Avalanches in bi-directional sandpile and burning models: A comparative study

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Abstract. We perform a statistical analysis of two one-dimensional avalanching models: the bi-directional sandpile and the burning model (described in detail in the companion paper by Gedalin et al., "Dynamics of the burning model"). Such a comparison helps understand whether very limited measurements done by a remote observer may provide sufficient information to distinguish between the two physically different avalanching systems. We show that the passive phase duration reflects the avalanching nature of the system. The cluster size analysis may provide some clues. The distribution of the active phase durations shows a clear difference between the two models, reflecting the dependence on the internal dynamics. Deeper insight into the active phase duration distribution even provides information about the system parameters.

1 Introduction

Avalanching models and the concept of self-organized criticality (SOC) (Bak et al., 1987, 1988; Jensen, 1998) made their way to space physics quite a while ago (see,e.g. Lu and Hamilton, 1991; Chang, 1992; Lu, 1995; Consolini, 1997; Chapman et al., 1998; Boffetta et al., 1999; Chang, 1999; Takalo et al., 1999; Consolini and De Michelis, 2001; Krasnoselskikh et al., 2002; Valdivia, 2003; Lui, 2004, for a by far incomplete list). SOC ideas have been extensively used for the explanation of the behavior of reconnecting systems (Chang, 1999; Chapman et al., 1998; Charbonneau et al., 2001; Boffetta et al., 1999; Consolini and De Michelis, 2001; Klimas et al., 2004; Krasnoselskikh et al., 2002; Lu and Hamilton, 1991; Takalo et al., 1999; Valdivia, 2003; Uritsky et al., 2001, 2002; Klimas et al., 2004). While attractive, these applications to space systems pose the serious problem of interpreting observations a remote observer makes within the context of the possible dynamical avalanching processes occurring at a place to which the observer has no direct access. Indeed, for example, localized reconnection throughout the current sheet cannot be studied directly without a net of spacecraft deployed in space which provides permanent measurements of the current sheet parameters. Since this does not seem possible in the nearest future, one has to rely on the observations of the consequences of the reconnection (going on, as it is widely assumed), that is, e.g., auroral activity, occuring far from the region where these activity factors were generated. Without firm knowledge of what happens between the generating and observation sites, one cannot establish a reliable mapping of the auroral enhancements to the places in the current sheet where the cause comes from. Yet, with some assumptions one may try to explain the current sheet dynamics from the observations of auroral activity. Some tentative studies in this direction have been done by Kozelov and Kozelova (2003). However, even in this case, there remains the question of whether usually performed simple statistical analyses (power spectrum search, active and passive time durations, etc.) allow one to distinguish between various models thus bringing us to the ultimate objective - the understanding of the underlying physics. Previous works on this topic (see, e.g. Kadanoff et al., 1989) evidenced that there are many different universality classes (each characterized by its own scaling rule) of cellular automata models for avalanching systems with different microscopic rules.

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In the present paper we analyze two different simple avalanching models from the point of view of a remote observer who is limited in his methods of observational data processing. The underlying physics in the two models is quite different. We show that, even if we ignore the lack of knowledge about the processes in the medium between the avalanching system and the observer, simple statistical analysis may still appear rather inconclusive as to what kind of physics governs the system and provides the observed results. Yet, certain observations may allow one to distinguish between the two systems.

2 Bi-directional sandpile model

The model is a bi-directional extension of the model described by Sanchez et al. (2002): a $2L+1$ site long sandpile is driven randomly. Into each site N_d grains are dropped at each times step with the probability p. If the height of (number of grains at) the site i exceeds by more than Z_c the height of at least one of the closest neighbors, $\theta(N(i)-N(i+1)-Z_c)+\theta(N(i)-N(i-1)-Z_c)>0$, the site becomes *active* and transfers its grains to its neighbors. The number of grains transferred to the neighbors, $2N_f$, is constant. If only one of the neighbors satisfies this condition it gets all the grains, if both neighbors are lower by more than Z_c the transferred grains are divided equally between the two. The grain redistribution is done numerically in two steps: first the whole array is scanned to identify the donating and receiving sites, and then the grain transfer occurs simultaneously at all relevant sites (parallel updating). Both boundaries are open, that is, $N(1)=N(L)=0$ is maintained throughout. In order to reduce the simulation time, we start with the distribution with a nearly critical slope and wait until the avalanching process becomes stationary. In the simulations the following parameters were used: $L=200$, $N_d=10$, $Z_c=200$, and $N_f=15$ (similar to Sanchez et al. (2002) so that the uni-directional results are known). The probability p was used to control the avalanching activity. A typical run was several million time steps after a steady state is established.

For a remote observer, who is limited in his ability to look into the avalanching system directly, the most straightforward and simple measurements would be those of the system activity. Let us assume that any giving or receiving (we shall call both active in what follows) site can be seen from a distance, so that a remote observer can reliably distinguish passive and active sites. This assumption implies, in fact, that there is a more or less well established one-to-one mapping of the avalanching region to the place where measurements are performed, like, for example, the often assumed mapping of the reconnecting current sheet in the magnetospheric tail to the auroral zone via the magnetic field lines (Klimas et al., 2004). Such a mapping requires a kind of energy transfer (for convenience we shall call it *radiation*) from the avalanching region to the measurement site, which means that the system should be non-conservative. In case the energy is not directly related to the grain number this non-conservation may be ignored during simulations. It should be understood, however, that a properly maintained one-to-one mapping is the essential ingredient of the model. Any non-linear propagation effects can, in principle, make the observed features non-resembling the true features of the avalanching system. Putting that simply, the observed radiation should reflect the features of the system we are going to analyze (e.g., current sheet) and not of some "black box" in between, otherwise there a danger of attributing the observed features to a wrong object. Assuming this does not happen, the first and most straightforward mode of measurement would be to detect whether the system is active (at least one of the sites is active) or passive (there are no active sites at all). Respectively, we would be interested in the *active phase duration* and *passive phase duration* distributions. In most cases we would be able also to measure the part of the system which is active at each moment, that is, the *number of active sites*.

Figure 1 and 2 show the number of active sites and the activity of the system as function of time, respectively. From both figures it is easily seen that the system is in the regime of non-weak driving and avalanche overlapping is substantial. Yet, cluster merging is negligible. The system is more active toward the edges and not active in the central part, which makes it rather similar to the directional model of Sanchez et al. (2002). Typical avalanches look as shown in the right panel. It is worth noting that a site, which was donating at some step, usually becomes receiving at the next step. If we defined as active only donating sites we would get *punctuated* avalanches. It also means that the length of a cluster, which does not touch a boundary, is always an even number.

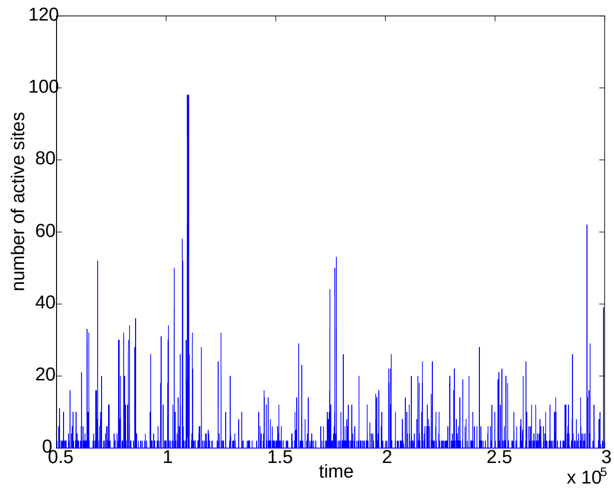

Fig. 1. Number of active sites as a function of time (bi-directional sandpile model).

In what follows we compare the cluster size distribution, the active phase duration distribution and the passive phase duration distribution for the two runs with the probabilities $p = 0.00015$ (high state) and $p = 0.00005$ (low state). This closely corresponds to the definition of the low drive $X=pL^2/N_f \ll 1$ and high drive $pL^2/N_f \gtrsim 1$ by Woodard et al. (2005)[1] (in our case the parameter $X=0.13$ and 0.4, respectively, for the low and high states). From the observation oriented point of view a more direct measure would be the percentage of time during which the system is in the active state, $\approx T_a/(T_a+T_p)$, where T_a is the mean active phase duration and T_p is the mean passive state duration. This percentage varies from about 10% for our low state to about 30% for our high state (see below). The cluster size is the width of the connected active area for $t=$ const. The distribution is obtained by collecting cluster sizes for all time steps.

[1]Woodard, R., Newman, D.E., Sanchez, R., and Carreras, B.A., Building blocks of self-organized criticality, http://arxiv.org./abs/cond-mat/0503159, 2005.

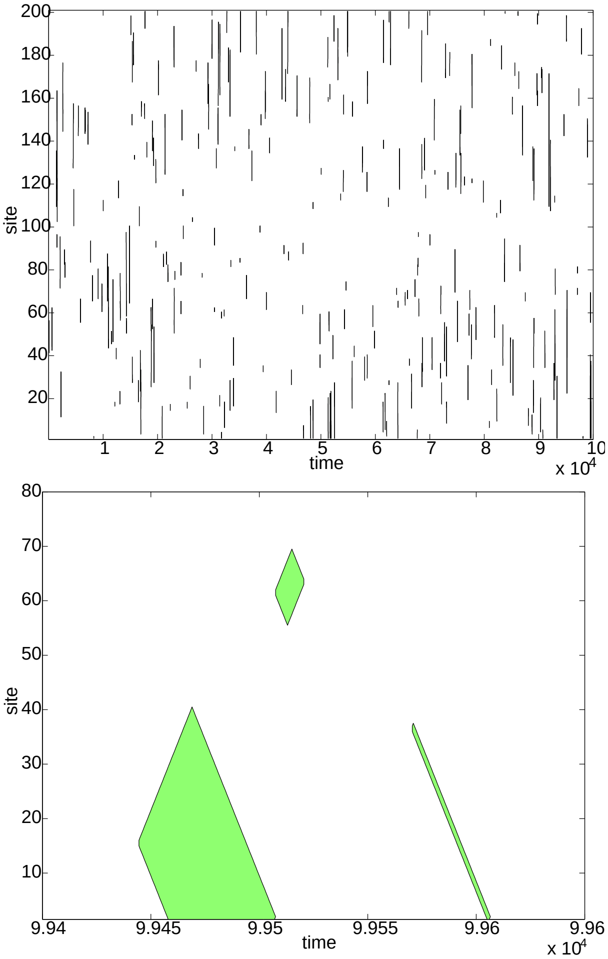

Fig. 2. View of the sand system activity: 10^6 time steps on the left, enlarged part on the right.

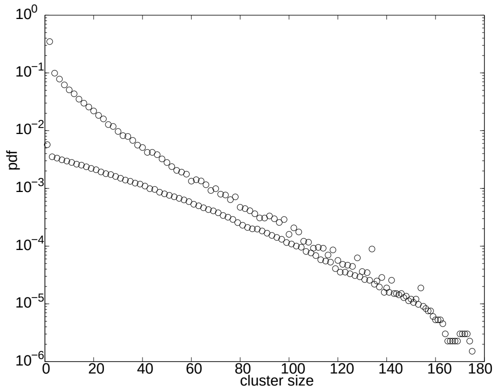

Fig. 3. Distribution of cluster sizes for the high state (bi-directional sandpile model). Log-linear scale. Upper trend is for even-size clusters, lower is for odd-size ones. See explanation in text.

The cluster size distribution in the high state is shown in Figure 3. The lower distribution corresponds to the odd-length clusters which are possible only when a cluster grows until it touches the edge of the sandpile (see Figure 2). Thus, the influence of the boundaries can be clearly seen. The distribution is Poisson-like with mean cluster size $\bar{w} = 12$. The corresponding distributions of active and passive phase durations are shown in Figure 4. While the passive phase durations are distributed according to Poisson statistics, the active phase durations show a clear transition from the Poisson shape at low durations to a power-law behavior at larger values. The mean passive and active durations, respectively, are $T_p = 58$ and $T_a = 23$. The transition from Poisson to power-law for active phase durations occurs near the mean value.

Instead of presenting separately the corresponding distributions for the low state we provide a comparative view, plotting the distributions against the normalized variables: $w/\bar{w}$ for the normalized cluster size, $T/T_{a,p}$ for the normalized active and passive phase duration, respectively. The corresponding mean values are given in Table 1.

Table 1. Mean values for bi-directional sandpile

	High state $p = 0.00015$	Low state $p = 0.00005$
Mean cluster size $\bar{w}$	12	12
Mean active phase duration T_a	23	21
Mean passive phase duration T_p	58	176

One can see that the cluster size is not affected by the driving strength, the active phase duration is affected only weakly, while the passive phase duration is affected substantially. This is in line with the understanding that driving affects primarily the chances of a new avalanche to start. The cluster size depends almost solely on the internal dynamics (mechanism of avalanching), as well as the avalanche duration. The latter may be (relatively weakly) affected by driving since input can occur during the avalanche development. Yet for moderate driving this effect should not be significant, and we expect that the microprocesses in the system determine the avalanche length.

The active and passive phase duration distributions are shown in Figure 5. The power-law parts of the active phase duration distributions, as well as the passive phase durations

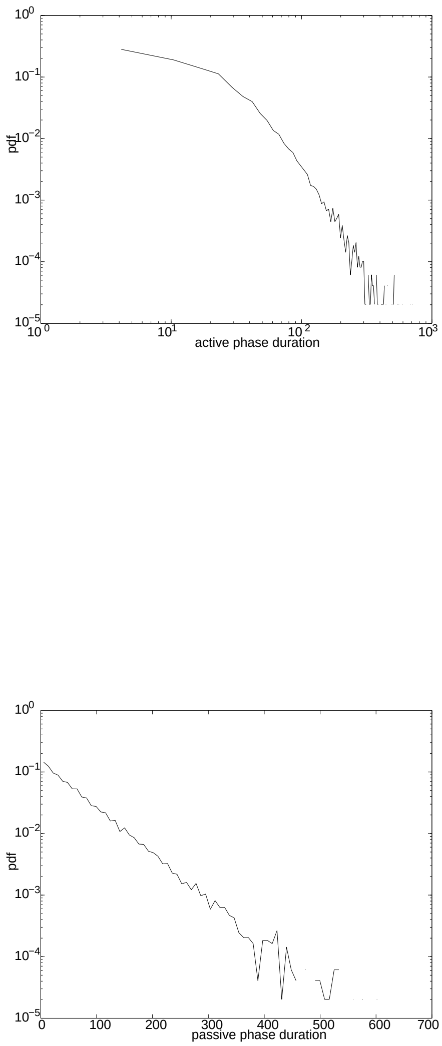

Fig. 4. Distributions of active (left, log scale) and passive (right, log-linear scale) phase durations (bi-directional sandpile model).

are almost identical (fluctuations at large durations are due to poor statistics) for both states.

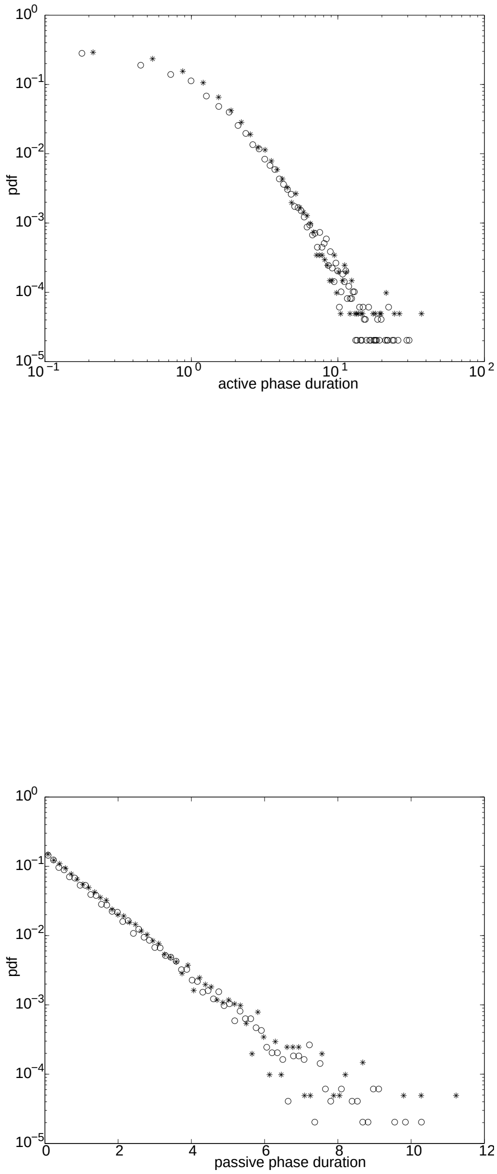

Fig. 5. Distribution of active (left) and passive (right) phase durations for high state (circles) and low state (stars) in the bi-directional sandpile model. The durations are normalized with the mean values.

3 Burning model

This model is described in detail in the companion paper. Here we provide only a brief description. Each of L sites is characterized by a *temperature* $T(i)$. Random heat input q with the probability p into each site is the external driving. When a site temperature exceeds the critical value, $T(i) >$

T_c, the site starts to burn, releasing isotropically the heat flux $J = kT$. The fraction a of the heat remains in the system (propagating to the neighbors), while $1 - a$ is radiated out (lost). Burning proceeds as long as $T(i) > T_l = sT_c$. For the simulations below the following parameters were used: $L = 400$, $T_c = 50$, $s = 0.3$, $k = 3$, and $q = 0.05T_c$. Again we study a high $p = 0.001$ and a low $p = 0.0003$ state. Since this model is dissipative we have also to analyze the effects of energy losses which is done varying the parameter $a = 0.9$ and $a = 0.97$.

Figure 6 shows the distributions of cluster size, active phase duration, and passive phase duration for the case $p = 0.0003$ and $a = 0.9$ (low driving with strong dissipation) for the burning model. The mean cluster size is $\bar{w} = 3.4$, the mean active phase duration is $T_a = 16$, and the mean passive phase duration is $T_p = 146$. The systems is active for about 10% of time.

Figure 7 provides the comparison for the four runs listed in Table 2. The behavior of the mean values is similar to

Table 2. Mean values for burning model

	Low state $p = 0.0003$ Strong dissipation $a = 0.9$	High state $p = 0.001$ Strong dissipation $a = 0.9$	Low state $p = 0.0003$ Weak dissipation $a = 0.97$	High state $p = 0.001$ Weak dissipation $a = 0.97$
Mean cluster size $\bar{w}$	3.4	3.4	11	12
Mean active phase duration T_a	16	18	66	77
Mean passive phase duration T_p	146	44	594	188

what we have seen in the case of the bi-directional sandpile and in agreement with our expectations. The passive phase duration distributions are essentially the same Poisson for all runs. The active phase duration slope is steeper for stronger dissipation (which limits the avalanche propagation).

4 Comparison and conclusions

The above results already show that there are clear differences in the remote observations of both systems. In order to make this more clear we visualize the distributions of passive and active phase durations for both models together. For this comparison we choose the low-state, $p = 0.00005$, bi-directional sandpile model and the weakly-driven, $p = 0.0003$, strongly dissipative, $a = 0.9$, burning model. For both models the system is active for about 10% of the time. The mean active and passive phase durations for the sandpile are $T_a = 21$ and $T_p = 176$. The mean active phase duration for the burning model, $T_a = 11$, is strongly affected by the lower limit ≈ 3 on the avalanche life time, so, in order to ensure proper visual comparison, we truncate the distribution from below, excluding from the analysis the shortest avalanches (which are most probable). With this truncation, the re-normalized mean values for the burn-

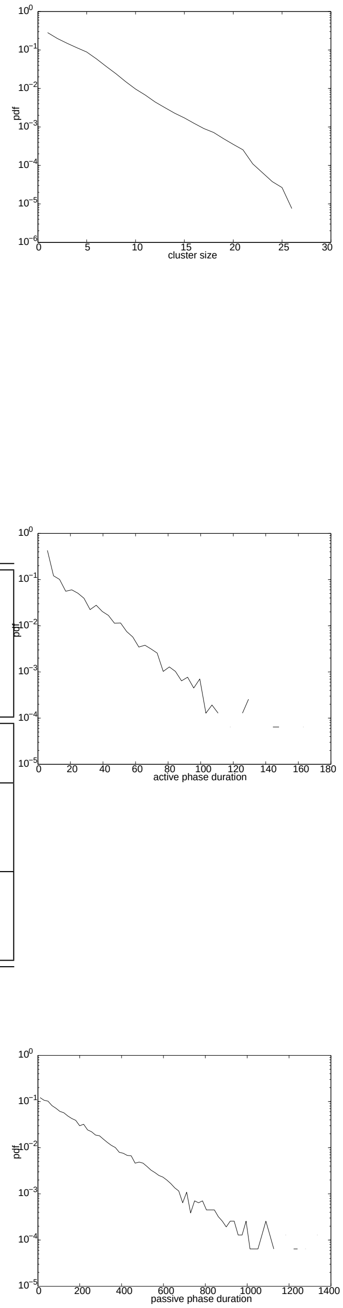

Fig. 6. Distributions of the cluster size, active phase duration, and passive phase duration for the case $p = 0.0003$ and $a = 0.9$.

ing model are $T_a = 23$ and $T_p = 145$, which is pretty similar to those for the sandpile. Thus, for the chosen parameters, both models exhibit similar levels of activity. Figure 8 shows the passive phase distributions for both models. The

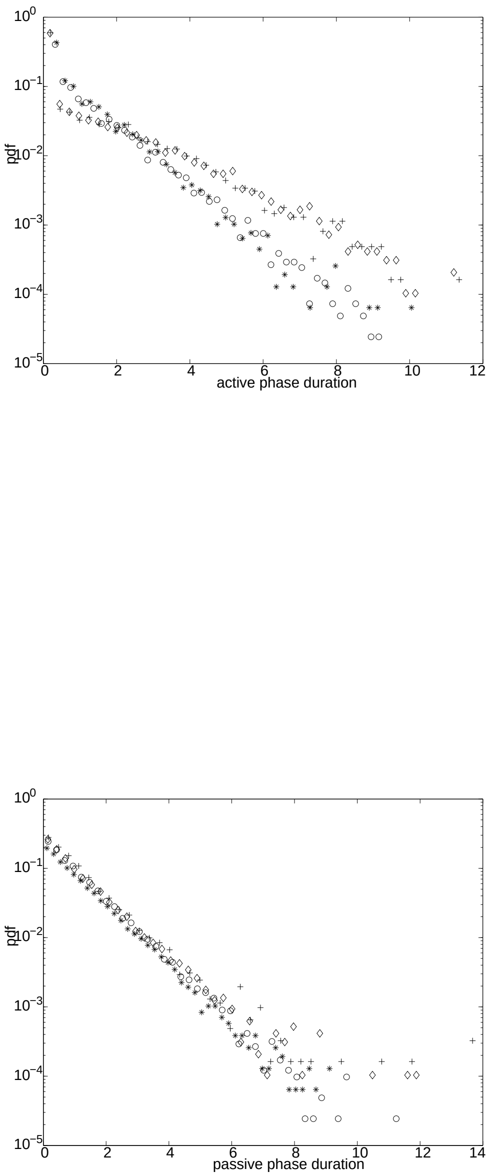

Fig. 7. Comparison of active (left) and passive (right) phase duration distributions (burning model). The durations are normalized with the corresponding mean values. Markers: stars - $p = 0.0003$, $a = 0.9$, circles - $p = 0.001$, $a = 0.9$, pluses - $p = 0.0003$, $a = 0.97$, and diamonds - $p = 0.001$, $a = 0.97$.

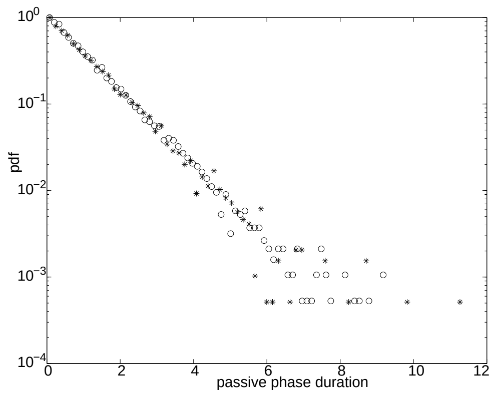

Fig. 8. Comparison of the normalized distributions for the passive phase durations, sandpile - stars, burning - circles.

plotted distributions are normalized as follows: $T \to T/T_p$, $P \to P/P(T_{\text{min}})$. The two distributions are identical (except for long avalanches where statistics becomes poor).

Figure 9 shows the active phase distributions for both models. The normalization is the same. While the two distributions coincide for short avalanches, at longer values the sandpile model clearly leaves the Poisson curve toward a power-law slope.

To summarize, we have studied two one-dimensional systems exhibiting avalanching behavior which could, in principle, be observed in a similar way by a remote observer. We have concentrated on two systems with similar random driving and similar remote observation modes. We have shown that simple direct analysis of the quiet time (passive phase duration) observations do not, in general, provide sufficient information which could be used to reliably distinguish between the two systems. More sophisticated methods, like thresholding, may be useful for the analysis (Sanchez et al., 2003; Laurson and Alava, 2004; Woodard et al., 2004), but they are outside of the scope of the present consideration. On the other hand, the avalanche lengths (active phase duration) may be useful in identifying internal properties of the physical system and the underlying mechanism of the avalanches. It is worth noting that the exponential active phase PDFs is

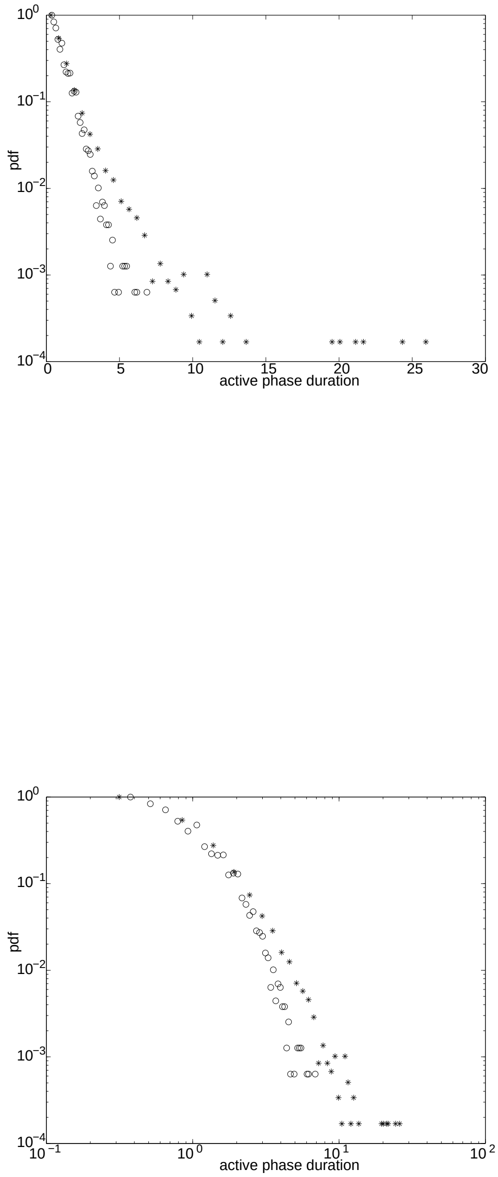

Fig. 9. Comparison of the normalized distributions for the active phase durations, sandpile - stars, burning - circles. Left panel - log-linear scale, right panel - log-log scale.

not expected to be unique for the burning model, and we do not claim that the proposed burning model is the most applicable but suggest it as a plausible possibility. Our analysis should be rather considered as a starting point for further studies of observable features of avalanching systems in the context of measurements made by a remote observer, in particular, for the problem of distinguishing between models on the basis of rather limited data.

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