Study of the energy storage process in the magnetosphere using a novel nonlinear frequency analysis

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OUTLINE

Energy storage process in the magnetosphere

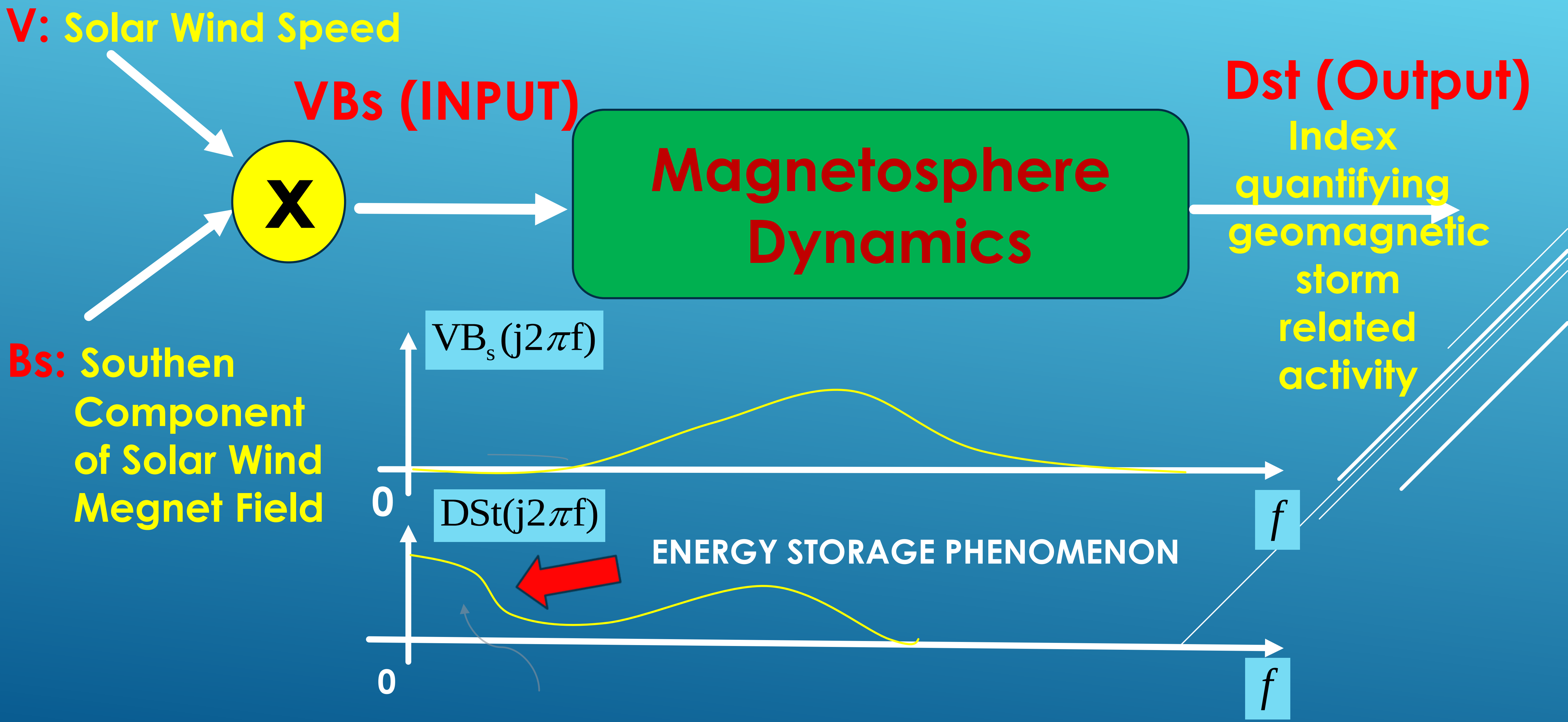
The Nonlinear AutoRegrersive with eXogenous (NARX) input model of a magnetosphere system

A novel analysis of the energy storage process of the magnetosphere system

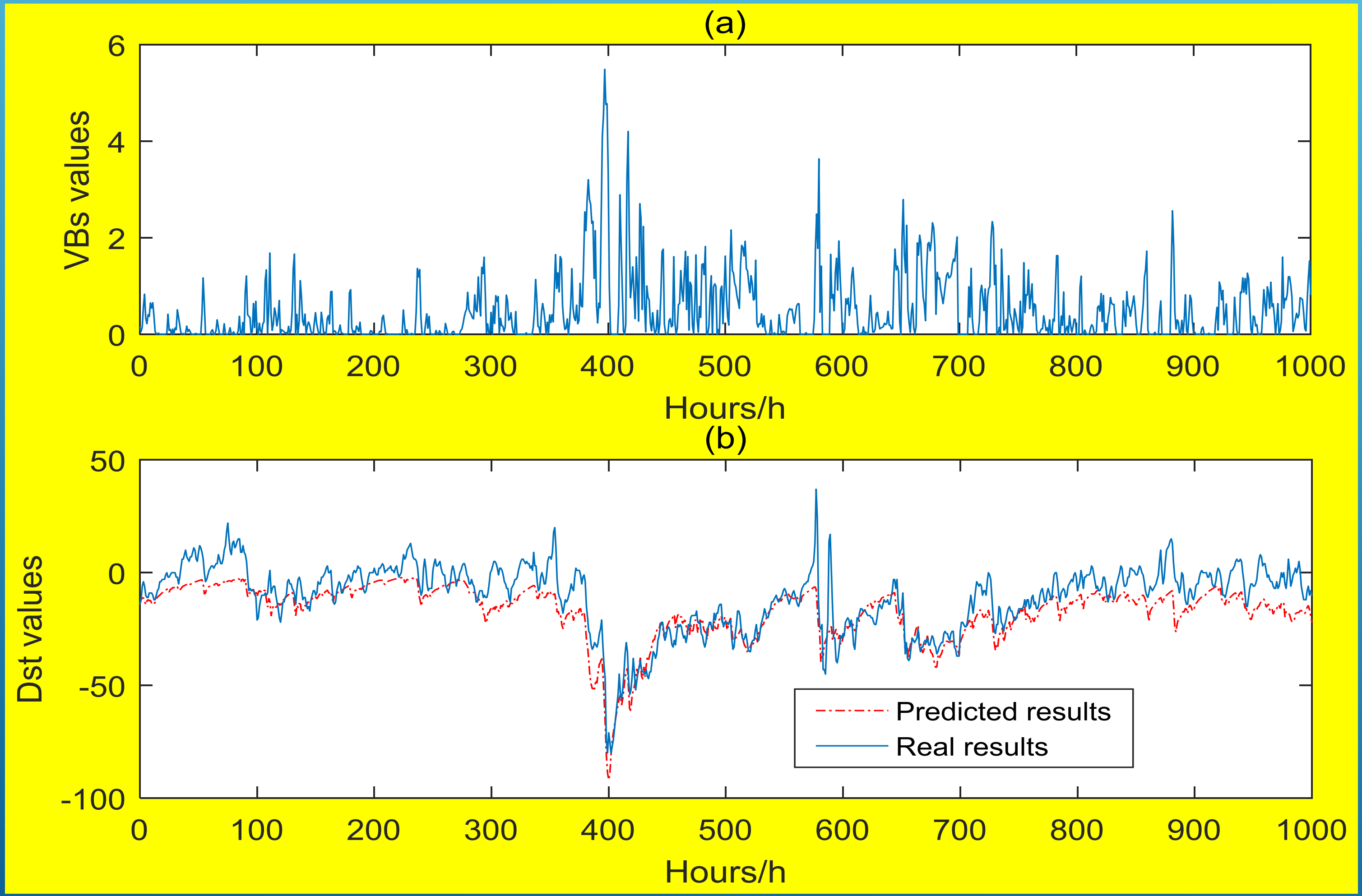
Revelation of energy storage phenomena with the magnetosphere system from satellite data analysis and possible significance

Conclusions

Energy storage process in the magnetosphere



The Nonlinear AutoRegrersive with eXogenous (NARX) input model of a magnetosphere system



$$
\begin{aligned}
y(k) = {} & 0.953y(k-1) - 4.86u(k-1) \\
& + 3.24u(k-2) - 0.5u(k-1)^2 \\
& - 0.06y(k-1)u(k-1) \\
& - 0.6u(k-2)^2
\end{aligned}
$$

y(k): DSt (measurement of geomagnetic storm activities)

u(k): VBs (V x Bs)

fs= 1/3600 Hz

A novel analysis of the energy storage process of the magnetosphere system: (i) Observations

VBs u(k)

$$u(k)=\cos\left(\frac{0.02\pi}{3600}\times 3600\times k\right)=\cos(0.02\pi k),\quad k=,....,.6000.$$

$$u(k)=\cos(0.02\pi k)+\cos(0.04\pi k)+\cos(0.06\pi k),\ k=0,\ldots,6000$$

$$u(k)=\frac{A}{\pi}\frac{\sin(0.06\pi(k-3000))-\cos(0.02\pi(k-3000))}{3600(k-3000)}\quad k=0,\ldots,6000$$

VBs frequencies

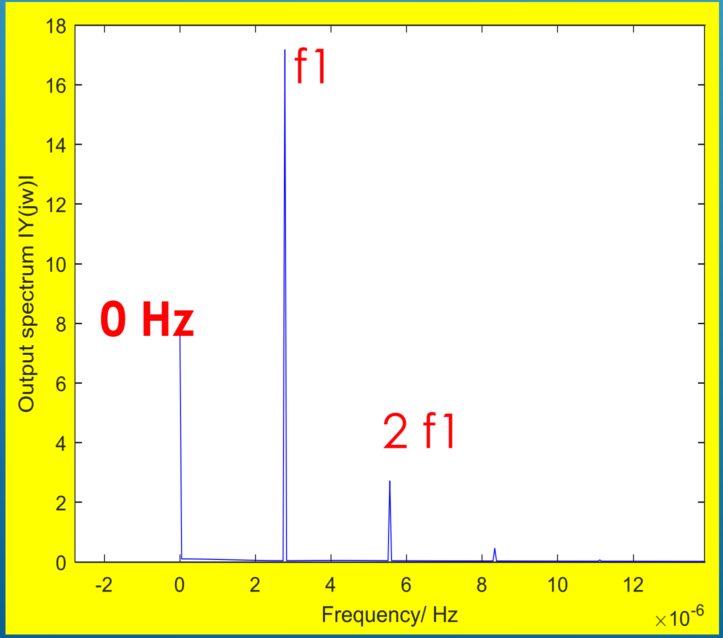
$$f_1=\frac{0.01}{3600}\ \text{Hz}$$

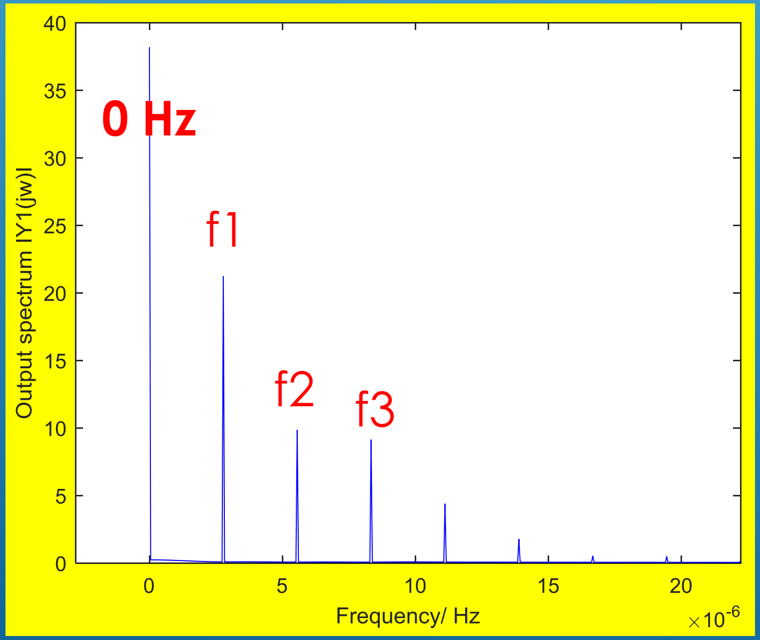
$$f_1=\frac{0.01}{3600}\ \text{Hz},\ f_2=\frac{0.02}{3600}\ \text{Hz},\ f_3=\frac{0.03}{3600}\ \text{Hz}$$

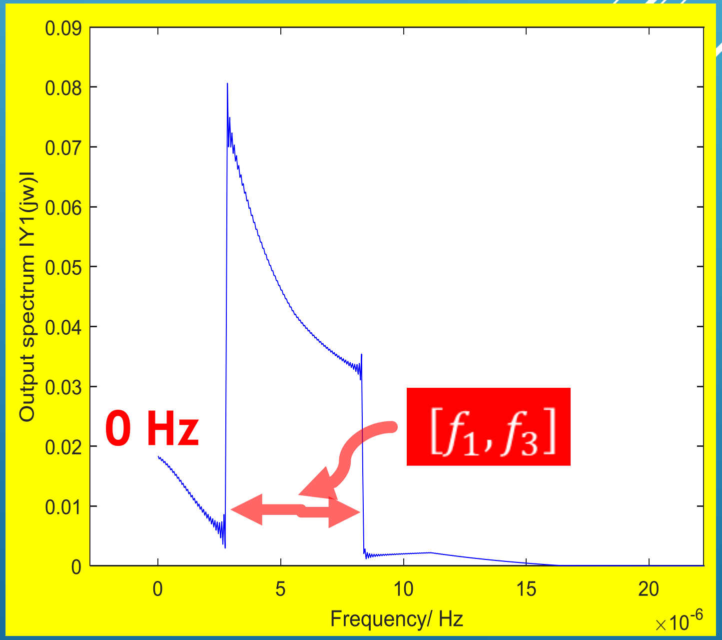
$$f\in[f_1,f_3]=\left[\frac{0.01}{3600},\frac{0.03}{3600}\right]\ \text{Hz}$$

Dst Spec-trum

$Y(j2\pi f)$







A novel analysis of the energy storage process of the magnetosphere system: (ii) Analysis

NARX model of the magnetosphere dynamics

$$y(k)=0.953y(k-1)-4.86u(k-1)+3.24u(k-2)-0.5u(k-1)^2-0.06y(k-1)u(k-1)-0.6u(k-2)^2$$

Generalized Associated Linear Equations (GALEs) of the NARX model

$$\begin{cases} y_1(k)=0.953y_1(k-1)-4.86u(k-1)+3.24u(k-2) \\ y_2(k)=0.953y_2(k-1)-0.5u^2(k-1) \\ \quad -0.06y_1(k-1)u(k-1)-0.6u^2(k-2) \\ y_3(k)=0.953y_3(k-1)-0.06y_2(k-1)u(k-1) \end{cases}$$

$y_1(k), y_2(k), y_3(k), \ldots$

components of $y(k)$ contributed by different orders nonlinearity

$Y_1(j2\pi f)$ $Y_2(j2\pi f)$ $Y_3(j2\pi f)$

Case 1

$$u(k) = \cos\left(\frac{0.02\pi}{3600} \times 3600 \times k\right) = \cos(0.02\pi k), \quad k = ,....,.6000.$$

$$f_1 = \frac{0.01}{3600}\ \text{Hz}$$

Case 2

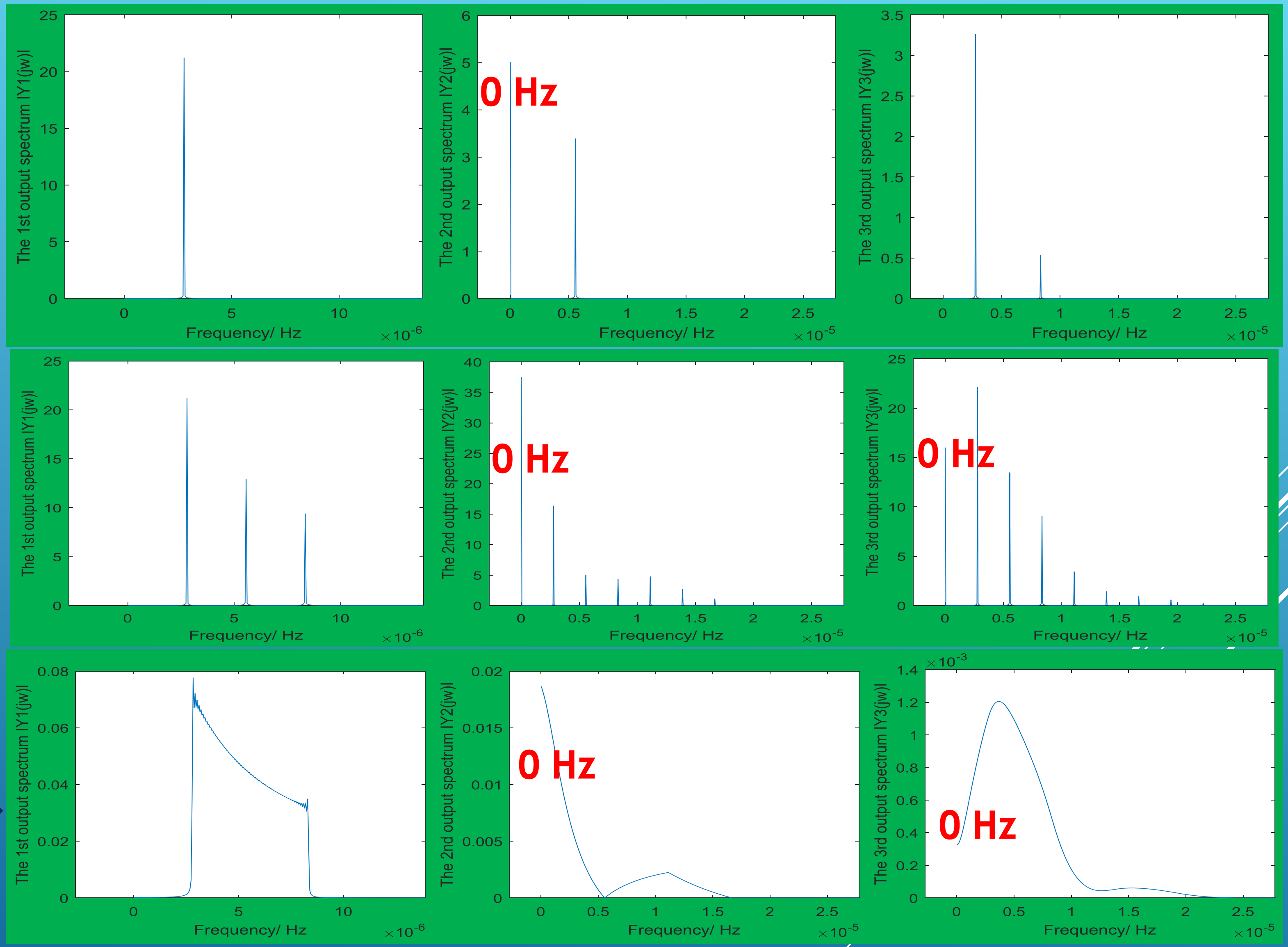
$$u(k) = \cos(0.02\pi k) + \cos(0.04\pi k) + \cos(0.06\pi k), \ k = 0,...,6000$$

$$f_1 = \frac{0.01}{3600}\ \text{Hz}, \ f_2 = \frac{0.02}{3600}\ \text{Hz}, \ f_3 = \frac{0.03}{3600}\ \text{Hz}$$

Case 3

$$u(k) = \frac{A}{\pi} \frac{\sin(0.06\pi(k-3000)) - \cos(0.02\pi(k-3000))}{3600(k-3000)}, \quad k = 0,...,6000$$

$$f \in [f_1, f_3] = \left[\frac{0.01}{3600}, \frac{0.03}{3600}\right]\ \text{Hz}$$



The analysis reveals which order of system nonlinearity contributes to the energy storage

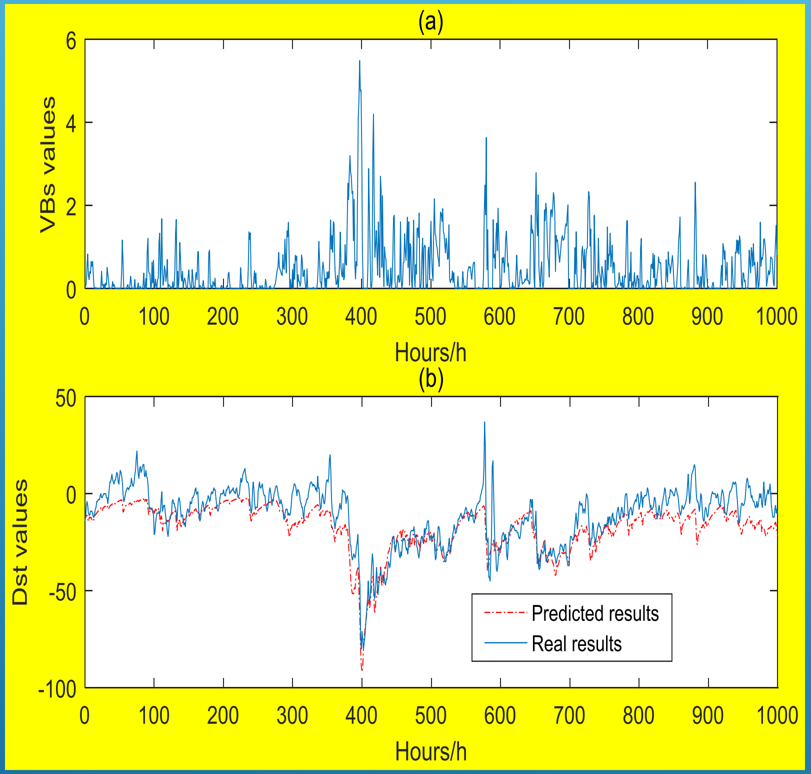
$$y(k)=0.953y(k-1)-4.86u(k-1)+3.24u(k-2)-0.5u(k-1)^2-0.06y(k-1)u(k-1)-0.6u(k-2)^2$$

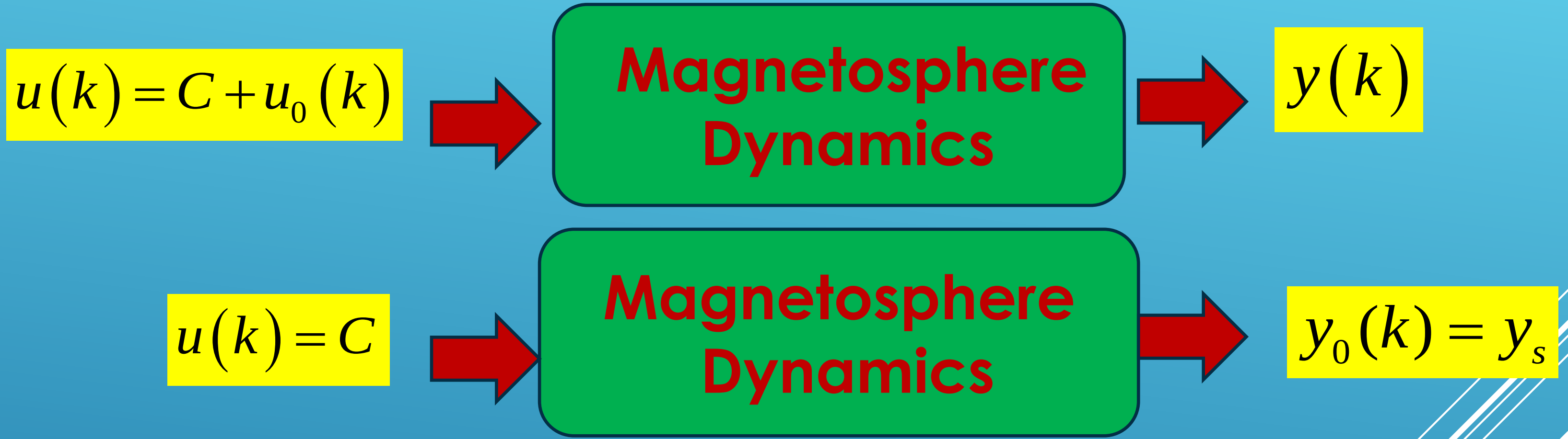
Case	Input frequencies	Order of nonlinearity making contribution to energy storage
Case 1	$f_1=\frac{0.01}{3600}$ Hz	2nd order
Case 2	$f_1=\frac{0.01}{3600}$ Hz, $f_2=\frac{0.02}{3600}$ Hz, $f_3=\frac{0.03}{3600}$ Hz	2nd and 3rd order
Case 3	$f\in[f_1, f_3]=\left[\frac{0.01}{3600},\frac{0.03}{3600}\right]$ Hz	2nd and 3rd order

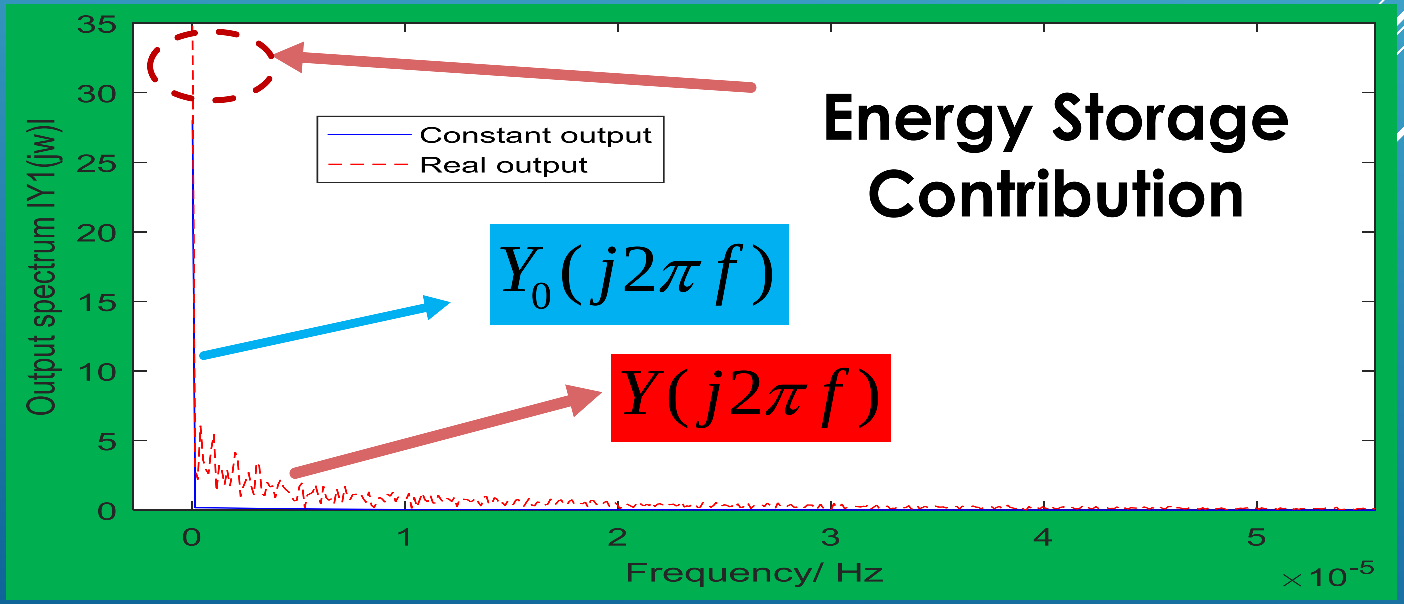
$$\begin{cases} y_1(k)=0.953y_1(k-1)-4.86u(k-1)+3.24u(k-2) \\ y_2(k)=0.953y_2(k-1)-0.5u^2(k-1) \\ \quad -0.06y_1(k-1)u(k-1)-0.6u^2(k-2) \\ y_3(k)=0.953y_3(k-1)-0.06y_2(k-1)u(k-1) \end{cases}$$

Coefficient 0.06 plays the most significant role making energy storage happen

Revelation of energy storage phenomena with the magnetosphere system from satellite data analysis

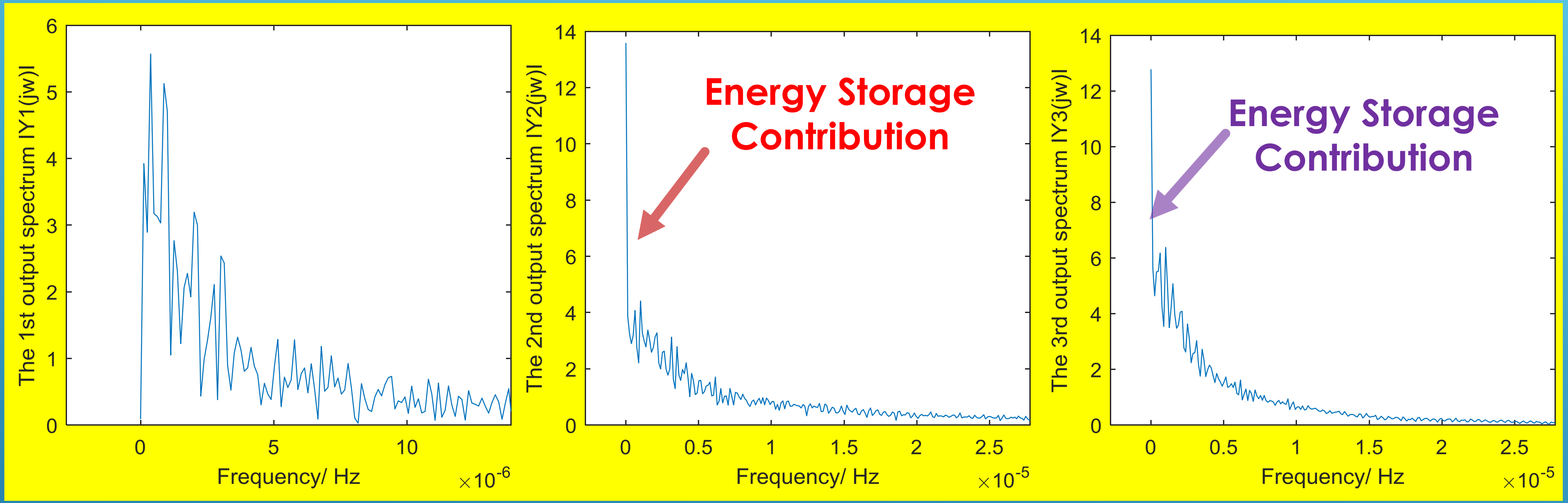






The result of the application of the novel analysis to the satellite data

$Y_1(j2\pi f)$ $Y_2(j2\pi f)$ $Y_3(j2\pi f)$



$$y(k)=0.953y(k-1)-4.86u(k-1)+3.24u(k-2)-0.5u(k-1)^2-0.06y(k-1)u(k-1)-0.6u(k-2)^2$$

Showing again the importance of Coefficient 0.06 in producing the energy storage

An Idea of investigation Of how the energy storage can be affected by relevant physical parameters in space

How the idea could work?

- **Build a parametric NARX model of magnetosphere dynamics as**

$$
\begin{aligned}
y(k) = {} & a_1(P)y(k-1) + b_1(P)u(k-1) \\
& + b_2(P)u(k-2) + c_1(P)u(k-1)^2 \\
& + c_2(P)y(k-1)u(k-1) \\
& + c_3(P)u(k-2)^2
\end{aligned}
$$

where

$P=[P_1,\cdots,P_N]$: Masurable parameters in space that can affect energy storage phenomenon such as. e.g., solar wind density, solar wind pressure, y and x components of the interplanetary magnetic field, and earths seasonal variation etc.

$a_1(P),\cdots,c_3(P)$: Konwn functions of P

- Study how P affects the value of the model coefficients especially $c_2(P)$ hence the energy storage

Conclusions

- Energy storage is the phenomena where the nonlinear magnetosphere dynamics transfer the energy of VBs to Dst at around zero frequency.

- A novel nonlinear frequency analysis has been applied to investigate which nonlinear terms in an identified model of magnetosphere dynamics can make most significant contributions to energy storage .

- The analyse result could be used to study the effects of other physical parameters in space on energy storage phenomena.