Physics of the saturation of particle acceleration in relativistic magnetic reconnection

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High-energy radiation and particle acceleration

- Many sources like GRBs, AGN, and Pulsar Wind Nebulae produce nonthermal X-rays and gamma-rays

- How do particles reach a high enough energy to emit this radiation?

- Two microphysical mechanisms known:
  - Fermi acceleration in shocks
    - Works well for kinetically dominated systems
  - Magnetic reconnection
    - Works well for magnetically dominated systems
Relativistic reconnection

- Magnetic energy $\gg$ total particle enthalpy
  \[
  \sigma = \frac{B^2}{8\pi (\rho + P)c^2} \gg 1
  \]

- Many applications in astrophysics
  - Gamma ray bursts
    (e.g. Narayan & Kumar 2009, Biniamini & Granot 2016)
  - Active galactic nucleus jets
    (Giannios et al 2008, Narayan & Piran 2012)
  - Pulsar Wind Nebulae (Crab flare)
Why is reconnection needed?

- Source is a plasma > electromagnetic acceleration

- The acceleration of a particle with mass m and charge q in an electromagnetic field is:
  \[
  \frac{dv}{dt} = \frac{q}{m\gamma} \left( E + \frac{v}{c} \times B \right)
  \]

- The magnetic field does no work, but causes particles to spiral around field lines

- \( E \parallel v \) is required to change the particle energy, but usually \( E = -\frac{\langle v \rangle}{c} \times B \) > flux freezing
Magnetic reconnection releases magnetic energy

- Re-connect the field lines in center (X-point) to dissipate energy
- Out-of-plane electric field produces linear acceleration
- Occurs at a universal rate $E/B_0 \sim v_{in}/v_{out} \sim 0.1$ - fast and efficient
Why is there saturation?

• Power laws in reconnection are hard, with 1<p<2

• A cutoff must be present from energy conservation

\[
\langle \gamma \rangle = \left( \frac{1-p}{2-p} \right) \frac{\gamma_f^{2-p} - \gamma_i^{2-p}}{\gamma_f^{1-p} - \gamma_i^{1-p}} \leq \sigma
\]

  • Werner et al (2016) found a cutoff at \( \gamma = 4\sigma \)

• Also, \( p \) varies significantly with \( \sigma \) (Guo+ 14, Werner+ 16)

  • In contrast, Fermi acceleration in shocks has a universal index of \( p \approx 2.2 \)

• Plan: Carry out simulations at many values of \( \sigma \) to understand if and why saturation occurs

  • Simulation setup similar to Werner+16
Particle in cell simulations

- Particle in cell method discretizes exact EM equations
- Fields only calculated on the grid
- Macroparticles represent many particles

From Surmin et al (2016)
Our PIC simulations of saturation

- Use Tristan-MP particle-in-cell code (Spitkovsky 2008) with current density filtering algorithm that reduces particle noise (use 8 macroparticles/cell/species)

- Simulation setup:
  - Pair plasma - easier to simulate and also directly applicable
  - Begin with flat Harris equilibrium which is susceptible to reconnection
    \[ \mathbf{B} = B_0 \tanh \frac{y}{\delta} \hat{x} \]
  - 2.5D setup with periodic boundary conditions

- Set background magnetizations of \( \sigma = 3, 10, 30, 100, 200, \) and 500

- Normalize quantities to the length scale \( \sigma m_e c^2 / eB_0 = \sigma r_L \) (eliminates \( \sigma \) dependence)
  - Length scales large enough to reach saturation \((320 \sigma r_L)^2\)
  - Set 32 cells per \( \sigma r_L \)
Schematic of Reconnection

- Magnetic Island
- Secondary Islands
- X-point
- Island Merger
- Outflows
Evidence of saturation for $\sigma=30$

Saturation occurs around $\tau=ct/\sigma r_\perp=84$
Consistent with Werner+16
Fitting the spectra at saturation

- We want to verify the results of Werner et al (2016)
- We fit particle spectra with a power law starting at $\gamma_i$ with a (super?)-exponential cutoff at $\gamma_f$:

$$N(\gamma) = A \left( \frac{\gamma}{\gamma_i} \right)^{-p} e^{-(\gamma/\gamma_f)^\xi}. $$

- We choose the lowest $\gamma_i$ that does not significantly worsen the fit
Saturated spectrum as a function of $\sigma$
We verify several results of Werner et al. (2016)
- The power law index decreases strongly with $\sigma$ to around 1.2
- The average Lorentz factor of accelerated particles is $\sim \sigma$
- The cutoff is at $\gamma_f = 4\sigma$

New results:
- We find that $p$ changes with $\sigma$ in just such a way that Equation (6) gives $\gamma_i = 4\sigma$ for $<\gamma> = \sigma$
- The dynamic range of the power law is very small: $\gamma_i / \gamma_f < 40$

Question: Why is the saturation occurring?

Table 1

<table>
<thead>
<tr>
<th>Run</th>
<th>$p$</th>
<th>$\gamma_i$</th>
<th>$\gamma_f$</th>
<th>$\zeta$</th>
<th>$f_{\text{acc}}$</th>
<th>$\langle \gamma \rangle_{\text{acc}}$</th>
<th>$\langle \gamma \rangle_{\text{pl}}$</th>
<th>$f_{\text{int}}$</th>
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<tbody>
<tr>
<td>S3</td>
<td>2.23</td>
<td>1.8</td>
<td>6.4</td>
<td>1.5</td>
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<td>3.1</td>
<td>0</td>
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<td>2.4</td>
<td>34</td>
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<td>132</td>
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<tr>
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<td>1.9</td>
<td>0.95</td>
<td>394</td>
<td>427</td>
<td>0.045</td>
</tr>
</tbody>
</table>

$^a$ The number for each run gives the value of $\sigma$.
$^b$ The fraction of the particle kinetic energy in accelerated particles (those with Lorentz factor above $\gamma_i$). This changes with time after saturation as well.
$^c$ The average Lorentz factor of accelerated particles.
$^d$ The average Lorentz factor predicted for accelerated particles based on our power law fits and Equation (6).
$^e$ The fraction of the particle kinetic energy in intermediate particles (those in the interval $1.8 < \gamma < \gamma_i$)

\[
\langle \gamma \rangle_{\text{pl}} = \left( \frac{1-p}{2-p} \right) \frac{\gamma_f^{2-p} - \gamma_i^{2-p}}{\gamma_f^{1-p} - \gamma_i^{1-p}}.
\]
Particle acceleration histories

- We trace particles entering an X-point both before (top) and after (bottom) saturation for $\sigma=30$
- Most acceleration comes from a single episode (~70% on average)
- Acceleration is linear and begins soon after the particle enters the acceleration region (true for >80% of all particles)
- Indicates that X-point acceleration is the most important mechanism
Consistent with previous work

● Other researchers find other mechanisms more important in terms of $\Delta \gamma$
  ● Island mergers or Fermi-type acceleration

● But spectral properties are determined by logarithmic acceleration $\gamma_{\text{end}}/\gamma_{\text{start}}$
  ● No disagreement on this!
  e.g, Guo et al 2015:
Current sheet analysis

- Hypothesis: Secondary islands in X-points cause saturation by disrupting acceleration

- Each structure in the current sheet has size $D_c$ and magnetic field $B_c$

- Compare the time for a particle to escape ($D_c/c$) to the time for it to undergo a quarter gyration ($\gamma mc^2/4qB_c$)

- Characterize each structure using the Lorentz factor at which these timescales are equal:

\[
\gamma_c = 4\sigma \frac{D_c}{\sigma r_L} \frac{B_c}{B_0}.
\]
Significance of $\gamma_c$

- Blue particle has $\gamma >> \gamma_c$
  - It is unaffected by the magnetic island

- Red particle has $\gamma << \gamma_c$
  - It is trapped by the magnetic island
Identification of structures using minima of $B$

\[
\frac{y - y_0}{\sigma r_L} \quad \frac{x - x_0}{\sigma r_L} \quad |B| \quad \frac{|B|}{B_0}
\]

- $1 < \gamma/\sigma < 10$
- $10 < \gamma/\sigma$
Calculating $\gamma_c$ for each structure

- Find maxima and minima of $|B_x|$ in the current sheet plane.
- Pair them hierarchically, starting with the lowest differences between maxima and minima and ending with the highest ones.
- Calculate $D_c = 2(y_{\text{max}} - y_{\text{min}})$ and $B_c = (B_{x,\text{max}} - B_{x,\text{min}})/2$ for each pair.
- Calculate $\gamma_c$

\[ \gamma_c = 4\sigma \frac{D_c}{\sigma r_L} \frac{B_c}{B_0}. \]
The distribution levels off at $\gamma_c = 4 \sigma$
This explains the spectral cutoff!
Typical acceleration

- The typical particle energy reached in a current sheet structure of size $D_c$ is
  \[ \gamma_t = \frac{qE}{mc} \frac{D_c}{\nu_y}. \]

- But particles ignore structures with smaller $\gamma_c$

- Define an implicit equation for maximum acceleration produced by structures larger than $\gamma_c$
  \[ \frac{\gamma_{\text{max}}(\gamma_c)}{\sigma} = 0.5 \frac{\langle D \rangle(\gamma_c)}{\sigma r_L}. \]

- Particle acceleration should stop when $\gamma_{\text{max}} < \gamma_c$
Graph of the implicit equation

- $\gamma_{\text{max}} < \gamma_c$ for $\gamma_c > 5\sigma$
- Implies a cutoff there, close to $4\sigma$
Implications for astrophysics

- Reconnection produces particle spectra with $1.15 < p < 2.3$, and corresponding radiation spectra $\nu F_\nu \sim \nu^{-0.15} - \nu^{0.9}$

- The dynamic range of the power law in frequency space is at most $40^2 = 1600$ or 3 decades

- Implications:
  - Prompt GRB emission is hard to produce in reconnection: its power laws are soft and its dynamic range up to 4 decades
  - AGN and PWN flares are much more promising: the spectra are much flatter and the dynamic range in frequency is typically only $\sim 100$ or 2 decades
  - $4\sigma$ limit makes it difficult to explain the most energetic flares in the Crab Nebula, which require $\gamma \sim 10^9$

- Need explosive reconnection? Stay tuned for Maxim’s talk
Conclusions

- Saturation of the power law produced in reconnection occurs quickly, producing a hard power law with a cutoff at $\gamma = 4\sigma$
- The dynamic range of the power law is only $\sim 40$
  - Too small to explain prompt GRBs
  - But OK for AGN and PWN flares
- The particle acceleration process is dominated by a single rapid phase of direct X-point acceleration
- The physical cause of the saturation is the spontaneous formation of magnetic islands in large X-points, which limits the size of acceleration regions.