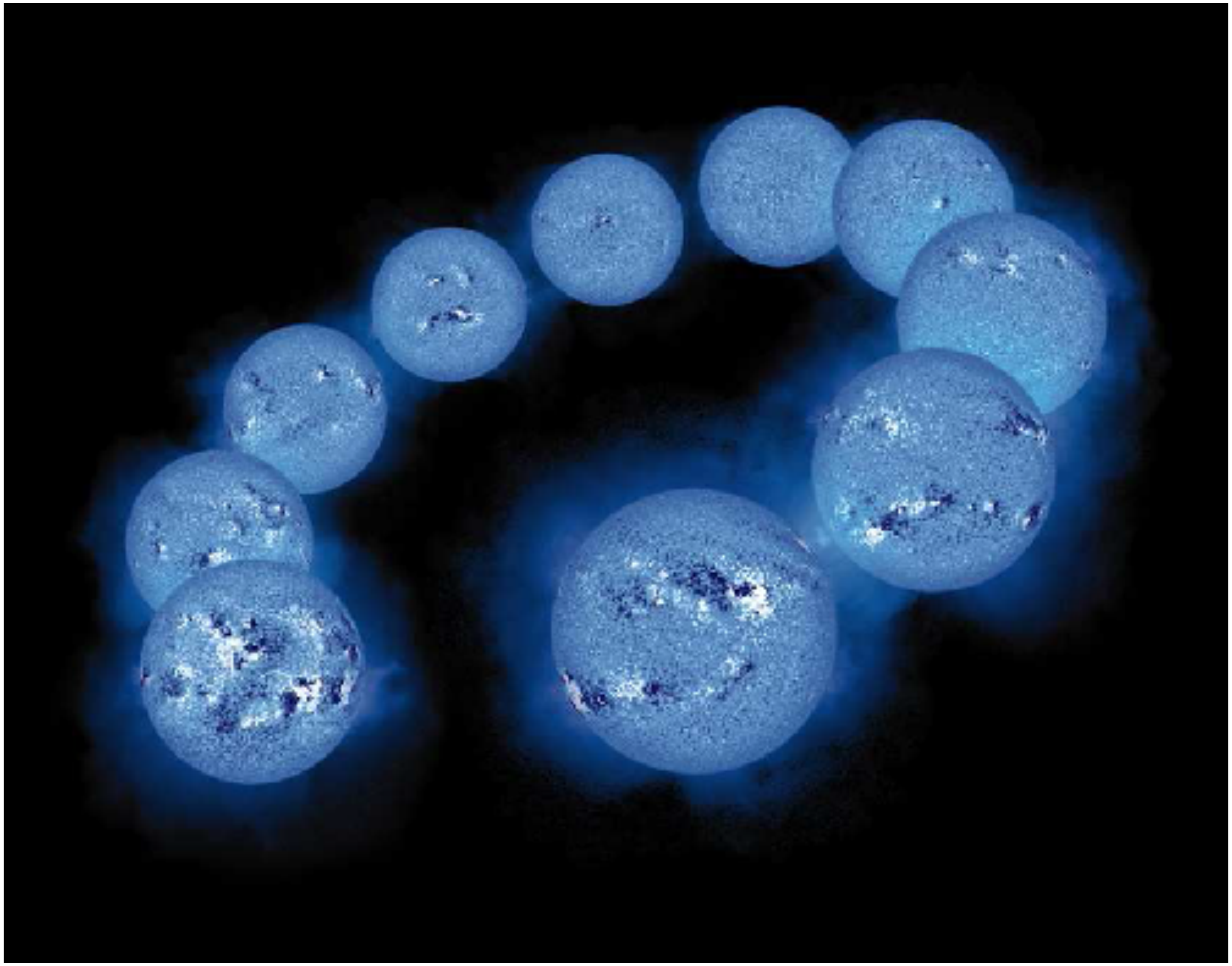
Shallow water MHD waves in the solar tachocline and solar activity variations

Teimuraz Zaqarashvili

Space Research Institute, Graz, Austria

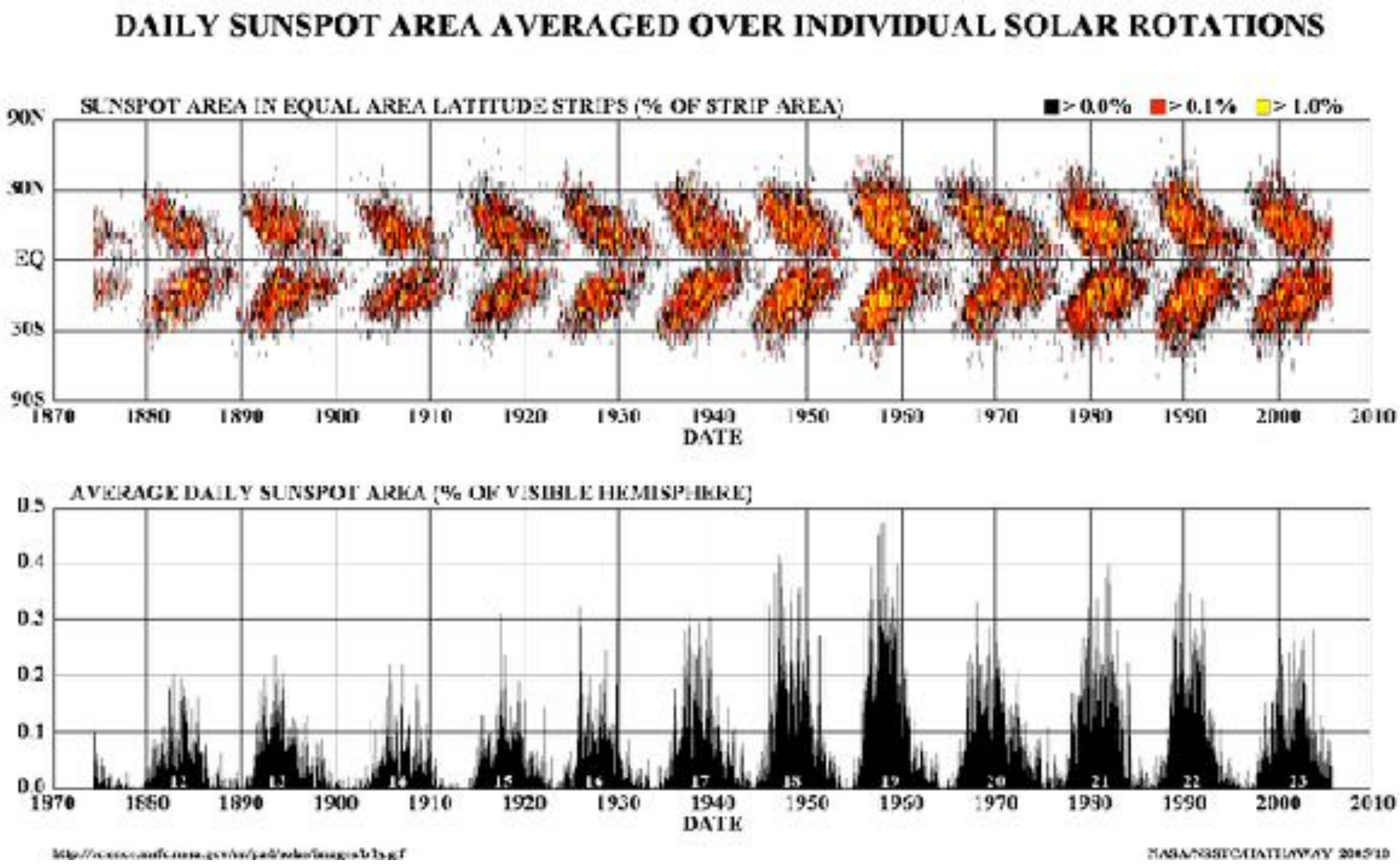
Abastumani Astrophysical Observatory at Ilia State University, Georgia

Solar activity is quasi-periodic (11 yrs).



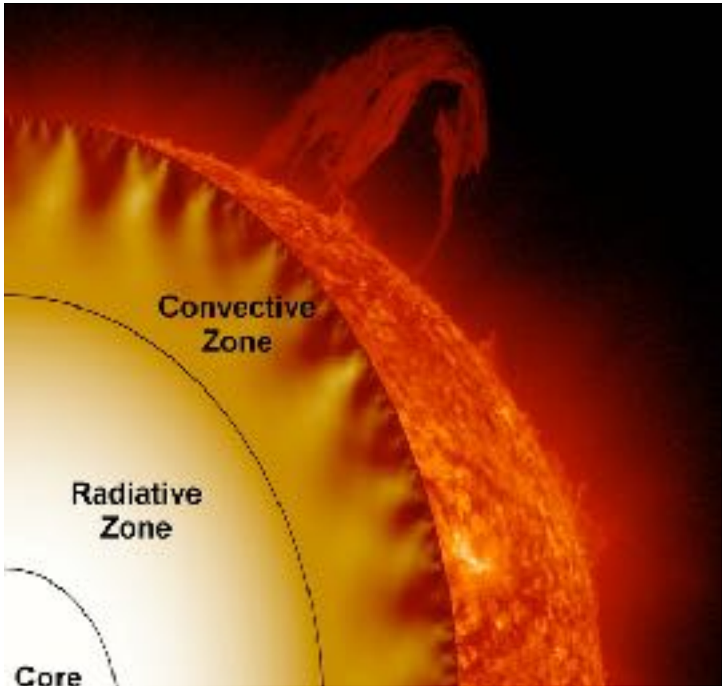
courtesy to NASA

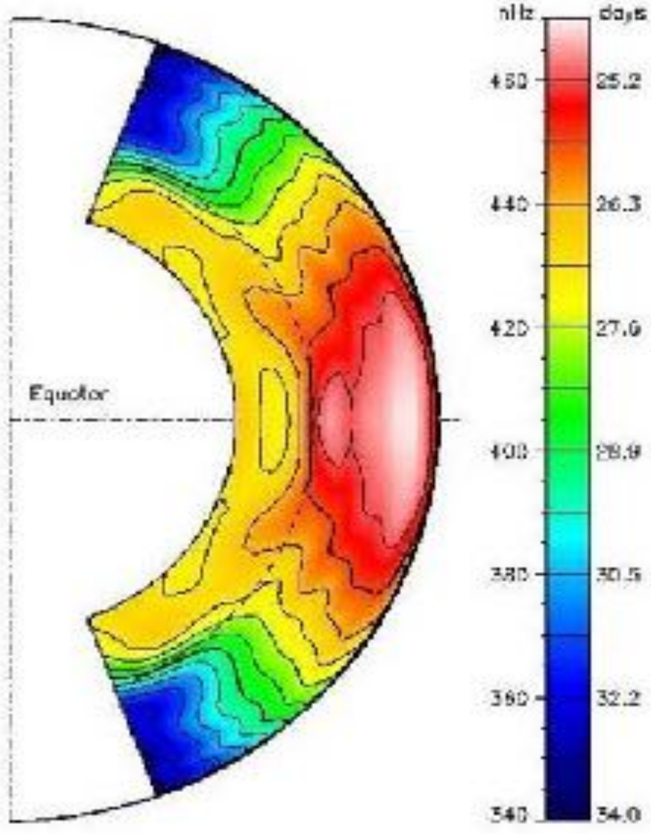
Solar activity undergoes the variations over shorter and longer time scales than the period of solar cycles.



Most of dynamo models suggest that the amplification of magnetic field occurs in the tachocline: a thin layer below the convection zone.

Shallow water equations can be used (Gilman 2000)





courtesy to Marshall Space Flight Center

Shallow water equations (derived from Laplace's tidal equations)

$$\frac{\partial u_x}{\partial t} - f u_y = -g \frac{\partial h}{\partial x}$$

$$\frac{\partial u_y}{\partial t} + f u_x = -g \frac{\partial h}{\partial y}$$

$$\frac{\partial h}{\partial t} + H \frac{\partial u_x}{\partial x} + H \frac{\partial u_y}{\partial y} = 0$$

$f = 2\Omega \sin\theta$ is the Coriolis parameter.

H is the layer thickness.

Near the equator the Coriolis parameter can be approximated as

$$f = \beta y \qquad \beta = \frac{2\Omega}{R}$$

Fourier expansion gives the equation of parabolic cylinder (Matsuno 1966)

$$\frac{d^2 u_y}{dy^2} + \left[\frac{\omega^2}{c^2} - k_x^2 + \frac{k_x \beta}{\omega} - \frac{\beta^2}{c^2} y^2\right] u_y = 0.$$

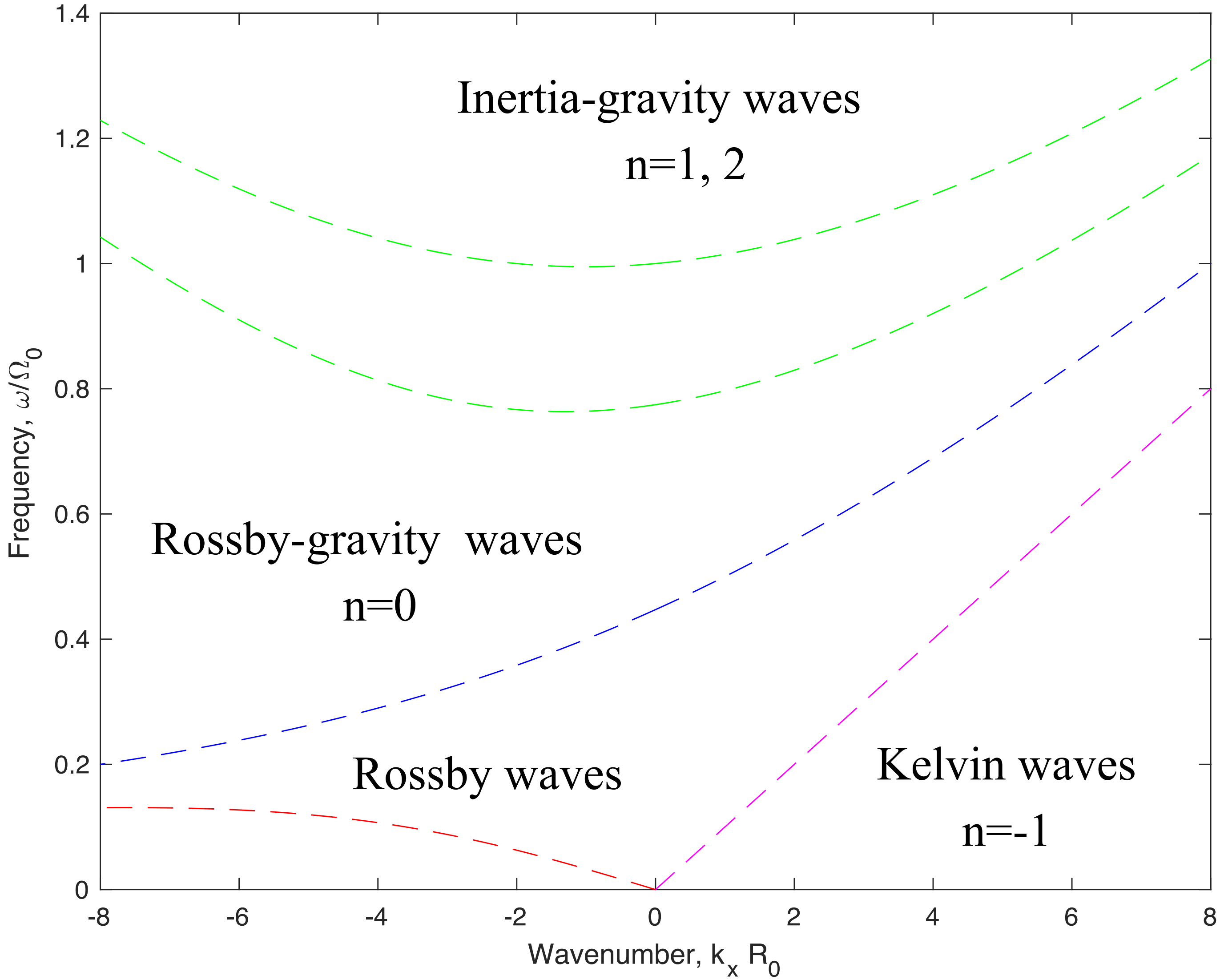
$c = \sqrt{gH}$ is the surface gravity speed.

The solution: $$u_y = C \exp\left[-\frac{\beta}{c}\frac{y^2}{2}\right] H_n\left(\sqrt{\frac{\beta}{c}}\, y\right)$$

Oscillatory inside the interval: $$y < \left|\sqrt{\frac{c}{\beta}(2n+1)}\right|$$

For $G = \frac{gH}{\Omega^2 R^2} \ll 1$ solutions are concentrated around the equator.

Dispersion relation: $$\omega^3 - \left[k_x^2 c^2 + \beta c(2n+1)\right]\omega + k_x \beta c^2 = 0.$$



Inertia-gravity waves
n=1, 2
Rossby-gravity waves
n=0
Rossby waves
Kelvin waves
n=-1
Frequency, ω/Ω0
Wavenumber, kx R0
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-8
-6
-4
-2
0
2
4
6
8

Reduced gravity in the solar tachocline

Sub-adiabatic temperature gradient provides a negative buoyancy force to the deformed upper surface of the tachocline (Gilman 2000), which is proportional to the fractional difference between the actual and adiabatic temperature gradients

$$|\nabla - \nabla_{ad}|$$

Upper tachocline	$10^{-4} - 10^{-6}$	Dikpati and Gilman (2001).
Lower tachocline	10^{-1}	

Then

Upper tachocline	$10^{-3} \leq G \leq 10^{-1}$
Lower tachocline	$G > 100$

MHD linear shallow water equations in the tachocline (Gilman 2000):

$$\frac{\partial u_x}{\partial t} - f u_y = -g\frac{\partial h}{\partial x} + \frac{B_x}{4\pi\rho}\frac{\partial b_x}{\partial x} + \frac{b_y}{4\pi\rho}\frac{\partial B_x}{\partial y},$$

$$\frac{\partial u_y}{\partial t} + f u_x = -g\frac{\partial h}{\partial y} + \frac{B_x}{4\pi\rho}\frac{\partial b_y}{\partial x},$$

$$\frac{\partial b_x}{\partial t} + u_y\frac{\partial B_x}{\partial y} = B_x\frac{\partial u_x}{\partial x},$$

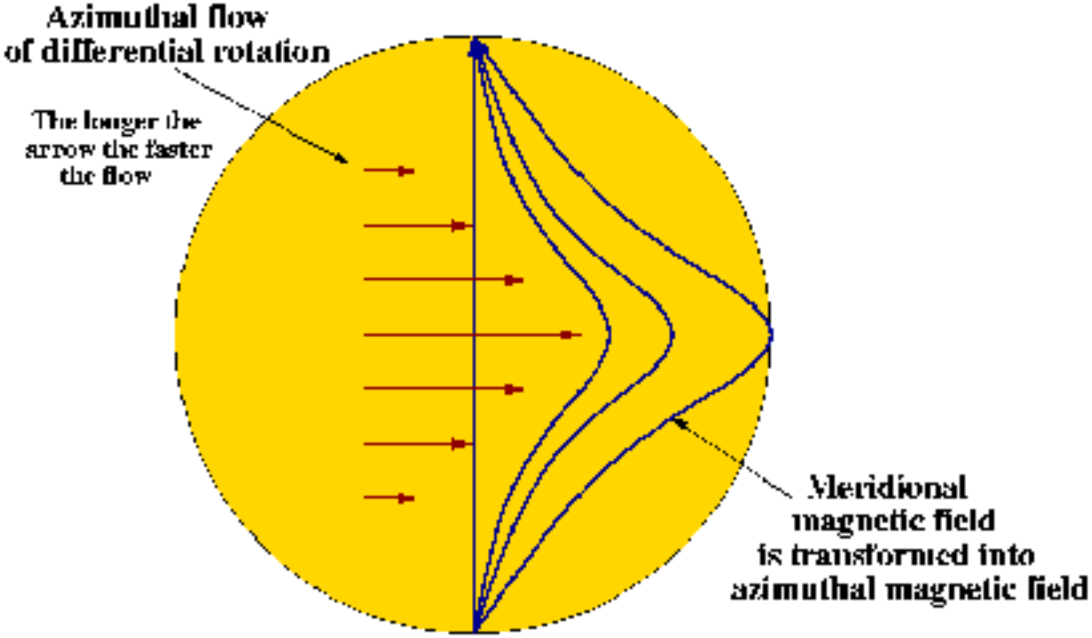
$$\frac{\partial b_y}{\partial t} = B_x\frac{\partial u_y}{\partial x},$$

$$\frac{\partial h}{\partial t} + H\frac{\partial u_x}{\partial x} + H\frac{\partial u_y}{\partial y} = 0,$$

Solar differential rotation $\Omega_d = \Omega(1 - s_2 \sin^2\theta - s_4 \sin^4\theta)$

The induction equation $$\frac{\partial B_\phi}{\partial t} \sim B_\theta \frac{\partial \Omega_d}{\partial \theta}$$

The toroidal magnetic field $B_\phi \sim B_\theta \cos\theta \sin\theta$



Near the equator:

$$B_x = B_0 \frac{y}{R}$$

Governing equation

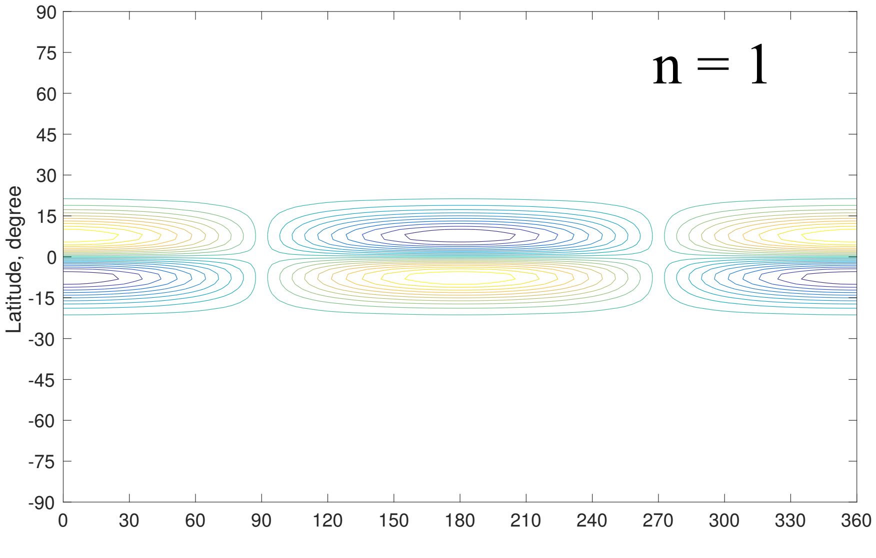
$$\frac{d^2\tilde{u}_y}{dy^2} + \left[\frac{\omega^2}{c^2} - k_x^2 - \frac{k_x\beta}{\omega} - \frac{k_x^4 c^2 v_{A0}^2}{R^2\omega^2(\omega^2 - c^2k_x^2)} - \tilde{\mu}^2 y^2\right]\tilde{u}_y = 0,$$

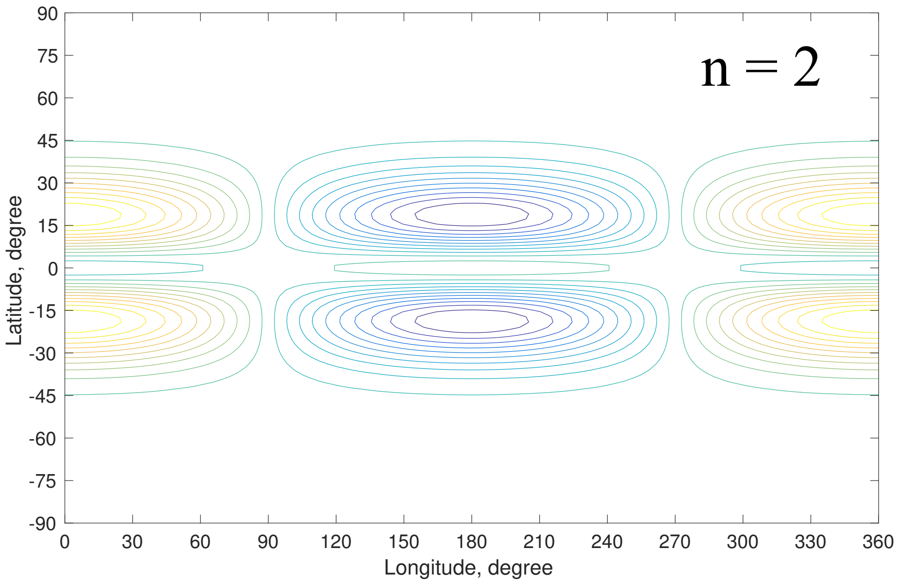
$$\tilde{\mu} = \sqrt{\frac{k_x^2 v_{A0}^2}{R^2c^2} + \frac{\beta^2}{c^2} + \frac{k_x^3\omega\beta v_{A0}^2}{\omega^4 R^2} + \frac{2k_x^3\omega\beta v_{A0}^2}{R^2\omega^2(\omega^2 - k_x^2c^2)} + \frac{k_x^8c^4v_{A0}^4}{R^4\omega^4(\omega^2 - c^2k_x^2)^2}}.$$

The dispersion relation

$$\frac{\omega^2}{c^2} - k_x^2 - \frac{k_x\beta}{\omega} - \frac{k_x^4c^2v_{A0}^2}{R^2\omega^2(\omega^2 - c^2k_x^2)} = |\tilde{\mu}|(2n+1)$$

Contour plot of toroidal magnetic field





$$G = gH/(R^2\Omega^2) = 0.001$$

$$B_0 = 10 \quad \text{kG}$$

The solution:

$$\tilde{u}_y = C \exp\left[-\frac{|\tilde{\mu}|y^2}{2}\right] H_n\left(\sqrt{|\tilde{\mu}|}y\right)$$

Oscillatory inside the interval:

$$y < \left|\sqrt{\frac{2n+1}{|\tilde{\mu}|}}\right|$$

Zaqarashvili (2018)

When $|\omega| \gg |k_x c|$

Magneto-inertia-gravity waves

$$\omega^3 - \left(k_x^2 c^2 + (2n+1)c\sqrt{\frac{k_x^2 v_{A0}^2}{R^2} + \beta^2}\right)\omega - k_x \beta c^2 = 0$$

When $|\omega| \ll |k_x c|$

Magneto-Rossby waves

$$k_x c\omega - \frac{k_x^2 c v_{A0}^2}{\beta R^2} = -(2n+1)\sqrt{\omega^4 - \frac{2k_x v_{A0}^2}{\beta R^2}\omega^3 + \frac{k_x^3 c^2 v_{A0}^2}{\beta R^2}\omega + \frac{k_x^4 c^2 v_{A0}^4}{\beta^2 R^4}}.$$

Fast magneto-Rossby waves

$$\omega_+ \approx -\frac{k_x c}{2n+1}$$

Slow magneto-Rossby waves

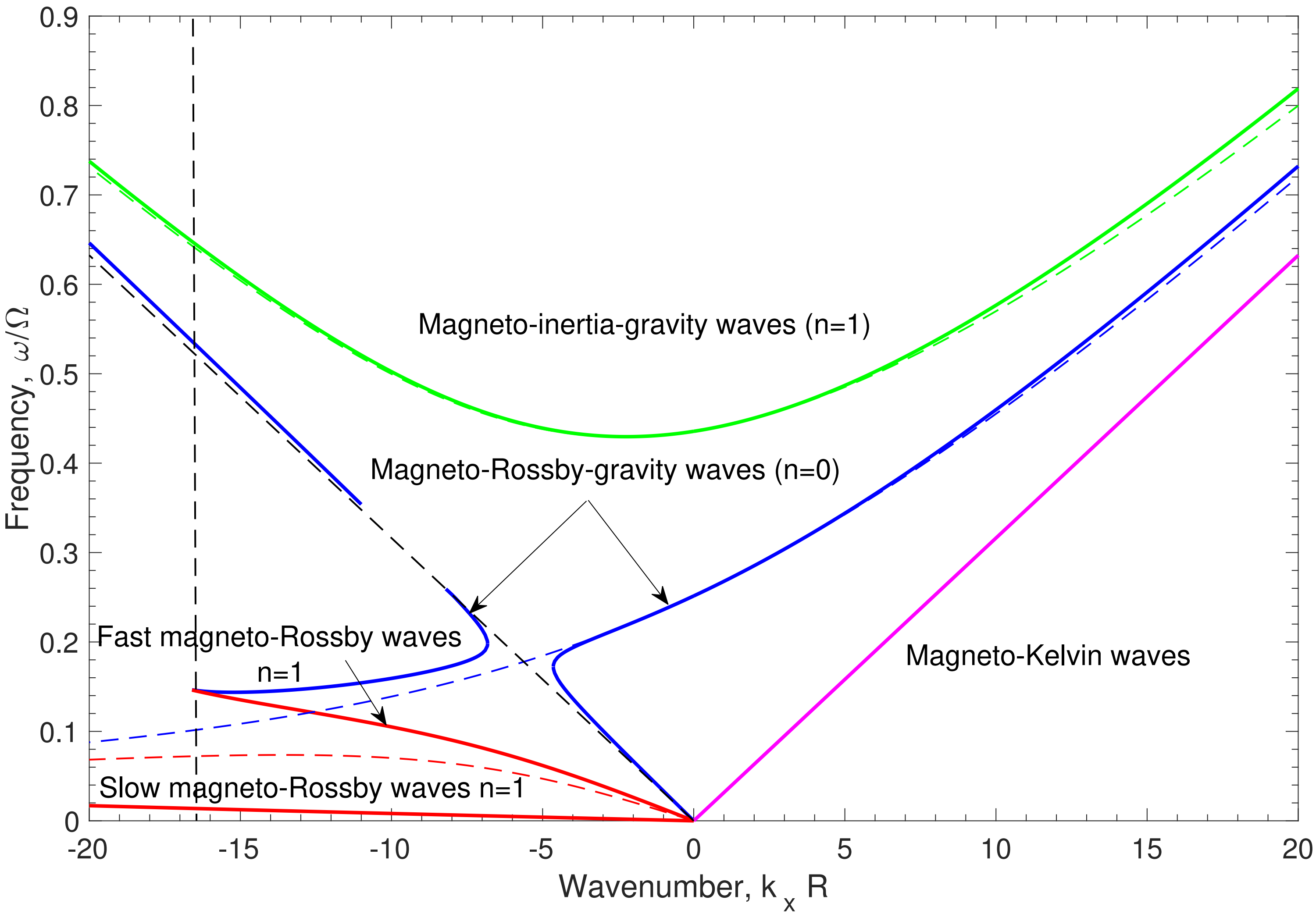
$$\omega_- \approx -\frac{(2n+1)^2 - 1}{(2n+1)^2 + 2} \frac{k_x v_{A0}^2}{\beta R^2}$$

Magneto-Kelvin waves

$$\omega = \sqrt{c^2 + v_{A0}^2 \frac{y^2}{R^2}} k_x$$

Near the equator $\omega = k_x c$

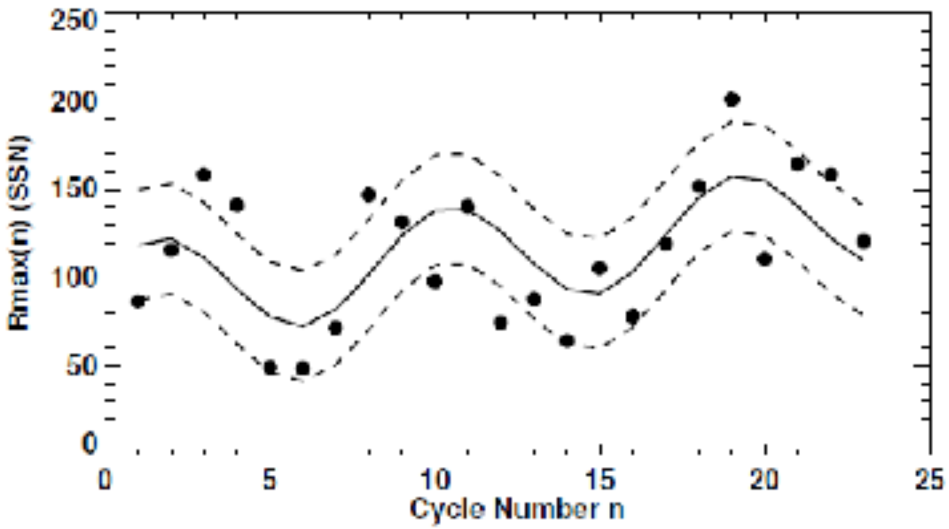
$G = gH/(R^2\Omega^2) = 0.001 \quad B_0 = 10$ kG



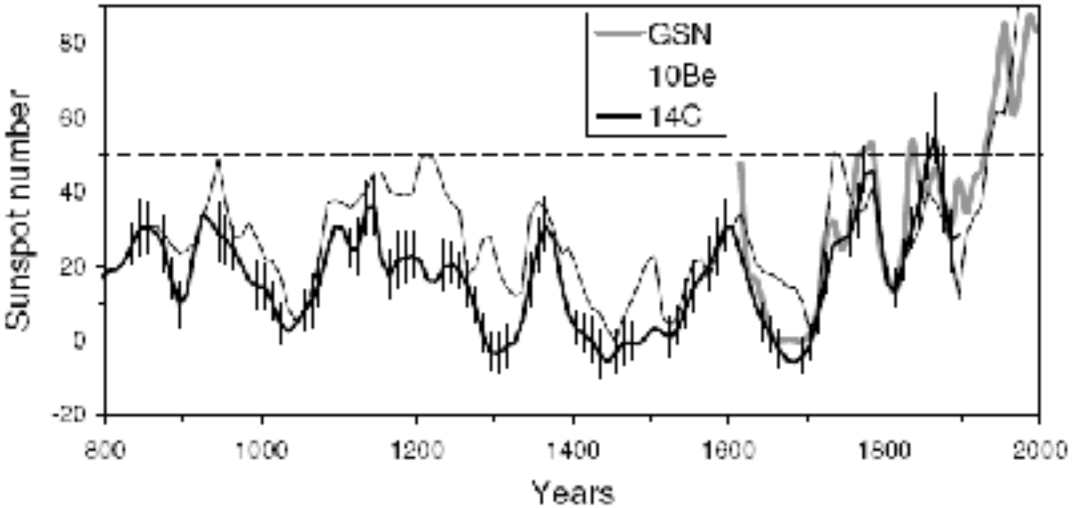
4 timescales:
hundreds of yrs,
tens of yrs,
1-2 yrs,
hundreds of days

Zaqarashvili (2018)

Gleissberg cycle (~100 yr)

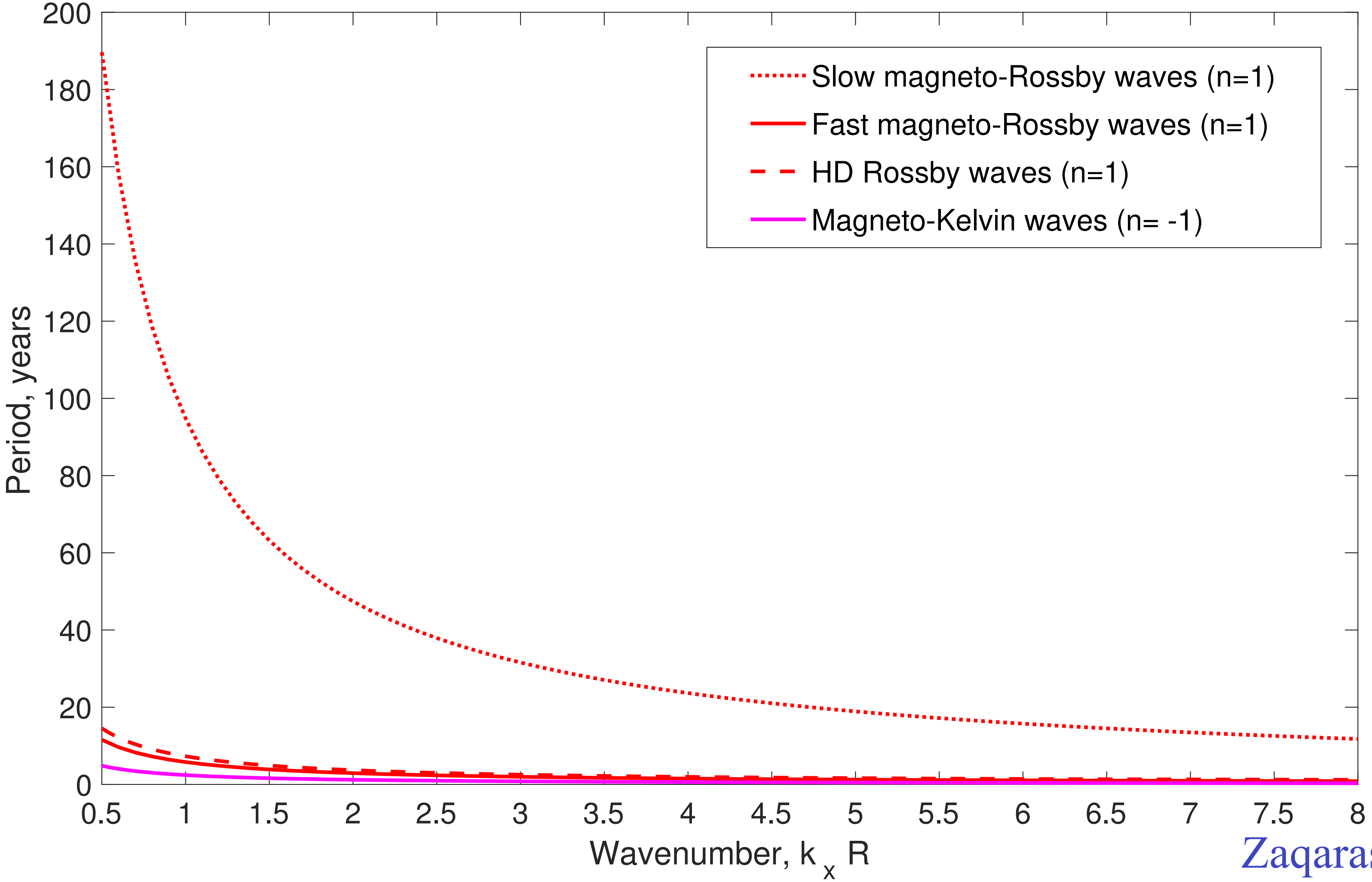


Hathaway (2010)



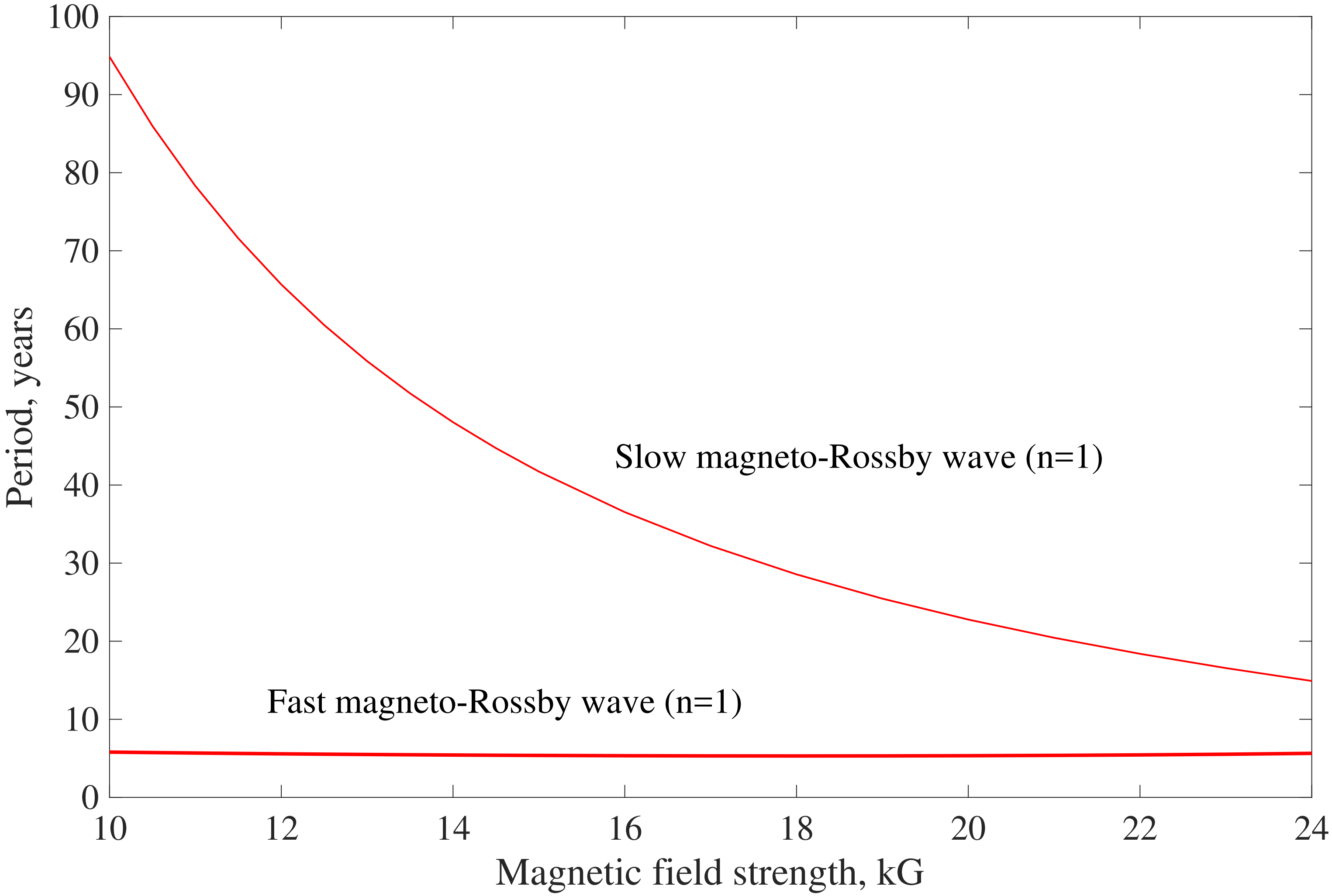
Usoskin et al. (2008)

$$G = gH/(R^2\Omega^2) = 0.001 \qquad B_0 = 10 \ \text{kG}$$



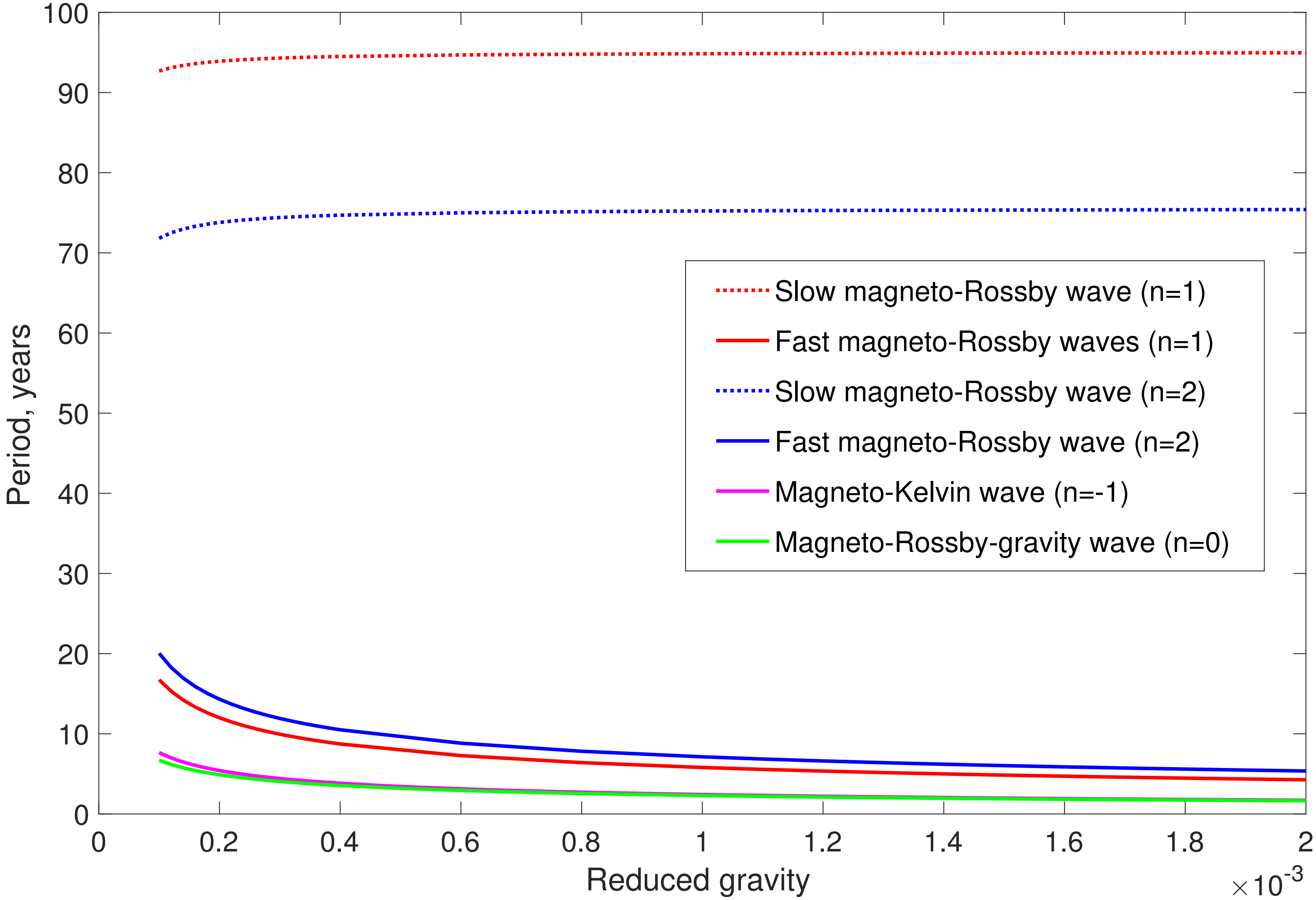
Zaqarashvili (2018)

$G = gH/(R^2\Omega^2) = 0.001$ $\quad k_x R = 1$



Zaqarashvili (2018)

$B_0 = 10$ kG $\qquad k_x R = 1$



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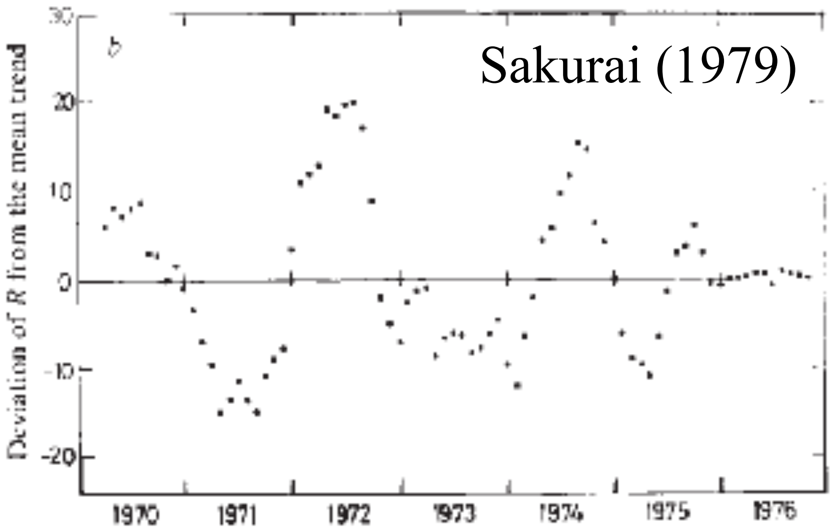
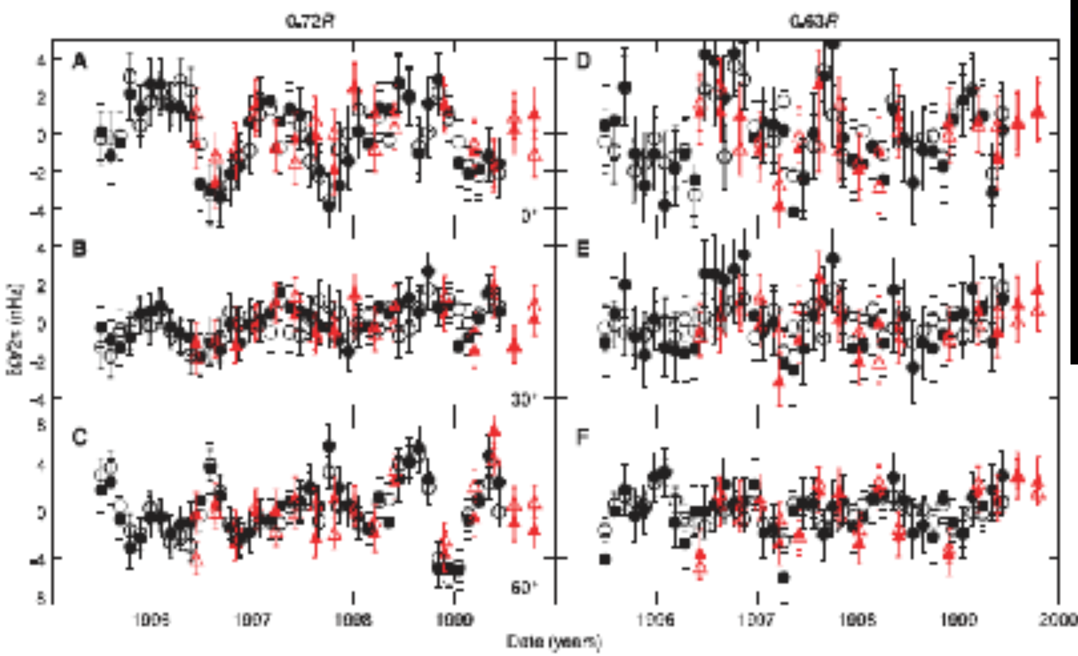
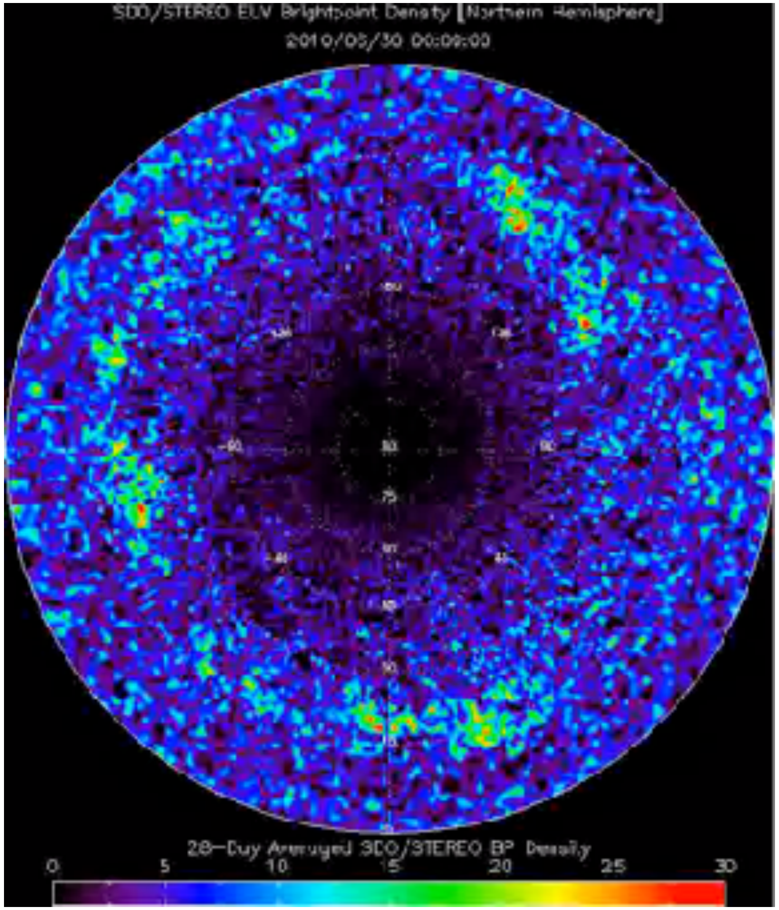
1-2 yr oscillations



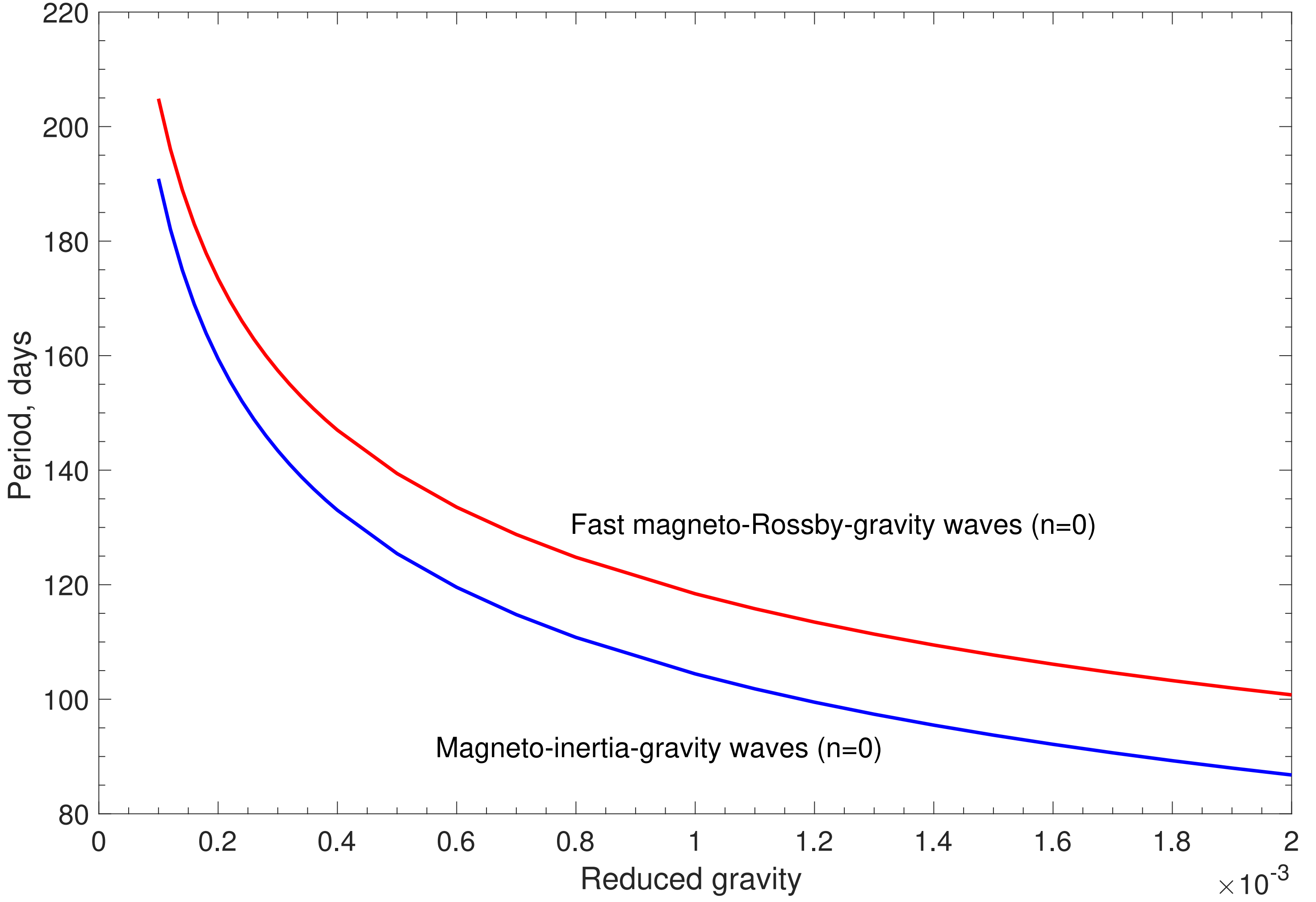




McIntosh et al. 2017

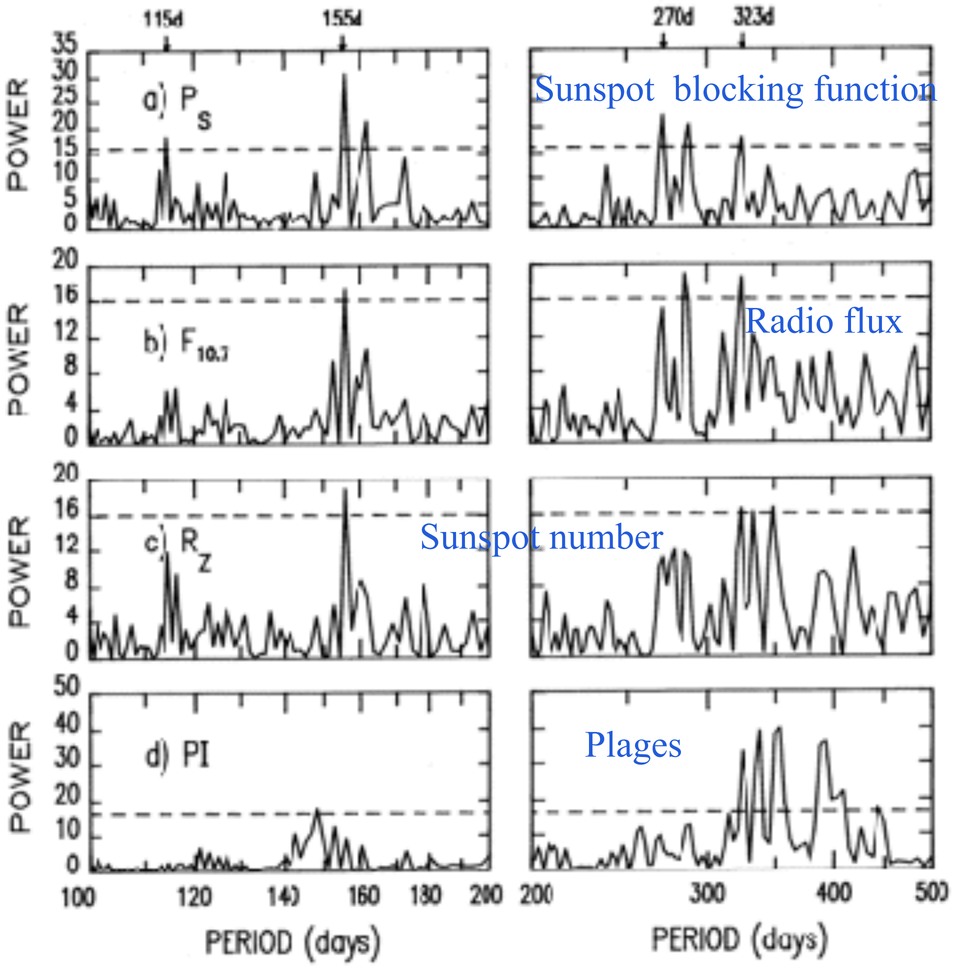
Howe et al. (2000)

$B_0 = 10$ kG $\qquad k_x R = 1$

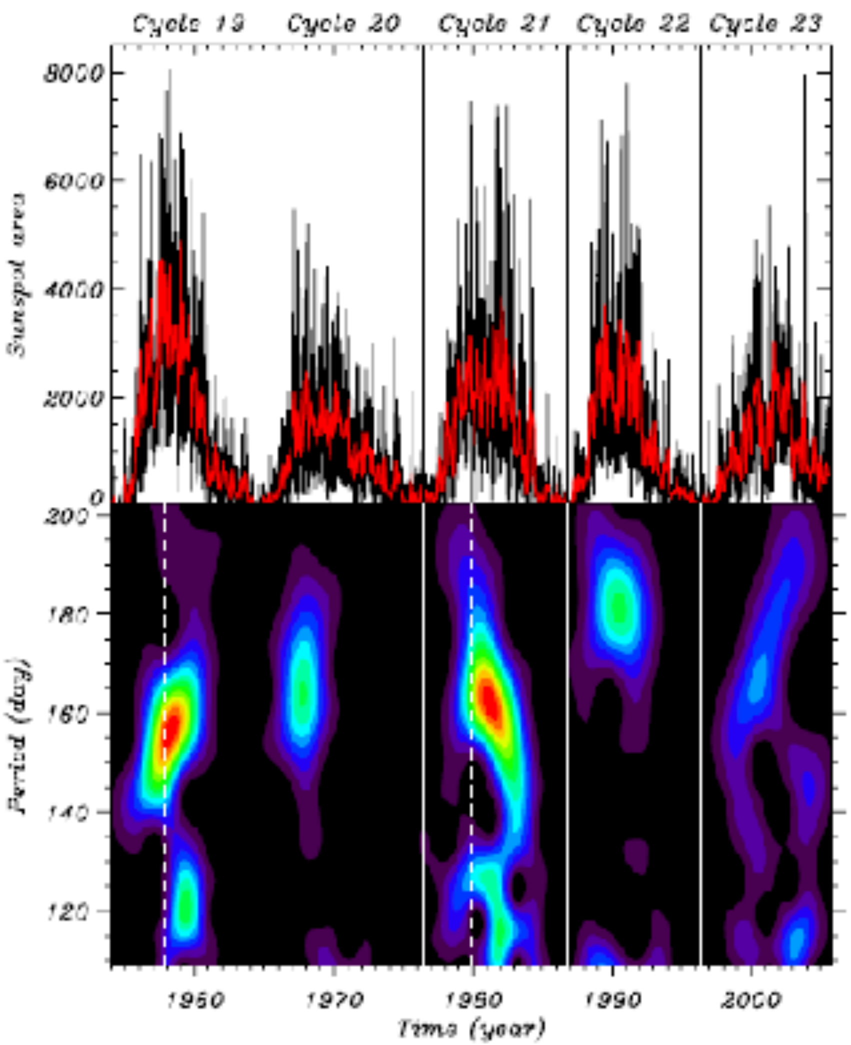


Zaqarashvili (2018)

Rieger-type periodicities



Lean and Brückner (1089)



Zaqarashvili et al. (2010)

Conclusions

➢ Shallow water MHD waves are trapped between latitudes $\pm 20^0 - 40^0$ in the upper overshoot tachocline.

➢ Global fast magneto-Rossby waves have periods of 11 yrs corresponding to Schwabe cycles.

➢ Global slow magneto-Rossby waves have periods of >100 yrs corresponding to Gleissberg cycles.

➢ Global magneto-Kelvin and slow magneto-Rossby-gravity waves have periods of 1-2 yrs corresponding to annual/quasi-biennial oscillations.

➢ Global fast magneto-Rossby-gravity and magneto-inertia-gravity waves have periods of 100-200 days corresponding to Rieger-type periodicity.

➢ Detailed analytical/numerical studies are necessary to make conclusion towards the connection of the shallow water waves to the solar activity and hence to the solar dynamo.