Monte Carlo methods and Dissipative Environments

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Dissipative environments

Path Integral

$$Z = \int \mathrm{e}^{-S[q,x_\alpha]} \mathcal{D}[q, x_\alpha]$$

Caldeira Legget Model

$$S[q, x_\alpha] = S_{\text{System}}[q] + S_{\text{Env}}[x_\alpha] + q \sum_\alpha c_\alpha x_\alpha$$

$$S_{\text{Env}} = \int \sum_\alpha \{m_\alpha \dot{x}_\alpha^2 + m_\alpha \omega_\alpha^2 x_\alpha\}$$

$$S_{\text{System}} = \int_\tau M \dot{q}_\tau^2$$

Integrating over the Environment Degrees of freedom

$$S_{\text{Dissipation}} = \eta \iint \frac{\pi^2 T^2 (q_\tau - q_{\tau'})^2}{\sin^2 \pi T(\tau - \tau')} d\tau d\tau'$$

For large T $S_{\text{Dissipation}} = \int_\tau \eta \dot{q}_\tau$

Model

Particle on a Ring $(R = 1)$

$$S[\theta] = \int_\tau M\dot{\theta}_\tau^2 + \eta \iint \frac{\pi^2 T^2 \sin^2\left(\theta_\tau - \theta_{\tau'}\right)}{\sin^2 \pi T(\tau - \tau')} d\tau d\tau'$$

Magnetic flux ϕ_x

$$Z = \sum_{m=-\infty}^{\infty} \mathrm{e}^{2\pi i m \phi_x} \int_{\mathcal{D}[\theta]} \mathrm{e}^{-S[\theta^{(m)}]}$$

Interest in the velocity correation function

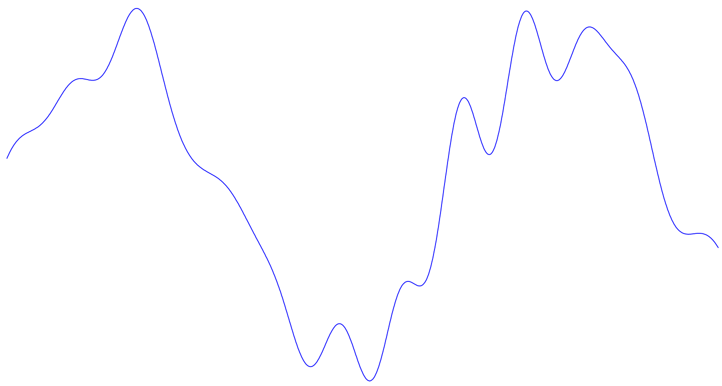
$$K(\tau) = \left\langle \dot{\theta}_t \dot{\theta}_{t-\tau} \right\rangle = \frac{\sum_m \mathrm{e}^{2\pi i m \phi_x} \int_{\mathcal{D}[\theta]} \dot{\theta}_t \dot{\theta}_{t-\tau} \mathrm{e}^{-S[\theta^{(m)}]}}{\sum_m \mathrm{e}^{2\pi i m \phi_x} \int_{\mathcal{D}[\theta]} \mathrm{e}^{-S[\theta^{(m)}]}}$$

$$K(\omega) = T\omega^2 \left\langle \left|\theta_\omega\right|^2 \right\rangle = K_0 + K_1 \left|\omega\right| + \mathcal{O}\left(\omega^2\right)$$

K_1 the dissipation term

Path Integral Monte Carlo

Minimization of the action by Markov Chain



Detail Balance

$$\pi(a)p(a \to b) = \pi(b)p(b \to a)$$

$\pi(a)$ stationary probability to be in state a

$p(a \to b)$ transition probabily states a and b

Metropolis algorithm, any step is accepted with

$$p(a \to b) = \begin{cases} 1 & \pi(b) > \pi(a) \\ \pi(b)/\pi(a) & \pi(b) < \pi(a) \end{cases}$$

Back to the Model

Stationary Probability

$$\pi(\theta) = \sum_m \mathrm{e}^{2\pi i m \phi_x} \mathrm{e}^{-S[\theta^{(m)}]}$$

Markov Chain

$$\theta_\tau \to \theta_\tau \pm A \quad ; \quad \theta_\omega \to \theta_\omega \pm A_\omega$$

Numerical Results for the velocity correlation