear fast waves. Only half-wavelength is presented.