2D Relativistic MHD simulations of the Kruskal–Schwarzschild instability in a relativistic striped wind

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Isradynamics 2018  Dead Sea
Outline of the Talk

- Relativistic jets in various sources (e.g. GRBs, AGNs, micro-quasars)
- Jet acceleration & energy dissipation mechanisms
- Kruskal-Schwarzschild Instability in a striped wind – analytic results
- Results from 2D Relativistic MHD simulations
- The role of buoyancy and the Kelvin-Helmholtz instability
- Conclusions
Jets Are Ubiquitous!

- VLA
  - $M_{BH} \gtrsim 3 \times 10^9 M_\odot$ M87
  - 1 kpc

- VLA
  - $M_{BH} \sim 2.5 \times 10^9 M_\odot$ Cygnus A
  - 70 kpc

- VLBA + VLA
  - Cygnus X1
  - $M_{BH} \sim 15 M_\odot$

- Gamma-ray bursts
  - $M_{BH} \gtrsim M_\odot$
  - (Nasa, artist’s rend.)

(Nasa, artist’s rend.) Gallo 2009
Jets Are Ubiquitous!

How are these jets accelerated to relativistic speeds and such large distances?

$M_{\text{BH}} \sim 3 \times 10^9 M_\odot$ M87

$M_{\text{BH}} \sim 2.5 \times 10^9 M_\odot$ Cygnus A

$M_{\text{BH}} \sim 15 M_\odot$ (Gallo 2009)

Gamma-ray bursts

$M_{\text{BH}} \gtrsim M_\odot$ (Nasa, artist’s rend.)
Jet Acceleration (in GRBs)

- There are two main options: **Thermal acceleration** and **magnetic acceleration** depending on the magnetization of the jet:

\[
\sigma \equiv \frac{w_B}{w_m} = \frac{B'^2}{4\pi (\rho' c^2 + 4p')} 
\]

- **Thermal acceleration** \((\sigma < 1)\), due to neutrino anti-neutrino annihilation, might play some role in the acceleration of GRB jets, where the flow is optically thick \((\tau_T \gg 1)\) near the central engine. It cannot work for AGN jets.

- It is widely believed that **magnetic acceleration** \((\sigma > 1)\) might be the dominant mechanism, in which case a Poynting-flux dominated jet is launched.

Rotational energy of the compact source is tapped by magnetic field lines

Poynting-flux dominated flow

(Mckinney & Uzdensky 2012)
Magnetic Energy Dissipation

Magnetic dissipation in Poynting-flux dominated flows

- Acceleration of the flow
- Non-thermal emission (e.g. prompt GRB emission)


**Striped Wind**

Similar to a pulsar wind, (e.g. Lyubarsky & Kirk 2001)

A fast reconnection process requires fast evacuation of hot plasma from current layer
Digression: Rayleigh-Taylor Instability (RTI)

- RTI occurs at the interface of two fluids with different densities, where the heavier fluid rests on top of the lighter fluid: $\rho_1 > \rho_2$

  \[
  \rho_1
  \begin{array}{c}
  \downarrow g \\
  \rho_2
  \end{array}
  \]

- This setup is always unstable to perturbations, which grow over time.

- Elementary linear stability analysis yields the dispersion relation (e.g. Chandrasekhar 1961):

  \[
  \omega^2 = \left( \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \right) g k < 0 
  \quad \text{for} \quad \rho_1 > \rho_2
  \]

- $g$ can also be the effective gravity that an accelerated system feels in the comoving frame.
Digression: Rayleigh-Taylor Instability (RTI)

- A bubble of relativistically hot plasma – a pulsar wind nebula – drives a shock wave, as it expands, into the much denser SN ejecta.

- The contact discontinuity between the two fluids is RTI unstable.

2D Hydro simulations of the RTI

(Li & Li, LANL)
Consider two relativistic fluids with different enthalpy densities, where again the heavier fluid rests on top of the lighter fluid. The enthalpy density is given by

\[ w = \rho c^2 + \frac{\hat{\gamma}}{\hat{\gamma} - 1} p \]

In equilibrium, the fluid pressure is vertically stratified to counter-balance the force of gravity, such that

\[ \partial_z p_0 = -\frac{w_0 g}{c^2} \]

\[ p_0(z) = p_0(0) \left[ 1 + \mathcal{O} \left( \frac{g z}{c^2} \right) \right] \]

Pressure is homogeneous on length scales:

\[ z \ll \frac{c^2}{g} \]
• The perturbation can only grow orthogonal to the magnetic field lines.

• Due to tension of magnetic field lines, the growth of modes in the direction parallel to field lines is suppressed.

\[ \hat{k} \parallel \hat{B} \] Mode parallel to the field lines

\[ \hat{k} \perp \hat{B} \] Mode normal to the field lines
The dynamical equations, in the ideal MHD limit, follow from conservation laws:

\[ \frac{\partial}{\partial t} (\rho u^\mu) = 0 \quad \frac{\partial}{\partial t} T^{\mu \nu} = 0 \]

The stress-energy tensor is a sum of the plasma and electromagnetic components:

\[ T^{\mu \nu} = T^{\mu \nu}_{\text{plasma}} + T^{\mu \nu}_{\text{em}} = \frac{wu^\mu u^\nu}{c^2} + p\eta^{\mu \nu} - b^\mu b^\nu \]

In ideal MHD, the flux-freezing condition applies:

\[ \frac{\partial B}{\partial t} = \nabla \times (v \times B) \]

Pressure and density are related to each other by the adiabatic condition:

\[ \frac{d}{dt} \left( \frac{p}{\gamma^2 \rho} \right) = 0 \]

By assuming harmonic perturbations, we find the dispersion relation (Lyubarsky 2010)

\[ \omega^2 = \pm g k \left( 5 + \frac{4}{\tanh(k \Delta)} \right)^{-1/2} \]
KSI in a Relativistic Fluid – Linear Stability Analysis

\[
\eta = \begin{cases} 
\sqrt{\frac{gk}{3}}, & k\Delta \gg 1 \\
(g)^{1/2}k^{3/4}\Delta^{1/4}, & k\Delta \ll 1
\end{cases}
\]
KSI in a Relativistic Striped Wind – Key Concept

Energy dissipation

\[ \frac{d\Gamma(r)}{d\ln r} \propto (\beta_{\text{in}}r)^{1/3} \]

Reconnection rate:

\[ g = -c^2 \frac{d\Gamma(r)}{dr} \hat{z} \]

KSI sets in and the hot plasma starts to drip out of the current layer

\[ \beta_{\text{in}} = \frac{v_{\text{in}}}{c} \]
Linear & Non-Linear Analysis with 2D Ideal-MHD Simulations in Athena
Large (Single) Wavelength Mode - Setup

- Initial density contrast: \( \rho_h = \psi \rho_c = \psi \)

- Magnetization: 
  \[
  \sigma_0 \equiv \frac{w_{B,0}}{w_{m,0}} = \frac{b_0^2}{\rho_{0,c} + 4\rho_{0,c}} \Rightarrow \frac{b_0^2}{\rho_{0,c}}
  \]
  [In Lorentz-Heaviside units]
  \( c = 4\pi = 1 \)

- Homogeneous pressure: 
  \( p_0 = \sigma_0/2 \) for \( \rho_{0,c} = 1 \)
Large (Single) Wavelength Mode - Setup

- Characteristic velocity: \( v_A = \sqrt{\frac{\sigma_0}{1 + \sigma_0}} \)
- Velocity perturbation:
  \[
  v_{1z}(y, z) = \frac{\kappa v_A}{2} \left[ \sin \left( \frac{2\pi m_0 y}{L_y} \right) \right] \left[ 1 + \cos \left( \frac{2\pi z}{L_z} \right) \right]
  \]
  Mode number \( m_0 = \frac{L_y}{\lambda_0} \)

- Initial density contrast: \( \rho_h = \psi \rho_c = \psi \)

- Magnetization: \( \sigma_0 \equiv \frac{w_{B,0}}{w_{m,0}} = \frac{b_0^2}{\rho_{0,c} + 4p_{0,c}} \quad \text{cold} \quad \frac{b_0^2}{\rho_{0,c}} \quad \text{[In Lorentz-Heaviside units]} \)
  \( c = 4\pi = 1 \)

- Homogeneous pressure: \( p_0 = \frac{\sigma_0}{2} \quad \text{for} \quad \rho_{0,c} = 1 \)
Large (Single) Wavelength Mode: \( k \Delta \ll 1, \ m_0 = 1 \)

\[
\kappa_{uv} = 10^{-6}
\]

\[
g = 10^{-2} \Rightarrow z_{\text{crit}} = g^{-1} = 10^2
\]
Large (Single) Wavelength Mode: \( k\Delta \ll 1 \), \( m_0 = 1 \)

**Linear Growth Rate: Theory Vs Simulations**

- There's good agreement between the linear theory and simulation in the linear stage.

\[ \langle f_m \rangle = \text{Spatially averaged Fourier amplitude of density perturbations for a given mode number } m \]
Large (Single) Wavelength Mode: $k \Delta \ll 1$, $m_0 = 1$

Magnetic field dissipation heats up the plasma
$k\Delta \approx 1$, $m_0 > 1$

- Mode with wavelength comparable to the thickness of the current sheet is the most destructive.

$k_{\nu} = 10^{-7}$, $m_0 = 10$, $g = 0.1$
$k\Delta \approx 1, \ m_0 > 1$

- Mode with wavelength comparable to the thickness of the current sheet is the most destructive.

$k_v = 10^{-7}, \ m_0 = 10, \ g = 0.1$
$k\Delta \approx 1, m_0 > 1 : \text{High Turbulent Velocity}$

- The turbulent velocity peaks in the region ($z = 0$) where the fluid undergoes maximum mixing.
- Does it mean that the reconnection rate proceeds at 10% of $c$? **NO!**
A better measure of the reconnection rate is obtained from the rate at which magnetic energy is dissipated, such that

\[
|\dot{E}_b| = 2v_{in} L_y \frac{b_0^2}{2} = \sigma_0 \rho_{0,c} L_y v_{in} = \left( \frac{\sigma_0}{10} \right) v_{in} \quad (\rho_{0,c} = 1, \ L_y = 0.1)
\]

Reconnection rate is two orders of magnitude smaller than that achieved in resistive MHD simulations!!
Why's the Reconnection Rate So Slow?

The blob test

\[ v_{\text{blob}} = 10^{-2} v_A , \quad g = 0.1 , \quad \sigma_0 = 10 \]

1) The blob is buoyant: The buoyancy force acts in the direction opposite to gravity, slowing down its descent.

\[ \mathbf{F}_b \approx -w_c V_b g \]

\[ g_{\text{eff}} = \frac{(w_h - w_c) g}{w_h} \]

\[ \approx \frac{(w_{h,0} - w_{c,0}) g}{w_{h,0}} \]

\[ = \frac{g}{2} \]
Why’s the Reconnection Rate So Slow?

The blob test

$$v_{\text{blob}} = 10^{-2}v_A, \quad g = 0.1, \quad \sigma_0 = 10$$

2) Kelvin-Helmholtz instability seeded by the RTI at the interface between the two fluids makes the ram-pressure flattened blob to curl upwards.

- The vortical motion prevents the blob from descending at a faster rate
Conclusions

- The KSI can potentially yield high reconnection rates in a striped wind outflow, which can lead to the bulk acceleration of the flow.

- 2D relativistic MHD simulation of the KSI show the same growth rate of the instability as predicted by the linear theory (Lyubarsky 2010).

- In 2D, the effective reconnection rate is two orders of magnitude smaller than that achieved in resistive MHD simulations.

- Buoyancy force and Kelvin-Helmholtz instability play a critical role in slowing down the evacuation of hot plasma from the reconnection layer, thus impeding the rate.

- Resistive MHD simulations of the KSI might yield faster reconnection rates.

Thanks!