Collective Processes in Collisional Plasma

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Two equivalent microscopic equations: invariant under $t \to -t$ and $\mathbf{v} \to -\mathbf{v}$ (time reversible)

- Method I

$$\dot{\mathbf{r}}_i^a(t) = \mathbf{v}_i^a(t),$$
$$\dot{\mathbf{v}}_i^a(t) = \frac{e_a}{m_a}\mathbf{E}[\mathbf{r}_i^a(t), t]$$
$$\nabla \cdot \mathbf{E}(\mathbf{r}, t) = \sum_{a=e,i} 4\pi e_a \sum_{i=1}^{N} \delta[\mathbf{r} - \mathbf{r}_i^a(t)].$$

- Method II

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} + \frac{e_a}{m_a}\mathbf{E} \cdot \frac{\partial}{\partial \mathbf{v}}\right) \times N_a(\mathbf{r}, \mathbf{v}, t) = 0,$$
$$\nabla \cdot \mathbf{E}(\mathbf{r}, t) = \sum_{a=e,i} 4\pi e_a \int d\mathbf{v} N_a(\mathbf{r}, \mathbf{v}, t),$$
$$N_a(\mathbf{r}, \mathbf{v}, t) = \sum_{i=1}^{N} \delta[\mathbf{r} - \mathbf{r}_i^a(t)]\delta[\mathbf{v} - \mathbf{v}_i^a(t)].$$

Only collisions

E.g., Helander & Sigma 2002; Zank 2014

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}} + \frac{e_a}{m_a}\mathbf{E}\cdot\frac{\partial}{\partial \mathbf{v}}\right) f_a(\mathbf{r},\mathbf{v},t) = C(f_a).$$

If $\mathbf{E}$ and f_a are *averaged* quantities,

$$\boxed{\mathbf{E} = \langle \mathbf{E} \rangle} \qquad \text{and} \qquad \boxed{f_a = \langle N_a \rangle},$$

then the theory is *CORRECT*, but then, it describes only collisional processes, and no collective processes!

Time Irreversibility of Collisional Transport Equation

Collisional transport equation represents irreversible process.

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}+\frac{e_a}{m_a}\mathbf{E}\cdot\frac{\partial}{\partial \mathbf{v}}\right)f_a=C(f_a)$$

$$=-\sum_b \frac{2\pi e_a^2 e_b^2 \ln\Lambda}{m_a}\frac{\partial}{\partial v_i}\int d\mathbf{v}'\,U_{ij}\left(\frac{f_a(\mathbf{v})}{m_b}\frac{\partial f_b(\mathbf{v}')}{\partial v_j'}-\frac{f_b(\mathbf{v}')}{m_a}\frac{\partial f_a(\mathbf{v})}{\partial v_j}\right),$$

$$U_{ij}=\frac{u^2\delta_{ij}-u_i u_j}{u^3},\qquad \mathbf{u}=\mathbf{v}-\mathbf{v}'.$$

If we replace $t\to -t$ ($\mathbf{v}\to-\mathbf{v}$ and $\mathbf{v}'\to-\mathbf{v}'$), then rhs is invariant, but lhs changes sign.

Incorrect collective + collisional theory

If $\mathbf{E}$ and f_a are *total* (average plus fluctuation),

$$\boxed{\mathbf{E} = \langle \mathbf{E} \rangle + \delta\mathbf{E} = \delta\mathbf{E}}, \qquad \boxed{f_a = \langle N_a \rangle + \delta f_a},$$

then

$$\boxed{\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} + \frac{e_a}{m_a} \mathbf{E} \cdot \frac{\partial}{\partial \mathbf{v}} \right) f_a(\mathbf{r}, \mathbf{v}, t) = C(f_a),}$$

is *INCORRECT* since microscopic equation should be reversible.

One cannot simply add the dissipation to the microscopic equation. One must *derive* the irreversible equation from first principles.

Nevertheless, e.g., Lifshitz and Pitaevskii assume

$$\boxed{C(f_a) \approx -\nu_{\rm coll} \langle N_a \rangle},$$

and linearize the equation,

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}}\right) \delta f_a + \frac{e_a}{m_a} \delta \mathbf{E} \cdot \frac{\partial \langle N_a \rangle}{\partial \mathbf{v}} = -\nu_{\rm coll} \langle N_a \rangle ,$$

to obtain collisional damping rate (Spitzer formula) for Langmuir wave,

$$\boxed{\gamma_{\rm coll} = -\frac{\pi n_e e^4 \ln \Lambda}{m_e^2 v_{Te}^3}.}$$

This is *heuristic* at best, if not *wrong!*

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COULOMB COLLISIONS OF PARTICLES IN A TURBULENT PLASMA

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Joint Institute for Nuclear Research

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The effect of Coulomb collisions on the intensity and direction of the spectral shift in a turbulent plasma is considered. A mixed kinetic-hydrodynamic approach is developed which permits the study of those cases when the virtual (difference) waves produced in nonlinear scattering are in the region of frequent collisions. It is shown that in this case Coulomb collisions significantly affect the intensity as well as the direction of the spectral transfer.

$$-i(\omega-\mathbf{kv})f_{\alpha k}^{(1)}+\frac{e_\alpha}{m_\alpha}E_k\frac{(\mathbf{k}\partial f_0/\partial\mathbf{v})}{k}=I_{\alpha k}(1,0)+I_{\alpha k}(0,1), \qquad (2.2)$$

$$-i(\omega-\mathbf{kv})f_{\alpha k}^{(2)}+\frac{e_\alpha}{m_\alpha}\int E_{k_1}\left(\frac{\mathbf{k}_1}{k_1}\frac{\partial f_{\alpha k_2}^{(1)}}{\partial\mathbf{v}}\right)\delta(k-k_1-k_2)\,dk_1\,dk_2$$
$$=I_{\alpha k}(2,0)+I_{\alpha k}(1,1)+I_{\alpha k}(0,2), \qquad (2.3)$$

$$-i(\omega-\mathbf{kv})f_{\alpha k}^{(3)}=-\frac{e_\alpha}{m_\alpha}\int E_{k_1}\left(\frac{\mathbf{k}_1}{k_1}\frac{\partial f_{\alpha k_2}^{(2)}}{\partial\mathbf{v}}\right)\delta(k-k_1-k_2)\,dk_1\,dk_2$$
$$+I_{\alpha k}(3,0)+I_{\alpha k}(2,1)+I_{\alpha k}(1,2)+I_{\alpha k}(0,3). \qquad (2.4)$$

Here

$$I_\alpha=\sum_{\alpha'}I_{\alpha\alpha'}, \qquad \alpha=e,i,$$

and $I_{\alpha\alpha'}$ is taken in the Landau form:[6]

$$I_{\alpha\alpha'}=-\frac{2\pi Le^4}{m_\alpha}\frac{\partial}{\partial v_i}\int\left\{\frac{f_\alpha(\mathbf{v})}{m_{\alpha'}}\frac{\partial f_{\alpha'}(\mathbf{v}')}{\partial v_j'}-\frac{f_{\alpha'}(\mathbf{v}')}{m_\alpha}\frac{\partial f_\alpha(\mathbf{v})}{\partial v_j}\right\}U_{ij}d\mathbf{v}, \qquad (2.5)$$

$$U_{ij}=\frac{1}{w^3}(w^2\delta_{ij}-w_iw_j), \qquad \mathbf{w}=\mathbf{v}-\mathbf{v}'; \qquad (2.6)$$

Irreversibility is the result of statistical averages and subsequent loss of information

Start from exact time reversible Klimontovich equation,

$$\boxed{\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}+\frac{e_a}{m_a}\mathbf{E}\cdot\frac{\partial}{\partial \mathbf{v}}\right)N_a(\mathbf{r},\mathbf{v},t)=0,}$$

$$\boxed{\nabla\cdot\mathbf{E}(\mathbf{r},t)=\sum_{a=e,i}4\pi e_a\int d\mathbf{v}N_a(\mathbf{r},\mathbf{v},t),}$$

$$N_a(\mathbf{r},\mathbf{v},t)=\sum_{i=1}^{N}\delta[\mathbf{r}-\mathbf{r}_i^a(t)]\delta[\mathbf{v}-\mathbf{v}_i^a(t)],$$

$$\dot{\mathbf{r}}_i^a(t)=\mathbf{v}_i^a(t),\qquad \dot{\mathbf{v}}_i^a(t)=\frac{e_a}{m_a}\mathbf{E}[\mathbf{r}_i^a(t),t].$$

Next consider free-particle Klimontovich equation,

$$\boxed{\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right)N_a^0(\mathbf{r},\mathbf{v},t)=0,}$$

$$N_a^0(\mathbf{r},\mathbf{v},t)=\sum_{i=1}^{N}\delta(\mathbf{r}-\mathbf{r}_i^a-\mathbf{v}_i^a t)\delta(\mathbf{v}-\mathbf{v}_i^a).$$

Subtract the two equations,

$$\boxed{\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right)[N_a(\mathbf{r},\mathbf{v},t)-N_a^0(\mathbf{r},\mathbf{v},t)]+\frac{e_a}{m_a}\mathbf{E}\cdot\frac{\partial}{\partial \mathbf{v}}N_a(\mathbf{r},\mathbf{v},t)=0.}$$

This equation describes collective processes where purely single particle dynamics are taken out of the picture.

Statistical Averages and Fluctuations

Then split total microscopic quantities into averages and fluctuations,

$$\boxed{N_a(\mathbf{r},\mathbf{v},t) = \langle N_a(\mathbf{r},\mathbf{v},t)\rangle + \delta N_a(\mathbf{r},\mathbf{v},t) \equiv f_a(\mathbf{r},\mathbf{v},t) + \delta N_a(\mathbf{r},\mathbf{v},t),}$$

$$\boxed{\mathbf{E}(\mathbf{r},t) = \delta\mathbf{E}(\mathbf{r},t),}$$

where ensemble averages $\langle\cdots\rangle$ of the fluctuations are zero, and $f_a(\mathbf{r},\mathbf{v},t)$ is the smoothed one particle distribution function.

Formal Development

Taking ensemble average we obtain formal particle kinetic equation,

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right) f_a(\mathbf{r},\mathbf{v},t) = -\frac{e_a}{m_a}\frac{\partial}{\partial \mathbf{v}}\cdot\left\langle \delta\mathbf{E}(\mathbf{r},t)\,\delta N_a(\mathbf{r},\mathbf{v},t)\right\rangle .$$

From the equation for collective processes we obtain

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right)\left[\delta N_a(\mathbf{r},\mathbf{v},t) - \delta N_a^0(\mathbf{r},\mathbf{v},t)\right] + \frac{e_a}{m_a}\,\delta\mathbf{E}\cdot\frac{\partial f_a(\mathbf{r},\mathbf{v},t)}{\partial \mathbf{v}}$$
$$+\frac{e_a}{m_a}\frac{\partial}{\partial \mathbf{v}}\cdot\left[\delta\mathbf{E}\delta N_a(\mathbf{r},\mathbf{v},t) - \left\langle\delta\mathbf{E}\delta N_a(\mathbf{r},\mathbf{v},t)\right\rangle\right] = 0.$$

The system of equations is closed by Poisson equation

$$\nabla\cdot\delta\mathbf{E}(\mathbf{r},t) = \sum_{a=e,i} 4\pi e_a \int d\mathbf{v}\,\delta N_a(\mathbf{r},\mathbf{v},t).$$

Source Fluctuation $\delta N_a^0(\mathbf{r}, \mathbf{v}, t)$

From the definition

$$N_a^0(\mathbf{r}, \mathbf{v}, t) = \sum_{i=1}^{N} \delta(\mathbf{r} - \mathbf{r}_i^a - \mathbf{v}_i^a t)\delta(\mathbf{v} - \mathbf{v}_i^a),$$

we may obtain

$$\begin{aligned} &\left\langle \delta N_a^0(\mathbf{r}, \mathbf{v}, t)\, \delta N_b^0(\mathbf{r}', \mathbf{v}', t') \right\rangle \\ &= \delta_{ab}\delta[\mathbf{r} - \mathbf{r}' - \mathbf{v}(t - t')]\delta(\mathbf{v} - \mathbf{v}')f_a(\mathbf{r}, \mathbf{v}, t). \end{aligned}$$

Nonlinear Spectral Balance Equation

After everything is said and done ... see, e.g., Yoon, Ziebell, Kontar, Schlickeiser (2016)
Particle kinetic equation;

$$\frac{\partial f_a}{\partial t} = \frac{\pi e_a^2}{m_a^2} \int d\mathbf{k} \int d\omega \left(\frac{\mathbf{k}}{k} \cdot \frac{\partial}{\partial \mathbf{v}} \right) \delta(\omega - \mathbf{k} \cdot \mathbf{v})$$
$$\times \left[\mathrm{Im} \frac{m_a \, \epsilon(\mathbf{k}, \omega)}{2\pi^3 k \, |\epsilon(\mathbf{k}, \omega)|^2} f_a + \langle \delta E^2 \rangle_{\mathbf{k},\omega} \left(\frac{\mathbf{k}}{k} \cdot \frac{\partial f_a}{\partial \mathbf{v}} \right) \right].$$

[Note that this is nothing new at the formal level. The novel aspect arises later as a result of properly dealing with $\langle \delta E^2 \rangle_{\mathbf{k},\omega}$]

Wave kinetic equation;

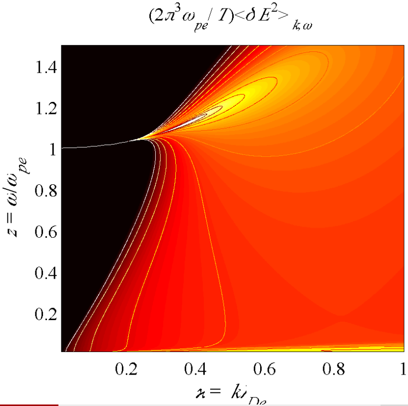
$$
\begin{aligned}
&\frac{i}{2}\,\frac{\partial\,\mathrm{Re}\,\epsilon(\mathbf{k},\omega)}{\partial\omega}\,\frac{\partial\langle\delta E^2\rangle_{\mathbf{k},\omega}}{\partial t}+\mathrm{Re}\,\epsilon(\mathbf{k},\omega)\,\langle\delta E^2\rangle_{\mathbf{k},\omega}+i\,\mathrm{Im}\,\epsilon(\mathbf{k},\omega)\,\langle\delta E^2\rangle_{\mathbf{k},\omega}\\
&+2\int d\mathbf{k}'\,d\omega'\,\Bigg[\{\,\chi^{(2)}(\mathbf{k}',\omega'|\mathbf{k}-\mathbf{k}',\omega-\omega')\,\}^2\left(\frac{\langle\delta E^2\rangle_{\mathbf{k}-\mathbf{k}',\omega-\omega'}}{\epsilon(\mathbf{k}',\omega')}+\frac{\langle\delta E^2\rangle_{\mathbf{k}',\omega'}}{\epsilon(\mathbf{k}-\mathbf{k}',\omega-\omega')}\right)\\
&-\bar{\chi}^{(3)}(\mathbf{k}',\omega'|-\mathbf{k}',-\omega'|\mathbf{k},\omega)\,\langle\delta E^2\rangle_{\mathbf{k}',\omega'}\Bigg]\,\langle\delta E^2\rangle_{\mathbf{k},\omega}\\
&-2\int d\mathbf{k}'\,d\omega'\,\frac{|\,\chi^{(2)}(\mathbf{k}',\omega'|\mathbf{k}-\mathbf{k}',\omega-\omega')\,|^2}{\epsilon^*(\mathbf{k},\omega)}\,\langle\delta E^2\rangle_{\mathbf{k}',\omega'}\,\langle\delta E^2\rangle_{\mathbf{k}-\mathbf{k}',\omega-\omega'}\\
&=\frac{2}{\pi}\,\frac{1}{k^2\,\epsilon^*(\mathbf{k},\omega)}\sum_a e_a^2\int d\mathbf{v}\;\delta(\omega-\mathbf{k}\cdot\mathbf{v})\,f_a(\mathbf{v})\\
&-\frac{4}{\pi}\int d\mathbf{k}'\,d\omega'\,\frac{1}{k'^2\,|\epsilon(\mathbf{k}',\omega')|^2}\left(\frac{\{\,\chi^{(2)}(\mathbf{k}',\omega'|\mathbf{k}-\mathbf{k}',\omega-\omega')\,\}^2}{\epsilon(\mathbf{k}-\mathbf{k}',\omega-\omega')}\,\langle\delta E^2\rangle_{\mathbf{k},\omega}\right.\\
&\left.-\frac{|\,\chi^{(2)}(\mathbf{k}',\omega'|\mathbf{k}-\mathbf{k}',\omega-\omega')\,|^2}{\epsilon^*(\mathbf{k},\omega)}\,\langle\delta E^2\rangle_{\mathbf{k}-\mathbf{k}',\omega-\omega'}\right)\sum_a e_a^2\int d\mathbf{v}\;\delta(\omega'-\mathbf{k}'\cdot\mathbf{v})\,f_a(\mathbf{v})\\
&-\frac{4}{\pi}\int d\mathbf{k}'\,d\omega'\,\frac{1}{|\mathbf{k}-\mathbf{k}'|^2\,|\epsilon(\mathbf{k}-\mathbf{k}',\omega-\omega')|^2}\left(\frac{\{\,\chi^{(2)}(\mathbf{k}',\omega'|\mathbf{k}-\mathbf{k}',\omega-\omega')\,\}^2}{\epsilon(\mathbf{k}',\omega')}\,\langle\delta E^2\rangle_{\mathbf{k},\omega}\right.\\
&\left.-\frac{|\,\chi^{(2)}(\mathbf{k}',\omega'|\mathbf{k}-\mathbf{k}',\omega-\omega')\,|^2}{\epsilon^*(\mathbf{k},\omega)}\,\langle\delta E^2\rangle_{\mathbf{k}',\omega'}\right)\sum_a e_a^2\int d\mathbf{v}\;\delta[\omega-\omega'-(\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}]\,f_a(\mathbf{v}).
\end{aligned}
$$

[Again this is nothing new at the formal level]

Spontaneous emission of electric field fluctuation

First approach is to treat $\langle E^2 \rangle_{\mathbf{k},\omega}$ as largely determined from thermal fluctuation,

$$\langle E^2 \rangle_{\mathbf{k},\omega} = \frac{2}{\pi} \frac{1}{k^2 |\epsilon(\mathbf{k},\omega)|^2} \sum_a e_a^2 \int d\mathbf{v} \delta(\omega - \mathbf{k}\cdot\mathbf{v}) f_a.$$



Collisional kinetic equation emerges

Time irreversible collisional kinetic equation emerges if we only consider spontaneous fluctuation from non-eigenmodes.

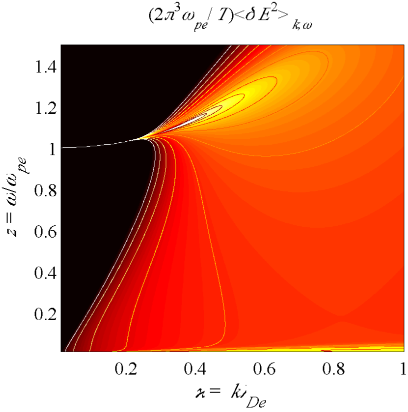
$$\frac{\partial f_a}{\partial t} = \frac{2e_a^2}{m_a^2}\sum_b e_b^2 \int d\mathbf{k} \int d\mathbf{v}' \frac{\mathbf{k}}{k}\cdot\frac{\partial}{\partial \mathbf{v}}\frac{\delta(\mathbf{k}\cdot\mathbf{v}-\mathbf{k}'\cdot\mathbf{v}')}{k^2|\epsilon(\mathbf{k},\mathbf{k}\cdot\mathbf{v})|^2} \times \left[\frac{\mathbf{k}}{k}\cdot\frac{\partial f_a(\mathbf{v})}{\partial \mathbf{v}} f_b(\mathbf{v}') - \frac{m_a}{m_b}\frac{\mathbf{k}}{k}\cdot\frac{\partial f_b(\mathbf{v}')}{\partial \mathbf{v}'} f_a(\mathbf{v})\right].$$

This equation describes *collisional* processes only.

Only Eigenmodes (Collective Processes Only)

If we are interested only in eigenmodes,

$$\left\langle E^2 \right\rangle_{\mathbf{k},\omega} = I_{\mathbf{k}} \delta(\omega - \omega_{\mathbf{k}}).$$



Quasilinear Kinetic Equation

Time irreversible quasilinear kinetic equation results from taking only eigenmodes into consideration.

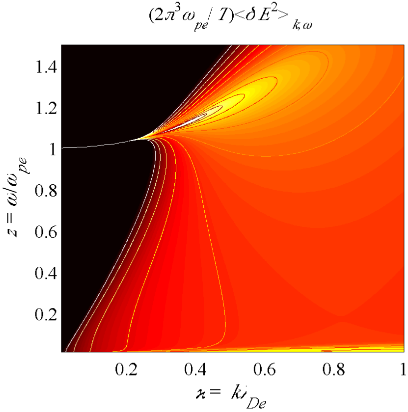
$$\begin{aligned}
\frac{\partial f_a}{\partial t} &= \frac{\partial}{\partial v_i}\left(A_i f_a + D_{ij}\frac{\partial f_a}{\partial v_j}\right). \\
A_i &= \frac{e^2}{4\pi m_e}\int d\mathbf{k}\,\frac{k_i}{k^2}\omega_{\mathbf{k}}\delta(\omega_{\mathbf{k}} - \mathbf{k}\cdot\mathbf{v}), \\
D_{ij} &= \frac{\pi e^2}{m_e^2}\int d\mathbf{k}\,\frac{k_i k_j}{k^2}\delta(\omega_{\mathbf{k}} - \mathbf{k}\cdot\mathbf{v}) I_{\mathbf{k}}.
\end{aligned}$$

This equation only describes *collective* processes.

Collective plus collisional processes

Both thermal fluctuations and eigenmodes

$$\left\langle E^2 \right\rangle_{\mathbf{k},\omega} = \boxed{\frac{2}{\pi}\frac{1}{k^2|\epsilon(\mathbf{k},\omega)|^2}\sum_a e_a^2 \int d\mathbf{v}\,\delta(\omega - \mathbf{k}\cdot\mathbf{v}) f_a} + \boxed{I_{\mathbf{k}}\delta(\omega - \omega_{\mathbf{k}})}.$$



Generalized Particle Kinetic Equation

The resulting particle kinetic equation describes both collisional and collective processes,

$$\begin{aligned}
\frac{\partial f_e(\mathbf{v})}{\partial t} &= \frac{2e^2}{m_e^2}\frac{\partial}{\partial v_i}\sum_b e_b^2 \int d\mathbf{k}\,\frac{k_i k_j}{k^2}\int d\mathbf{v}'\,\frac{\delta(\mathbf{k}\cdot\mathbf{v}-\mathbf{k}'\cdot\mathbf{v}')}{k^2|\epsilon(\mathbf{k},\mathbf{k}\cdot\mathbf{v})|^2} \\
&\quad\times\left[\frac{\partial f_e(\mathbf{v})}{\partial v_j} f_b(\mathbf{v}') - \frac{m_e}{m_b}\frac{\partial f_b(\mathbf{v}')}{\partial v_j'} f_e(\mathbf{v})\right] \\
&\quad+\frac{\pi e^2}{m_e^2}\frac{\partial}{\partial v_i}\int d\mathbf{k}\,\delta(\omega_{\mathbf{k}}-\mathbf{k}\cdot\mathbf{v}) \\
&\quad\times\left(\frac{m_e}{4\pi^2}\frac{k_i}{k^2}\,\omega_{\mathbf{k}} f_e(\mathbf{v}) + \frac{k_i k_j}{k^2}\, I_{\mathbf{k}}\frac{\partial f_e(\mathbf{v})}{\partial v_j}\right).
\end{aligned}$$

Generalized Wave Kinetic Equation

The generalized wave kinetic equation (derivation omitted) also describes both collective and collisional processes,

$$\begin{aligned}
\frac{\partial I_{\mathbf{k}}^L}{\partial t} &= \frac{\pi\omega_p^2}{k^2}\int d\mathbf{v}\,\delta\left(\omega_{\mathbf{k}}^L-\mathbf{k}\cdot\mathbf{v}\right)\left(\frac{n_0e^2}{\pi}\,f_e(\mathbf{v})+\omega_{\mathbf{k}}^L\,I_{\mathbf{k}}^L\,\mathbf{k}\cdot\frac{\partial f_e(\mathbf{v})}{\partial\mathbf{v}}\right) \\
&\quad +2\gamma_{\mathbf{k}}^{\mathrm{coll}L}I_{\mathbf{k}}^L+P_{\mathbf{k}}^L, \\
\gamma_{\mathbf{k}}^{\mathrm{coll}L} &= \omega_{\mathbf{k}}^L\frac{4n_ee^4\omega_p^2}{T_e^2}\int d\mathbf{k}'\,\frac{(\mathbf{k}\cdot\mathbf{k}')^2\lambda_D^4}{k^2k'^4|\epsilon(\mathbf{k}',\omega_{\mathbf{k}}^L)|^2} \\
&\quad\times\left(1+\frac{T_e}{T_i}+(\mathbf{k}-\mathbf{k}')^2\lambda_D^2\right)^{-2}\int d\mathbf{v}\,\mathbf{k}'\cdot\frac{\partial f_e(\mathbf{v})}{\partial\mathbf{v}}\delta(\omega_{\mathbf{k}}^L-\mathbf{k}'\cdot\mathbf{v}), \\
P_{\mathbf{k}}^L &= \frac{3e^2}{4\pi^3}\frac{1}{(\omega_{\mathbf{k}}^L)^2}\left(1-\frac{m_e}{m_i}\frac{T_e}{T_i}\right)^2\frac{v_e^4}{k^2}\int d\mathbf{k}'\,k'^2|\mathbf{k}-\mathbf{k}'|^2 \\
&\quad\times\left(1+\frac{T_e}{T_i}+k'^2\lambda_D^2\right)^{-2}\left(1+\frac{T_e}{T_i}+(\mathbf{k}-\mathbf{k}')^2\lambda_D^2\right)^{-2} \\
&\quad\times\int d\mathbf{v}\int d\mathbf{v}'\sum_{a,b}f_a(\mathbf{v})f_b(\mathbf{v}')\delta[\omega_{\mathbf{k}}^L-\mathbf{k}\cdot\mathbf{v}+\mathbf{k}'\cdot(\mathbf{v}-\mathbf{v}')].
\end{aligned}$$

Spitzer (incorrect) vs Correct Collisional Damping Rates

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Collisional damping rates for plasma waves

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Incorrect Spitzer formula

$$\bar{\gamma}_{\rm coll} \equiv \frac{\gamma_{\rm coll}}{\omega_{pe}} = -\frac{\pi n_e e^4 \ln \Lambda}{m_e^2 v_{Te}^3 \omega_{pe}} = -\pi g \ln\left(\frac{1}{2^{3/2}(4\pi g)}\right).$$

Correct collisional damping rate

$$\gamma_{\mathbf{q}}^{L({\rm coll})} \equiv \frac{\gamma_{\mathbf{k}}^{L({\rm coll})}}{\omega_{pe}} = \frac{2 g z_{\mathbf{q}}^L}{q^2} \int d\mathbf{q}' \frac{(\mathbf{q}\cdot\mathbf{q}')^2}{q'^4 |\epsilon(\mathbf{q}', z_{\mathbf{q}}^L)|^2}$$
$$\times \left(1 + \frac{T_e}{T_i} + \frac{(\mathbf{q}-\mathbf{q}')^2}{2}\right)^{-2}$$
$$\times \int d\mathbf{u}\, \mathbf{q}' \cdot \frac{\partial \Phi_e(\mathbf{u})}{\partial \mathbf{u}} \delta\left(z_{\mathbf{q}}^L - \mathbf{q}'\cdot\mathbf{u}\right),$$

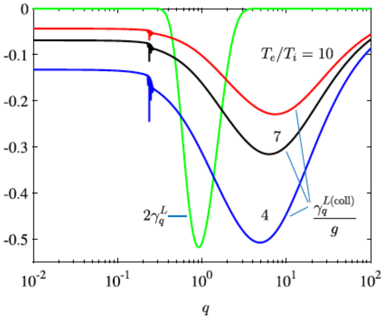

FIG. 1. Normalized collisional damping for L waves, $\gamma_q^{L(\mathrm{coll})}/g$, vs normalized wavenumber q, for three values of the ratio T_e/T_i. The dimensionless Landau damping rate γ_q^L is also plotted in green. Note that the Landau damping rate is *not* divided by the factor g. The factor 2, which multiplies γ_q^L is for the sake of visual presentation.

plasma parameter g must be small by definition, so we consider several different choices, $g = 10^{-10}$, 10^{-8}, 10^{-6}, and 10^{-4}. For these choices, we find that $\bar{\gamma}_{\mathrm{coll}}/g \sim -61.12$, -46.6524, -32.1849, and -17.7173, which are all far higher in absolute value than those depicted in Fig. 1. This shows that the use of incorrect collisional damping rate may greatly over-estimate the actual damping rate.

Collective Processes in Collisional Plasmas

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Generation of Suprathermal Electrons by Collective Processes in Collisional Plasma

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Wave kinetic equation

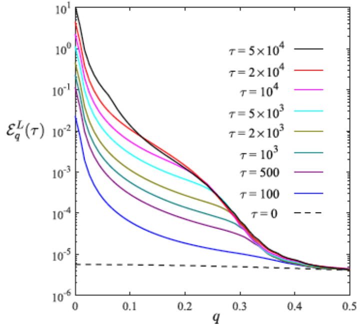
$$\frac{\partial I_{\mathbf{k}}^{\sigma L}}{\partial t} = \frac{\pi\omega_p^2}{k^2}\int d\mathbf{v}\,\delta(\sigma\omega_{\mathbf{k}}^L - \mathbf{k}\cdot\mathbf{v}) \times \left(\frac{n_0 e^2}{\pi}F_e(\mathbf{v}) + \sigma\omega_{\mathbf{k}}^L I_{\mathbf{k}}^{\sigma L}\,\mathbf{k}\cdot\frac{\partial F_e(\mathbf{v})}{\partial \mathbf{v}}\right) + 2\gamma_{\mathbf{k}}^{\sigma L} I_{\mathbf{k}}^{\sigma L} + P_{\mathbf{k}}^{\sigma L},$$

Particle kinetic equation

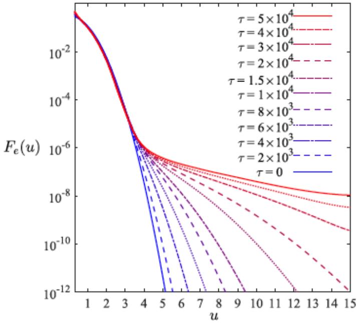
$$\frac{\partial F_a(\mathbf{v})}{\partial t} = \frac{\partial}{\partial v_i}\left(A_i(\mathbf{v})F_a(\mathbf{v}) + D_{ij}(\mathbf{v})\frac{\partial F_a(\mathbf{v})}{\partial v_j}\right) + \sum_b \theta_{ab}(F_a, F_b),$$

Collective Processes in Collisional Plasmas

Wave intensity



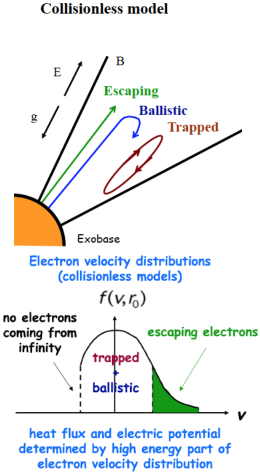
Particle distribution



High energy tail is spontaneously generated in highly collisional plasma.

Kappa Distribution and Velocity Filtration

$$
\begin{aligned}
f(r,\mathbf{v}) &\propto \left(1+\frac{mv^2}{2\kappa T}+q(r)\right)^{-\kappa+1}, \\
q(r) &= \frac{m\Phi_g(r)+Ze\Phi(r)}{T}, \\
n(r) &= n_0\left(1+\frac{q(r)}{\kappa}\right)^{-\kappa+\frac{1}{2}}, \\
T(r) &= T_0\frac{\kappa}{\kappa-\frac{3}{2}}\left(1+\frac{q(r)}{\kappa}\right).
\end{aligned}
$$



Conclusion

- Correct formulation of combined collisional and collective theory is presented.
- It is shown that highly collisional plasma (such as in the chromospheric plasma) spontaneously generates high energy tail.
- Such a feature is required in the velocity filtration model of the coronal heating, but never demonstrated hitherto.