Some aspects of magnetic reconnection

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Outline

1. Collisionless reconnection:
   - electron inertia versus non-scalar pressure tensor

2. Energetics of forced magnetic reconnection

3. Formation of plasmoids in the resistive MHD reconnection models
Role of a non-scalar electron thermal pressure tensor

the simplest case: electron MHD (EMHD) \rightarrow \text{immobile heavy ions}

Basic equations

\[
\frac{∂B}{∂t} = -c (\mathbf{J} \times \mathbf{E})
\]

\[E = \frac{1}{c} (V \times B) + \frac{1}{ne} \frac{∂\rho_e}{∂t} + \frac{m_e V_e}{c^2} + \frac{\mathbf{j}}{c} \text{ collisionless}
\]

"Reconnective" terms

\[\mathbf{j} = -neV = \frac{e}{4\pi} (\mathbf{J} \times \mathbf{B})
\]

\[V = \frac{1}{n} \int V d^3V
\]

\[P_{\parallel} = m \int \frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \mathbf{f} d^3V
\]

\[W_z = V_z - V_e
\]
Evolution eq. for $p_{ix}$ follows from electron kinetic eq.

\[
\frac{\partial p_{ix}}{\partial t} + \frac{\partial}{\partial x} \left( v_i v_x p_{ix} \right) + \frac{\partial}{\partial v_x} \left( \frac{m}{2} v_x^2 p_{ix} \right) = 0
\]

\[
\mathbf{F} = -e \left[ \mathbf{E} + \frac{1}{c} (\mathbf{v} \times \mathbf{B}) \right]
\]

\[
\frac{\partial p_{ix}}{\partial t} = -\frac{2}{\gamma_{2,0}} (p_{ix} v_{ix}) - \frac{\partial}{\partial v_x} \left( \frac{m}{2} v_x^2 p_{ix} \right) - \frac{\partial}{\partial x} \left( \frac{m}{2} v_x^2 p_{ix} \right)
\]

\[
- \frac{e}{m} \left( \varepsilon_{ix} B_x (p_{ix} v_{ix} + E_{ix} Q_{ix}) + E_{ix} Q_{ix} \right) - \frac{2}{\gamma_{2,0}} \frac{\partial p_{ix}}{\partial v_x}
\]

\[
\mathbf{Q}_{ix} = m \int \mathbf{v}_x \mathbf{v}_x p_{ix} + \frac{2}{\gamma_{2,0}} \frac{\partial p_{ix}}{\partial v_x}
\]

Heat flux tensor

\[
\frac{\partial \mathbf{Q}_{ix}}{\partial t} = -\frac{2}{\gamma_{2,0}} (p_{ix} \mathbf{v}_{ix})
\]

\[
- \mathbf{Q}_{ix} \frac{\partial v_{ix}}{\partial x} - \mathbf{Q}_{ix} \frac{\partial v_{ix}}{\partial v_x} - \mathbf{Q}_{ix} \frac{\partial v_{ix}}{\partial x}
\]

\[
+ \frac{1}{\gamma_{2,0}} \left( \frac{p_{ix} \partial v_{ix}}{\partial x} + \frac{p_{ix} \partial v_{ix}}{\partial v_x} + \frac{p_{ix} \partial v_{ix}}{\partial x} \right)
\]

\[
- \frac{e}{m c} B_k (\varepsilon_{ix} Q_{ix} + E_{ix} Q_{ix} + E_{ix} Q_{ix})
\]

\[
- \frac{2}{\gamma_{2,0}} \frac{\partial p_{ix}}{\partial v_x}
\]
Implications for a planar force-free magnetic equilibrium

\[ B_{0y}(x) + B_{0z}(x) = \text{const} = B_0 \]

Near the field reversal:

\[ B_{0y}(x) = B_0 \ln x \Rightarrow j_y = \frac{c B_0}{2 \pi} x^2 x_e \Rightarrow \]

\[ \frac{d V_{0y}}{dx} = \frac{c B_0 x^2}{4 \mu_0 e} + 0 \]

**Non-gyrotropic tensor**

\[ P_{0y} = P_{0x} - \text{ankrosropy in the plane perpendicular to the magnetic field } B_{0z} = B_0 \]

\[ P_{yy} = P_{zz} = P_{xx}(1 + \frac{\omega_e^2 \frac{c^2}{\omega^2}}{\omega_e^2}) \]

Normally \( \frac{c}{\omega_e} \ll 1 \rightarrow \text{magnetic shear length exceeds electron inertial skin depth } \gamma_e \)

Note, that \( q_{yy}^{(0)} = 0 \)

Tearing instability in such a field
pressure tensor linear perturbation

\[ P_{ij}^{\prime\prime} \] inside the current sheet

(neglecting first \( Q_{\phi} \))

\[ \nabla \cdot \mathbf{P}^{\prime\prime} = -\frac{\partial (\mathbf{B} \cdot \mathbf{V}_I)}{\partial t} - \sum_{k} \mathbf{E}_k \times \mathbf{B}_k (\nabla \times \mathbf{P}^{\prime\prime} + \mathbf{E}_k \mathbf{P}^{\prime\prime}) \]

separation into \( (P_{xx}^{\prime\prime}, P_{xy}^{\prime\prime}) \) and \( (P_{yx}^{\prime\prime}, P_{yy}^{\prime\prime}) \)

contributing to

(Reconnection) \( \psi / t \)

(Advection of \( \psi / t \))

\[ \frac{\partial \psi}{\partial t} = \text{c} E_2 = -\frac{\text{c}}{\text{ne}} \left( \frac{\partial P_{xx}^{\prime\prime}}{\partial x} + \frac{\partial P_{yy}^{\prime\prime}}{\partial y} \right) + \ldots \]

\[ \frac{\partial B_{z2}}{\partial t} = \text{c} \left( \frac{\partial E_2}{\partial y} - \frac{\partial E_2}{\partial x} \right) = \frac{\text{c}}{\text{ne}} \left[ \frac{\text{c}}{\text{ne}} \left( \frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial y^2} \right) - \frac{\partial}{\partial x} \left( \frac{\partial B_x}{\partial y} + \frac{\partial B_y}{\partial x} \right) \right] + \ldots \]

\[ Y_1 = L_1 \psi + \beta \psi + \frac{i \kappa \alpha}{r \text{c}} \frac{\delta}{\partial t} + \frac{i \kappa \alpha}{r \text{c}} \frac{\delta}{\partial t} \]

inertial term

Hall term

\[ \mathbf{B} = \frac{\text{d} \psi}{\text{d} t} + \beta \frac{\text{d} \psi}{\text{d} t} + \frac{\kappa \alpha}{r \text{c}} \frac{\delta}{\partial t} + \frac{\kappa \alpha}{r \text{c}} \frac{\delta}{\partial t} \]
\[ \beta \to 0 \text{ limit } \Rightarrow p^{\text{sw}} \to 0 \]

"Standard" EMHD inertial regime

\[ x \approx \frac{1}{2} \left( \frac{\beta}{\alpha} \right) \sim \frac{1}{2} \frac{d^2}{d \phi^2} \quad \text{de} = \frac{c}{4\pi e} \ll 1 \]

Prinvar tensor effect becomes significant for \( \beta > \frac{d^2}{\alpha} \ll 1 \)

Does \( \Omega \) play a role?

\[ \begin{align*}
\gamma p_x'' &= -p_0 \frac{\Omega^2}{2x} + \omega_2^2 p_x'' - \frac{\partial}{\partial x} \xi_x x^2 \\
\gamma p_y'' &= -p_0 \frac{2\Omega}{2x} - \omega_2 p_y'' - \frac{\partial}{\partial x} \xi_y y^2
\end{align*} \]

Contribution of \( \Omega \) becomes important when \( \xi_x x^2, \xi_y y^2 > p_0 v_x \)

Linearized eqs. for \( \Omega \)

\[ \gamma \Omega'' = \frac{p_0}{n m_e} \left[ \partial_p \frac{\partial^2 \Omega''}{\partial x^2} + \partial_p \frac{\partial^2 \Omega''}{\partial y^2} + \partial_p \frac{\partial^2 \Omega''}{\partial z^2} \right] - \omega_8 \left( \xi_0 + \xi_2 \Omega'' + \xi_2 \Omega'' + \xi_2 \Omega'' + \xi_4 \Omega'' \right) \]

Balanced by magnetic isotropisation
\[ q_{xy} = 0; \quad q_{xx} = q_{yy} = \frac{\beta^2}{1 + \frac{\omega^2}{\gamma^2}} \] 

significant for \( \beta \geq 2 \epsilon \)

electron Larmor radius \( \beta_e \sim (\Delta x) \)

Conclusions

1) \( \beta_e \ll (\Delta x) \Rightarrow \) fluid-type description works
   - plasma-tenor effects are small (including electron gyroviscosity)
   - reconnection is due to the bulk inertia of electrons

2) \( \beta \gg 2 \epsilon \Rightarrow \) no truncation of fluid equation
   - kinetic description is necessary
   - what to expect? numerical simulation point to \( (\Delta x) \sim \beta_e \sim \beta_e^{1/2} \epsilon \)

Why?

(\( \epsilon \)) \( \Rightarrow \) \( \beta_e \Rightarrow \) \( \beta \) approach holds \( \Rightarrow \) contradiction

(\( \Delta x \)) \( \ll \beta_e \Rightarrow \) guide field play no role \( \Rightarrow \)

(\( \Delta x \)) if determined by "meandering" of electrons \( \Rightarrow \)

(\( \Delta x \)) \( \sim (\beta_e \epsilon) \frac{1}{2} \Rightarrow \beta_e \Rightarrow \) contradiction
Energetics of forced magnetic Reconnection

Two types of Reconnection:

i) spontaneous - via resistive MHD instability (tearing mode)

ii) forced - under external deformation of an MHD stable configuration

\[ \mathbf{B}^{\text{eq}} = \left( 0, B_0 \sin \pi x, B_0 \cos \pi x \right), \]
\[ -a \leq x \leq a \]

\[ \mathbf{B}^{\text{on}} = (\nabla \phi \times \mathbf{e}) + \mathbf{B}^{\text{eq}} \]
\[ \psi_0 (x) = B_0 \cos \pi x ; \]
\[ \mathbf{B}_2 = \nabla \psi_0 \]

Simple example: sheared force-free magnetic field

Linear force-free field
Fearing perturbation

a new free-free equilibrium
\[ y(x, y) = \frac{y(x)}{y} + \frac{y(x)}{c_0} \cos ky \]

\[ \Delta x = \left[ \frac{y}{y(x)} \right]^{1/2} \text{ width of islands} \]

\[ y'' + \kappa^2 y = 0 \quad \kappa^2 = \kappa^2 x > 0 \]

Stability parameter \[ \Delta' = -2 \kappa \csc(\kappa a) \]

\[ \Delta' > 0 \rightarrow \text{instability} \quad \text{for} \quad \kappa a > \frac{\pi}{2} \rightarrow \kappa a > \frac{\pi}{2a} \]

Physical explanation:
magnetic energy is reduced at \( \Delta' > 0 \)

\[ \Delta W_{\text{M}} \propto \Delta' (y'^2(x)) \]

Fearing-stable field (\( \kappa a < \frac{\pi}{2a} \)) still has electric current and, hence, possesses some free magnetic energy

Can it be released by reconnection?

Yes, if reconnection is triggered externally.
**Forced magnetic reconnection**
(Hahl and Kulsrud, 1985)

\[ X^* = \frac{1}{2} (a + \delta a \cos y) \]

\( \delta a \ll a \) - linear treatment for a slightly deformed new equilibrium

\[ y(x,y) = \frac{B_0}{2} \cos 2x + y(a) \cos y \]

**Boundary condition:**
\[ y(x = \pm a) = \frac{y_0}{a} (a) \]
\[ y_0(a) = B_0 \sin (ka) \]

**Force-free equilibrium condition:**
\[ y'' + \gamma^2 y = 0 \]

2 possible solutions with different magnetic topology

1) Regular solution:
\[ y''(x) = B_0 \frac{\sin (ka)}{C_0 (\cos a)} \cos (x + a) \]

magnetic islands are present
\[ y(0) \neq 0 \]

not possible in ideal MHD
21. Ideal MHD solution: \( \psi_i(0) = 0 \Rightarrow \)

\[ \psi_i(\infty) = B_0 \sin(2\alpha) \sin(\alpha) \]

\[ \Rightarrow \text{discontinuity of} \quad \frac{d\psi}{dx} \text{at} \quad x=0 \Rightarrow \]

\[ \text{Current sheet} \quad \iota \quad \text{formed at} \quad x=0 \]

\[ \text{Forced reconnection; transition from} \quad \psi_i(0) \quad \text{to} \quad \psi_i(\infty) \quad \text{due to} \]

\[ \text{a finite plasma resistivity on a} \]

\[ \text{tearing time-scale.} \]

**How does magnetic energy change?**

\[ \Delta W_M = -2 \int_{0}^{t} \left[ \frac{\partial}{\partial t} \frac{\partial}{\partial x} \sin^2(\alpha x) \right] dx \]

\[ \Rightarrow \]

1) \[ \Delta W_M^{(i)} = \frac{B_0^2}{2 \mu_0} \left[ 2 \sin(2\alpha \omega) \cos(\omega x) - 2 \cos(2\alpha \omega) \sin(\omega x) \right] \]

\[ \Delta W_M^{(i)} \quad \text{is positive} \Rightarrow \text{magnetic energy of} \]

\[ \text{the system initially} \]

\[ \text{increases due to the} \]

\[ \text{external deformation} \]

**Weak shear:** \( \alpha \ll 1, \quad k \alpha \ll 1 \)

\[ \Delta W_M^{(i)} = \frac{B_0^2}{2 \mu_0} \frac{\partial}{\partial x} \frac{2}{3} \left( \frac{\kappa_0}{2} \right)^2 \]

\[ \Delta W_M^{(i)} = \frac{B_0^2}{2 \mu_0} \frac{\partial}{\partial x} \frac{2}{3} \left( \frac{\kappa_0}{2} \right)^2 \]
2) \[ \Delta W_H^{(r)} = -\frac{B_0^2}{2\mu_0} \int_0^L \left( \frac{\sin^2(\kappa z) \sin(\kappa z)}{\cosh^2(\kappa L)} \right) dx \]

\( \Delta W_H^{(r)} \) is negative \( \Rightarrow \) magnetic energy relaxes below its initial magnitude

\( L < l_0 \Rightarrow \Delta W_H^{(r)} = -\frac{B_0^2}{2\mu_0} x^2 \Rightarrow |\Delta W_H^{(r)}| > |\Delta W_H^{(r)}| \)

External perturbation acts as a trigger for internal magnetic relaxation

Temporal evolution of magnetic energy

[Graph showing the temporal evolution of magnetic energy with \( t \)-axis and \( W_H \)-axis, labeled as \( \tau_r \) - reconnection time and given as \( \tau_r = \tau_s \frac{S_{15}}{S} \) for the standard reconnection MHD.]
 Released magnetic energy:

$$\Delta M_r = \frac{B^2}{2\mu_0} a(L)^2 f(\Delta a)$$

dependence on magnetic shear

The energy effect of forced reconnection is strongly amplified for a marginally stable magnetic configuration.

A link between the forced and spontaneous reconnection.
Possible implications for solar corona

Another scenario of how magnetic energy could be stored in the corona without being quickly released by reconnection.

1) Magnetic energy is accumulated in the corona due to one kind of deformation (for example, shearing).

2) Then, at some instant, another deformation occurs (for example, emergence of new flux).

Force reconnection takes place, and initially stored magnetic energy is released.
Formation of plasmoids: "comeback" of the resistive MHD reconnection

Classical Sweet-Parker model → highly elongated current sheet

\[ l \sim L S^{-\frac{1}{2}} \]

\[ S = \frac{l^2}{\mu_0} \quad \text{Landau number} \]

\[ S \sim 10^{12} \text{ in the solar corona} \]

Reconnection time \[ T \sim T_s S^{\frac{1}{4}} \]

is too large!!!

Tearing instability of S-P current sheet

Loureiro et al. ...


Phys. Plasmas, 14, 100703, 2007

Current sheet breaks up into a large number of magnetic islands (plasmoids) very quickly (tens: Alfvénic time-scale)

\[ N \sim S^{\frac{3}{4}}, \quad T \sim T_s S^{\frac{1}{8}}, \quad T_a = \frac{1}{V_a} \]

much faster reconnection for \( S \gg 1 \)
How could it manifest itself in a dynamically evolving system?

**Simple model:**

Forced reconnection

\[ x_t = t(a + \text{decay}) \]

external deformation of amplitude \( S \) on the time scale \( \Delta t \approx \tau_{a} = a/\Delta x_{a} \)

**Subsequent evolution of the system**

1. \( \Delta t \ll t \ll \tau_{a} \tau_{a} ; S_{a} = a/\Delta x \)

"ideal" evolution \( \Rightarrow \)

ideal solution \( \psi^{(0)} \) formed with the current sheet width

\[ \psi^{(0)} = a \Delta x/\Delta t \]

2. Magnetic reconnection, i.e. transition from \( \psi^{(0)} \) to \( \psi^{(1)} \) configuration with magnetic islands, proceeds at

\[ \Delta t \ll \tau_{a} \ll \tau_{a} S_{a}^{3/5} \]

reconnection time
Ideal phase: current sheet with
length $L \sim 1/k \sim a$ and
width $e \sim a \sqrt{t}$
\[ \Rightarrow \]
aspect ratio $L/e \sim t/a$ is increasing
with time \[ \Rightarrow \]
C.S. should become tearing-unstable
\[ \Rightarrow \]
The effect of it become noticeable,
when $y(t) / t_c \sim 1$
\[ \Rightarrow \]
For tearing mode:
$y_c \sim T_a / \tilde{E}^{1/35} \tilde{(kx)^{1/5} (A')^{3/5}}$
For long-wave perturbation (most unstable)
$K \approx \frac{1}{a} \Rightarrow kx \sim \frac{e}{a}$; \[ \Rightarrow \]
$\Delta' \approx \frac{1}{kx} \approx \frac{a}{e}$
\[ \Rightarrow \]
$T_a \approx T_a \cdot \frac{e}{a}$; \[ \Rightarrow \]
$S_a \approx S_a \cdot \frac{e}{a}$
\[ \Rightarrow \]
$y_c \approx T_a / \tilde{E}^{1/35} \approx T_a / S_a (\theta_{SA})^{2}$
$y_c(t) \cdot t \sim S_a (\theta_{SA})^{2}$
\[ \Rightarrow \]
$T_k \sim T_a \cdot S_a^{1/5}$
\[ \Rightarrow \]
Such a C.S. cannot survive
for $t \geq T_a \cdot S_a^{1/5}$
No stabilization by the plasma flow
(Bulanov et al, 1979)

$$\frac{1}{t} < \frac{1}{\delta} \text{ for } t > t^*$$

C. S. breakdown if

$$t^* \sim t_a \text{ for } \delta_a < \delta_a$$

This is the case for asymptotically large $\delta_a$

This could explain why this effect has not been observed in numerical simulations with

$$\delta_a \sim 10^4$$

Transition to a faster regime of reconnection could be anticipated for larger $\delta_a$