Particle Dynamics on a Ring Affected by Noisy Environments

Yoav Etzioni, Ben-Gurion University

In collaboration with Prof. B.Horovitz

Introduction

For a spin-boson model the dephasing of a non equilibrium state.

$$\rho_{01}(\tau) \sim \mathrm{e}^{-K(\tau)}$$

The same in equilibrium

$$\langle \sigma_x(t)\sigma_x(t+\tau) \rangle \sim \mathrm{e}^{-K(\tau)}$$

With

$$K(\tau) = \begin{cases} T\,\tau & T > 0 \\ \log \tau & T = 0 \end{cases}$$

Similar behavior applies for the Brownian particle with CL environment

We study the semiclassical dynamics, in particular the dephasing of a particle on a ring.

Caldeira-Leggett environment

For the CL environment the Langevin equation of motion

$$m\ddot{\vec{\mathbf{x}}}(t) + \eta\dot{\vec{\mathbf{x}}}(t) = \vec{\xi}(t)$$

Projected on a ring

$$mR\ddot{\theta}(t) + \eta R\dot{\theta}(t) = \xi_x(t)\cos\theta(t) + \xi_y(t)\sin\theta(t) + F$$

The noise correlation by FDT

$$\langle \xi_i(\omega)\xi_i(\omega')\rangle = \hbar\eta\omega \coth\beta\hbar\omega \;\delta(\omega+\omega') \qquad i = x, y$$

We consider the $T = 0$ limit, $\omega \coth\beta\hbar\omega \to |\omega|$, the correlation in time

$$B(\tau) = \langle \xi_i(t)\xi_i(t+\tau)\rangle = -\frac{1}{\pi\tau^2} \qquad \tau > 0 \qquad i = x, y$$

Correlation Function

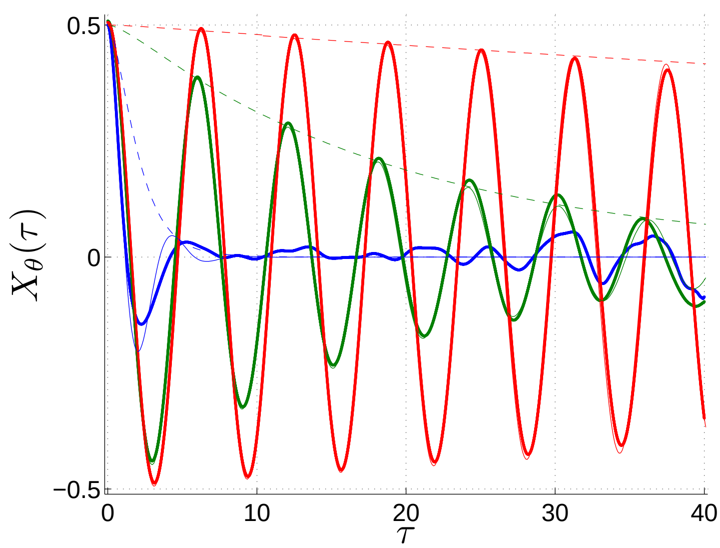
Define dephasing via the correlation functions with $\tau = t - t'$

$$X(\tau) = \langle \cos\theta(t)\cos\theta(t+\tau)\rangle$$

A perturbative results of the correlation function with $1/\bar{\eta} = \frac{\hbar}{\eta R^2}$

$$X(\tau) \approx \begin{cases} \frac{1}{2}\tau^{-1/\bar{\eta}} & v = 0 \\ \cos v\tau \mathrm{e}^{-v\tau/\bar{\eta}} & v \neq 0,\ \tau > \frac{1}{v} \end{cases}$$

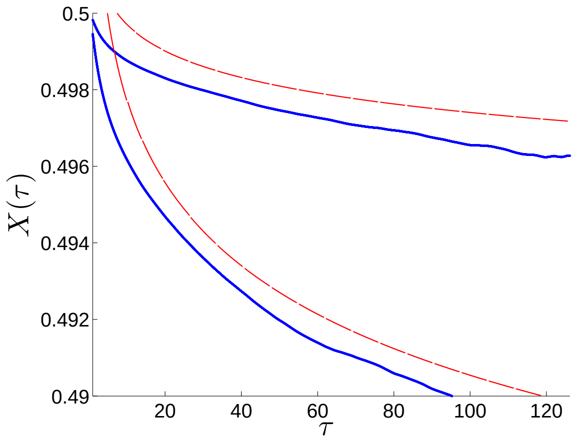
The correlation functions for finite v for various $\bar{\eta}$

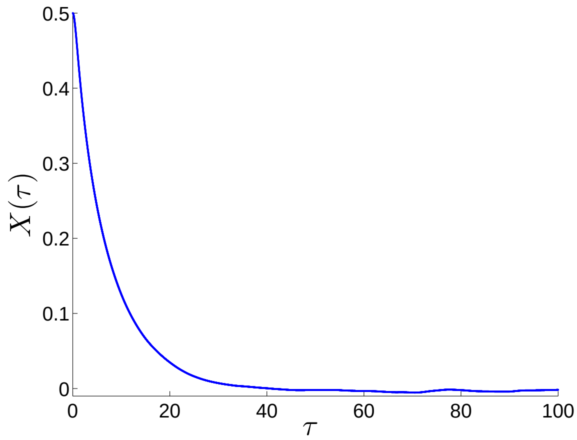


$$F = 0$$

For $\bar{\eta} \gg 1$ the decay is close to a power law

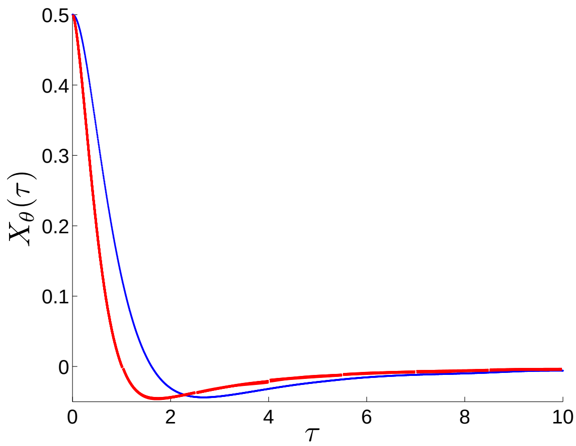
For $\bar{\eta} \sim 1$ an exponential decay





$\bar{\eta} \ll 1$ the behaviour determined by ϕ

$$mR\ddot{\theta}(t) + \eta R\dot{\theta}(t) = |\xi(t)| \sin\left(\theta(t) - \phi(t)\right)$$



$$X(\tau) \sim -\tau^{-2}$$

Current

Langevin equation with spatial temporal noise

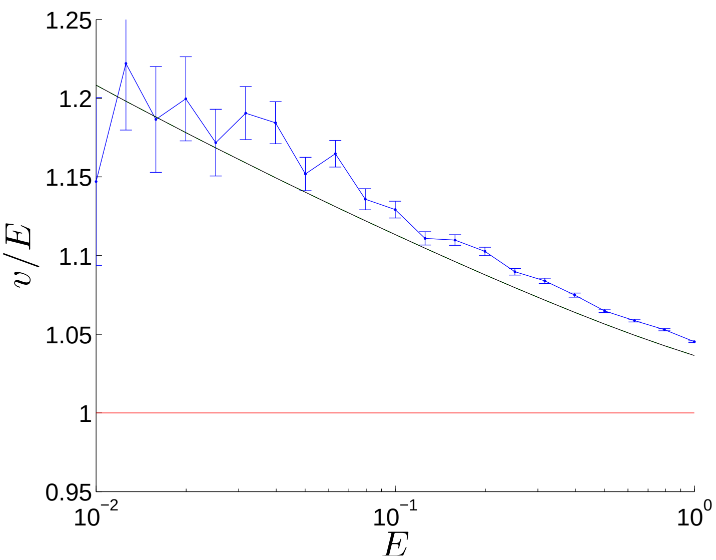
$$mR\ddot{\theta}(t) + \eta R\dot{\theta}(t) = f(\theta, t) + F$$

An expression for the velocity

$$v = \frac{F}{\eta} - \int_0^\infty \left(1 - \mathrm{e}^{-\tau\eta/m}\right) \partial_\theta \left\langle f(\theta, t) f(\theta', \tau + t)\right\rangle d\tau$$

In out case with $B(\tau) < 0$ then $v > F$

$$v = \frac{F}{\bar{\eta}} - \frac{1}{\bar{\eta}} \log v$$



Dirty Metal

The Dielectric function

$$\varepsilon(\mathbf{q},\omega) = \frac{4\pi\sigma}{-i\omega + Dq^2}$$

With FDT, the force correlation function

$$K_f(\mathbf{X},\tau) = \sum_n \sqrt{\beta_n(r)} \cos\left(n(\theta(t) - \theta(t'))\right) B(\tau)$$

Langevin equation for the particle dynamics with $g = \hbar/\ell^2\eta$

$$mR\ddot{\theta}(t) + \eta R\dot{\theta}(t) = \sum_n \beta_n(r) \left\{ \xi_x^n(t) \cos n\theta(t) + \xi_y^n(t) \sin n\theta(t) \right\}$$

$$\langle \xi_i^n(\omega)\xi_i^m(\omega')\rangle = g\,|\omega|\,\delta(\omega+\omega')\delta_{n,m} \qquad i = x, y$$

Where the Fourier coefficients $r = R/\ell$

$$\beta_n(r) \approx \begin{cases} \frac{n}{r}\mathrm{e}^{-n/2r} & r \gg 1 \\ \\ \delta_{1n} & r \ll 1 \end{cases}$$

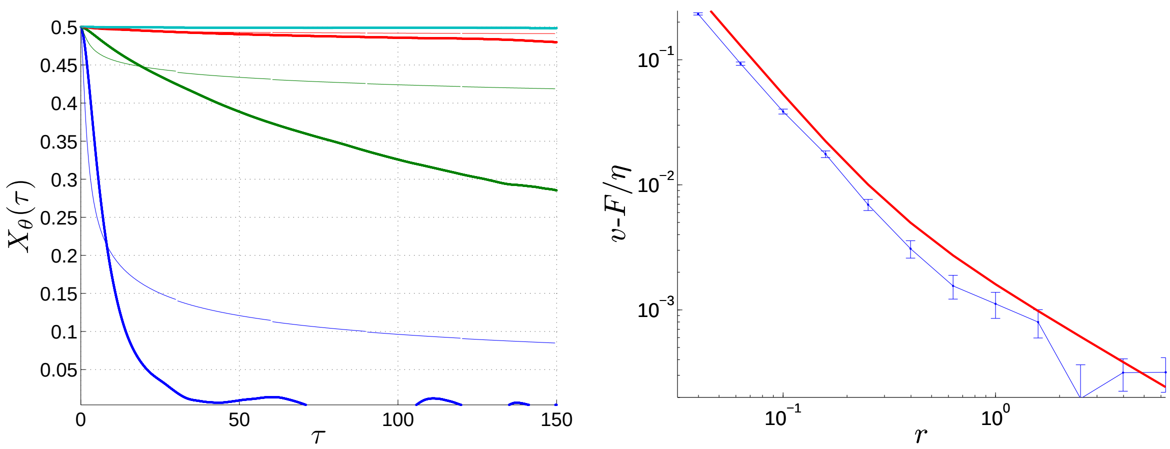
In the $R \ll \ell$ limit it reduces to the CL equation

The perturbation expression is as the CL results

$$X(\tau) = \frac{1}{2}\tau^{-g/\pi r^2}$$

The current dependence on r

$$v \approx \frac{F}{\eta} + g \begin{cases} 1/r^2 & r \ll 1 \\ \\ 1/r & r \gg 1 \end{cases}$$



The dephasing decreases as r increases

Summary

In this semiclassical model dephasing exists at the $T \to 0$ limit.

The dephasing decreases as r increases.

The Noise enhances the current in the ring.