MHD Instabilities in Poynting dominated jets

E. Sobacchi & Y.E. Lyubarsky
Universality of relativistic jets

- PWNe
- microquasars
- GRBs
- AGN

Scale $\sim 1$ parsec

Scale $\sim 10^5$ parsec
How (do we think) jets are launched?

- Rotation of the compact object (NS or BH) twists the magnetic field lines.
- Plasma accelerated by magnetic tension.

Extraction of rotational energy via the Blandford-Znajek process:

- Rotation of the compact object (NS or BH) twists the magnetic field lines.
- Plasma accelerated by magnetic tension.
How is the observed radiation emitted?

The jet is initially Poynting dominated

How to convert the Poynting flux into the observed radiation?

Since gradual acceleration is inefficient, one possibility is energy dissipation driven by MHD instabilities.
Two flow regimes:

Causally connected jets

\[ \theta \gamma < 1: \text{light crossing time shorter than expansion time} \]

\[ \theta = \text{opening angle}, \gamma = \text{Lorentz factor} \]

Causally disconnected jets

\[ \theta \gamma > 1: \text{light crossing time longer than expansion time} \]

Their stability properties are different
Two flow regimes:

Causally connected jets

$\theta \gamma < 1$: light crossing time shorter than expansion time

strong causal connection ($\theta \gamma < 1$)

Causally disconnected jets

$\theta \gamma > 1$: light crossing time longer than expansion time

if there is even no weak causal connection ($\theta \gamma > \sigma^{1/2}$)

shocks arise and weak causal connection ($1 < \theta \gamma < \sigma^{1/2}$) is restored

Their stability properties are different
Causally connected jets

$\theta \gamma < 1$: light crossing time shorter than expansion time

$E \approx B_\phi \approx \gamma B_p$

The fields are of the same order in the proper frame → residual of hoop stress and electric force balanced by $B_p$

The flow structure at any distance from the source is relaxed to an appropriate cylindrical equilibrium configuration
Global perturbations can grow

In cylindrical jets the most dangerous instability is due to kink (m=1) modes, but

- growth rate is suppressed in the ultra-relativistic regime ($\gamma \gg 1$)
- growth rate is suppressed when the radial profile of the poloidal magnetic field is flat (which might be the case in a realistic scenario; see Narayan et al. 2009)
- GRB and PWN outflows are not causally connected (i.e. they have $\theta \gamma \gg 1$)

Mizuno et al. (2012)
Causally disconnected jets

$\theta \gamma > 1$: light crossing time longer than expansion time

$E \approx B_\phi > \gamma B_\rho$

the poloidal field can be neglected $\rightarrow$
flow can be conceived as coaxial magnetic loops

The flow structure is far from cylindrical equilibrium configuration
Causally disconnected jets - instabilities

Global perturbation cannot grow unless the flow is converging due to confinement by external pressure.

Simulations show dissipation at the recollimation nozzle: what are the fundamental physical parameters driving the instability?

Bromberg & Tchekhovskoy (2016)
Causally disconnected jets - instabilities

A simple picture

magnetic coils with small displacement \( \delta R \ll R_1 \)

\( \delta R \approx R_2 \)

\( \delta R/R \approx B/\phi \)

R shrinks approaching the recollimation nozzle

Since \( \delta B/B \approx \delta R/R \), the perturbations grow by a factor \( R_1/R_2 \) as the outer radius \( R \) shrinks. Effect depends only on the geometry
Causally disconnected jets - instabilities

force-free equilibrium solution

oscillating outer radius:

\[ R = R_0 \left| \cos \left( \frac{z}{L} \right) \right| \]

toroidal magnetic field:

\[ B_0 = \alpha \frac{r}{R^2} \left[ 1 + \frac{1}{6} \left( 1 + 3\tau^2 \right) \frac{r^2}{L^2} \right] e_\phi \]

\[ E_0 = \alpha \frac{r}{R^2} \left[ e_r + \tau \frac{r}{L} e_z \right] \]

with \( \tau \equiv \tan \left( \frac{z}{L} \right) \)

drift velocity:

\[ \gamma^2 = \frac{B_0^2}{B_0^2 - E_0^2} \approx 3 \frac{L^2}{r^2} \gg 1 \]
Causally disconnected jets - instabilities

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The calculation

- linearised Maxwell’s and Euler’s equations
- perturbed fields of the form
  \[ f(r, z) \exp [i\omega (z - t) + im\phi] \]

slowly varying
(on a scale \( \sim L \) along \( z \))

moves approximately at the speed of light
Causally disconnected jets - instabilities

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The calculation

Self-similar solution with two independent modes close to the recollimation nozzle (\( R \ll R_0 \))

\[ \delta B = \frac{f_1 (r/R)}{R} + \frac{f_2 (r/R)}{R^2} \]

The unperturbed field scales as

\[ B_0 \propto \frac{1}{R} \]

- **converging flow** \( \rightarrow \delta B/B \propto R^{-1} \rightarrow \text{unstable} \)
- **diverging flow** \( \rightarrow \delta B/B \sim \text{const} \rightarrow \text{stable} \)
Causally disconnected jets - instabilities

Perturbations grow by a factor $R_1/R_2$ as the outer radius $R$ shrinks
Effect depends only on the geometry $\rightarrow R_1/R_2 \gg 1$ is required

$R_1/R_2 \gg 1$ is likely when
(i) central engine radiates most of the Poynting flux on the equatorial plane
(ii) plasma magnetic pressure exceeds confining pressure

Recollimation-driven instability promising explanation for PWN flares (Lyubarsky 2012) and possibly GRB high-energy emission (Bromberg & Tchekhovskoy 2016)
Conclusions

Causally connected outflows ($\theta\gamma < 1$):
- global MHD instabilities can always grow
- the growth rate is suppressed if the flow is ultra-relativistic ($\gamma \gg 1$)
- the growth rate is suppressed if the radial profile of the poloidal magnetic field is flat
- kink instability may be relevant for AGN

Causally disconnected outflows ($\theta\gamma > 1$):
- global MHD instabilities can grow only if the flow is recollimated by the external pressure
- if the outer radius of the jet shrinks from $R_1$ to $R_2 < R_1$, perturbations grow by a factor $R_1 / R_2$. Impact of the instability depends only on the geometry of the flow
- recollimation-driven instabilities promising explanation for gamma-ray emission in PWNe and possibly in GRBs