Particle Dynamics on a Ring Affected by Noisy Environments

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Introduction

For a spin-boson model the dephasing of a non equilibrium state.

$$\rho_{01}(\tau) \sim \mathrm{e}^{-K(\tau)}$$

The same in equilibrium

$$\langle \sigma_x(t)\sigma_x(t+\tau)\rangle \sim \mathrm{e}^{-K(\tau)}$$

With

$$K(\tau) = \begin{cases} T\,\tau & T > 0 \\ \log\tau & T = 0 \end{cases}$$

Similar behavior apply for the Brownian particle with CL environment

We study the semiclassical dynamics, in particular the dephasing, of a particle on a ring.

Caldeira-Leggett environment

For the CL environment the Langevin equation of motion

$$m\ddot{\vec{\mathbf{x}}}(t) + \eta\dot{\vec{\mathbf{x}}}(t) = \vec{\xi}(t)$$

Projected on a ring

$$mR\ddot{\theta}(t) + \eta R\dot{\theta}(t) = \xi_x(t)\cos\theta(t) + \xi_y(t)\sin\theta(t) + E$$

The noise correlation by FDT

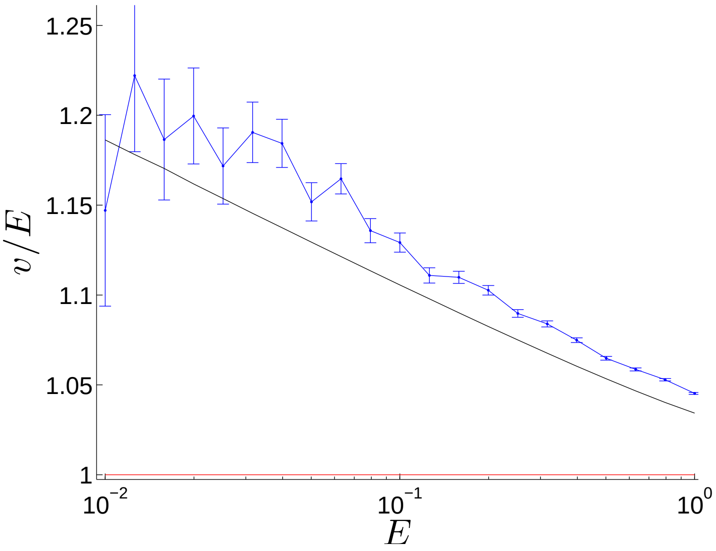
$$\langle \xi_i(\omega)\xi_i(\omega')\rangle = \hbar\eta\omega \coth\beta\hbar\omega \;\delta(\omega+\omega') \qquad i = x, y$$

We consider the $T = 0$ limit, $\omega\coth\beta\hbar\omega \to |\omega|$, the correlation in time

$$\langle \xi_i(t)\xi_i(t+\tau)\rangle = -\frac{1}{\pi\tau^2} \qquad \tau > 0 \qquad i = x, y$$

Results

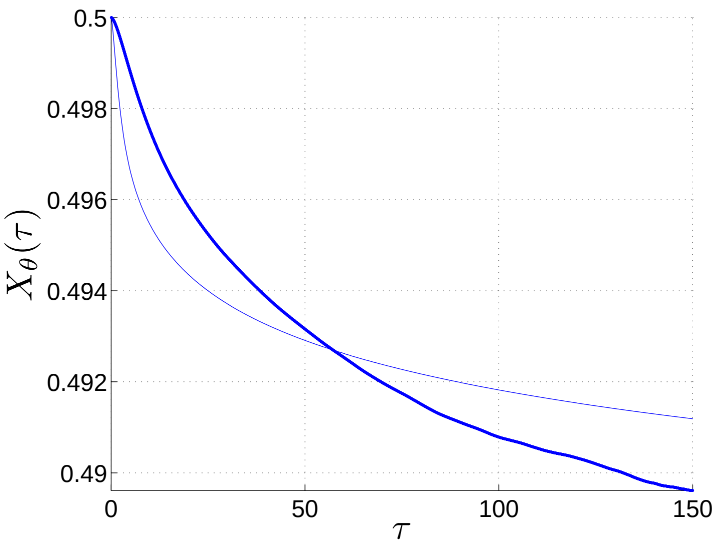
The current $\left\langle \dot{\theta}(t) \right\rangle$ is enhanced by the noise



Define dephasing via the correlation function

$$X(\tau) = \langle \cos\theta(t) \cos\theta(t+\tau) \rangle$$

Decay of the correlation $\Rightarrow$ dephasing



The numerical results and $\sim \tau^{\hbar}$ perturbation result (thin line) with $E = 0$

Dirty Metal

The response of the potential to a charged particle

$$\phi(\mathbf{q},\omega) = \alpha(\mathbf{q},\omega)\rho(\mathbf{q},\omega)$$

$$\alpha(\mathbf{q},\omega) = \frac{4\pi}{q^2\varepsilon(\mathbf{q},\omega)}$$

Where the Dielectric function

$$\varepsilon(\mathbf{q},\omega) = \frac{4\pi\sigma}{-i\omega + Dq^2}$$

The correlation function of the potential

$$K_\phi(\mathbf{q},\omega) = \hbar\coth(\hbar\beta\omega)\ \mathrm{Im}\alpha(\mathbf{q},\omega)$$

with $|\mathbf{q}| < 1/l$

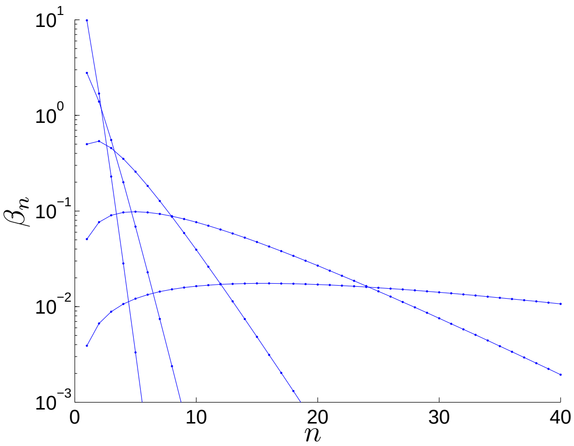
$$K_\phi(\mathbf{r}(t),\tau) = \hbar\eta\left(\frac{R^2}{l^2}\sin^2\left(\frac{\theta(t)-\theta(t')}{2}\right)+1\right)^{-1/2}$$

Langevin equation for the particle dynamics

$$m R \ddot{\theta}(t) + \eta R \dot{\theta}(t) = \sum_n \beta_n(R) \left\{ \xi_x^n(t) \cos n\theta(t) + \xi_y^n(t) \sin n\theta(t) \right\}$$

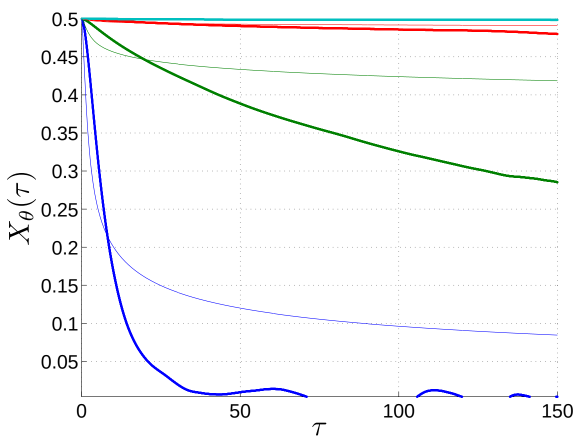
$$\langle \xi_i^n(\omega) \xi_i^m(\omega') \rangle = \hbar \eta \, |\omega| \, \delta(\omega + \omega') \delta_{n,m} \qquad i = x, y$$

Where the Fourier coefficients for various R



In the $R \ll l$ limit it reduces to the CL equation

The dephasing decreases as R increases



The numerical result and $\sim \tau^{\hbar/R^2}$ perturbation (thin line)