

# VIOLATION OF A VANISHING SECOND ADIABATIC INVARIANT BY A WARPED MAGNETIC-EQUATORIAL SURFACE

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This study addresses a long-standing question in magnetospheric physics: How are electrons that mirror at (or too near) the magnetic equator ever lost from radiation belts? This puzzle arises because the resonance conditions for Landau resonance and for Doppler-shifted cyclotron resonance with actual electromagnetic or electrostatic cyclotron waves (normally invoked to account for causing pitch-angle diffusion) are difficult to satisfy in the absence of a Doppler shift corresponding to particle motion along the planets magnetic field. Various suggestions for scattering such equatorially mirroring particles invoke relativistic electron mass (such that a wave spectrum band-limited by the nonrelativistic electron gyrofrequency may still overlap with a radiation-belt electrons relativistic gyrofrequency), collisions with atmospheric or plasmaspheric constituents (usually regarded as almost negligible except at the lowest  $L$  values of interest), or bounce resonance with a spectrum of magnetosonic (or other compressional) waves. The present study takes one step back and asks how near to  $90^\circ$  an electrons equatorial pitch angle would be maintained in the absence of wave-particle interactions and other scattering mechanisms. For the case of a purely dipolar magnetic field the minimum deviation from a  $90^\circ$  equatorial pitch angle would indeed be very small (except that quantum uncertainty would impose half of Plancks constant as a minimum value for the second adiabatic invariant  $J$ ). However, the geomagnetic equatorial surface (locus of minima in  $B$  along field lines) typically shows a quadrupolar warp with amplitude  $\sim 400$  km ( $\sim 0.06$  Earth radii) positive and negative relative to the offset-dipoles equatorial plane, and there is no reason to expect that the minimum- $B$  surface is necessarily perpendicular to  $B$  itself. Thus, the grad- $B$  drift of an equatorially mirroring particle should carry the particles guiding center off the magnetic-equatorial surface. There would be a restoring force  $\mu(\partial B/\partial s)$ , where  $\mu$  is the particles magnetic moment and  $s$  is the coordinate of arc length along a magnetic field line, but the particle would no longer mirror at the magnetic equator. The process described above is not bounce-resonant (unless the bounce frequency happens to be twice the drift frequency) and therefore is not diffusive, but the resulting increase in  $J$  can be calculated (or at

least estimated) from the equation of a forced simple harmonic oscillator. The question is whether the equatorial pitch angle thereby varies from  $90^\circ$  by enough to bring the particle into cyclotron resonance with waves of the sort usually invoked as causes of pitch-angle diffusion. We will work out the simple case of a dipole + quadrupole representation of the Earth's main magnetic field, with expansion coefficients determined from a recent IGRF model.