Quantum Stochastic Transport Along Chains

Statistical Mechanics Day XI

Dekel Shapira, BGU

[1] D. Shapira and D. Cohen (arXiv:1907.01993)

Rate equation: $\dot{\vec{p}} = W\vec{p}$

Chain

- Clean system: Diffusion and drift
- Electric field disorder $\rightarrow$ **Sliding transition** (Derrida)

Ring ($W\vec{p} = -\lambda\vec{p}$)

- Clean system: λ-complex (non zero bias)
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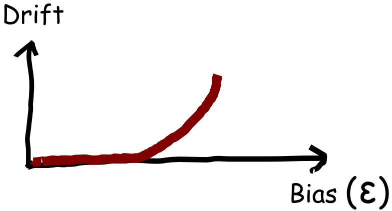

Sinai, Theory Probab. Appl. (1983)
Derrida, J. Stat. Phys. (1983)

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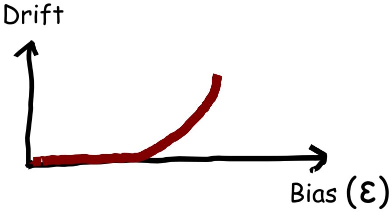
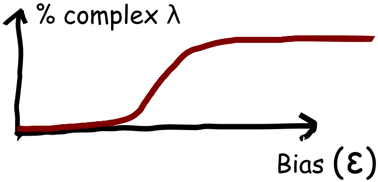




Sinai, Theory Probab. Appl. (1983)
Derrida, J. Stat. Phys. (1983)

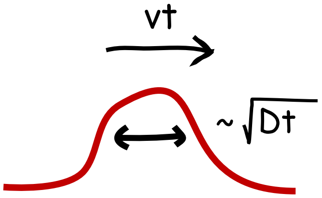
Hatano, Nelson, PRL (1996)
Hurowitz, Cohen, Scientific Reports (2016).

Dynamics of a particle in a tight-binding chain

Stochastic

Rate equation

$$\langle x \rangle = vt$$
$$\left\langle x^2 \right\rangle = 2Dt$$



Hamiltonian

$$H = -\frac{c}{2}\sum_x \left(|x{+}1\rangle\langle x| + h.c.\right) - \mathcal{E}\boldsymbol{x}$$
$$= -c\cos\left(\boldsymbol{p}\right) - \mathcal{E}\boldsymbol{x}$$

Bloch oscillation

$$\dot{x} = c\sin\left(p_0 + \mathcal{E}t\right)$$

We would like to bridge between the stochastic and coherent models

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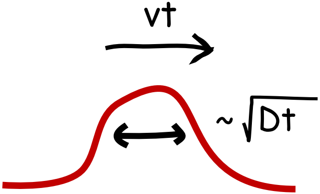

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$$\begin{aligned} H &= -\frac{c}{2} \sum_x \left(|x{+}1\rangle\langle x| + h.c. \right) - \mathcal{E}\boldsymbol{x} \\ &= -c\cos(\boldsymbol{p}) - \mathcal{E}\boldsymbol{x} \end{aligned}$$

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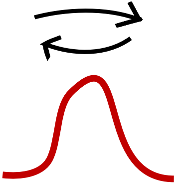


We would like to bridge between the stochastic and coherent models

Ohmic master equation

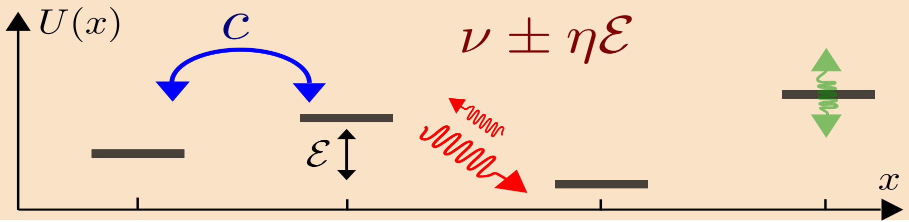
Particle Hamiltonian:

$$\boldsymbol{H}^{(\boldsymbol{c})} = -\boldsymbol{c}\cos(\boldsymbol{p}) + U(\boldsymbol{x}) = -\boldsymbol{c}\cos(\boldsymbol{p}) - \boldsymbol{\mathcal{E}x}$$

Particle-Bath coupling:

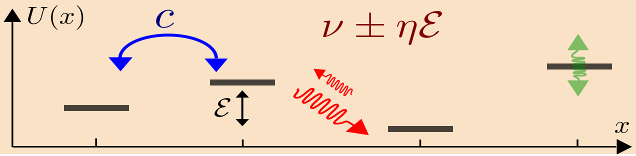
$$\boldsymbol{H}^{(\text{int})} = -\sum_x f_x(t)\boldsymbol{W_x}$$

$$\boldsymbol{W_x} = |x+1\rangle\langle x| + h.c$$



- ν - Intensity of the bath fluctuations
- Friction: $\eta = \nu/2T$
- f_x - Bath on bond x
- T - Temperature

Ohmic master equation



Dynamics of density matrix $\boldsymbol{\rho}$ is governed by the master equation:

$$\frac{d\boldsymbol{\rho}}{dt} = \mathcal{L}\boldsymbol{\rho} = -i[\boldsymbol{H}^{(\boldsymbol{c})}, \boldsymbol{\rho}] + \left(\mathcal{L}^{(\mathrm{B})} + \mathcal{L}^{(\mathrm{S})}\right)\boldsymbol{\rho}$$

$$\boldsymbol{H}^{(\boldsymbol{c})} = -\boldsymbol{c}\cos(\boldsymbol{p}) + U(\boldsymbol{x}) = -\boldsymbol{c}\cos(\boldsymbol{p}) - \mathcal{E}\boldsymbol{x}$$

$$\mathcal{L}^{(\mathrm{B})}\boldsymbol{\rho} = -\sum_x \left(\frac{\boldsymbol{\nu}}{2}[\boldsymbol{W}_x, [\boldsymbol{W}_x, \boldsymbol{\rho}]] + \frac{\boldsymbol{\eta}}{2} i[\boldsymbol{W}_x, \{\boldsymbol{V}_x, \boldsymbol{\rho}\}]\right)$$

Model parameters:

Bias and hopping amplitude: $\mathcal{E}, \boldsymbol{c}$

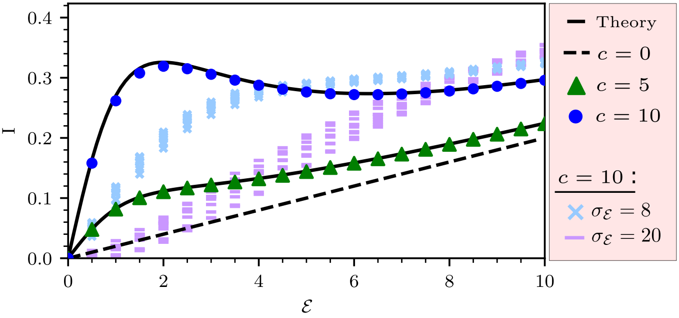
Noise and friction: $\boldsymbol{\nu}, \boldsymbol{\eta}$

Disordered system: $\mathcal{E}_x \sim \mathcal{E} + [-\sigma_{\mathcal{E}}, \sigma_{\mathcal{E}}]$.

$\mathcal{E}_x \equiv -(U(x+1) - U(x))$

$\boldsymbol{V_x} = -i[\boldsymbol{H}^{(\boldsymbol{c})}, \boldsymbol{W_x}]$

- $c = 0$: Stochastic current
  $c > 0$: Becomes **non-monotonic** in $\mathcal{E}$ as c increases
- Disorder may **increase** current
- Sinai-Derrida transition is **blurred** as a result of an additional transport channel



$I_x = \frac{1}{L}\left((w_x^+ - w_x^-) - c\,\mathrm{Im}(\alpha_0)\right) = \frac{1}{L}\left[1 + \frac{c^2}{6\nu^2 + 2\mathcal{E}^2}\right] 2\eta\mathcal{E} \equiv \frac{1}{L} v \qquad \mathcal{E}_x \sim \mathcal{E} + [-\sigma_{\mathcal{E}}, \sigma_{\mathcal{E}}] \qquad L$ — Number of sites

Results - Steady state current

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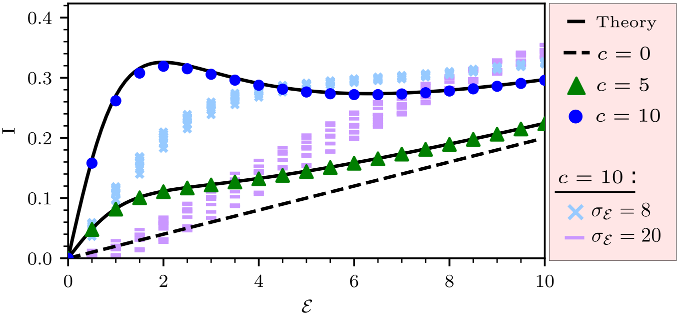

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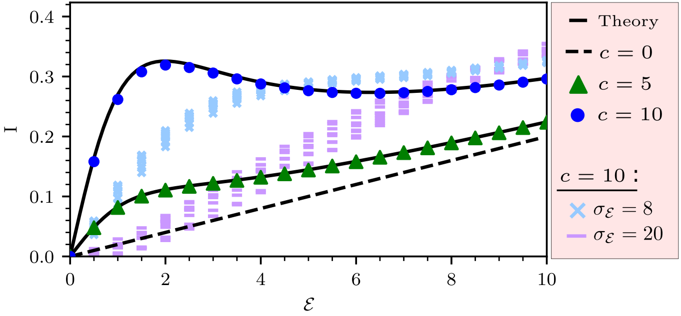

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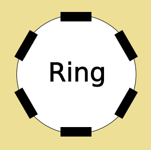
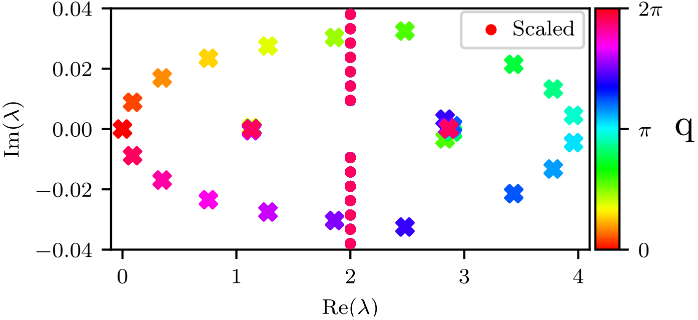
- Closing the chain into a ring
- Spectrum $\mathcal{L}\boldsymbol{\rho} = -\lambda\boldsymbol{\rho}$
- Dynamics $\boldsymbol{\rho(t)} = \sum_{q,s} \exp\left(-\lambda_{q,s} t\right)\boldsymbol{\rho_{q,s}}$





$\lambda_{0,0} = 0 \qquad \lambda_{0,\pm} = 2\nu \pm \sqrt{\nu^2 - \mathcal{E}^2} \qquad \lambda_{0,s} = 2\nu + i\mathcal{E}s, \ \ (s = \pm 2, \pm 3, \dots)$

Results - Eigenvalues

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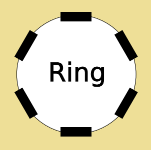

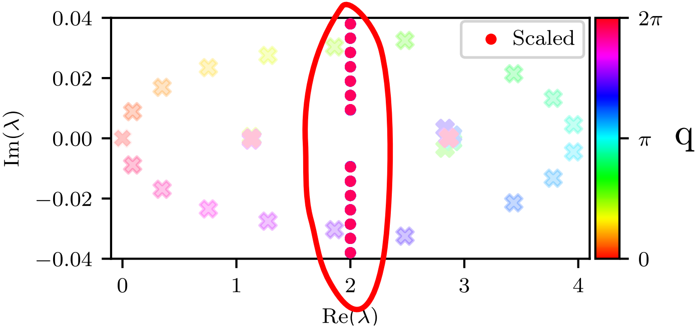

Under-damped decoherence modes

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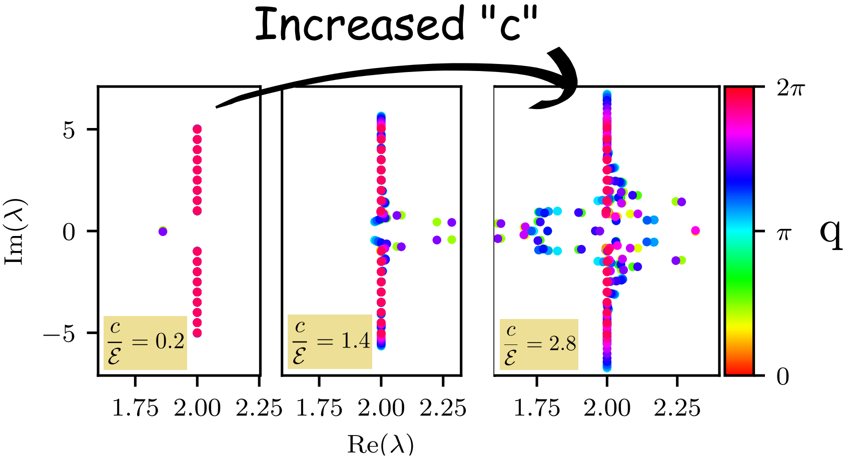
Results - Eigenvalues - Bloch

- Without the external bath: Eigenvalues form a "Wannier-Stark Ladder"
  ⇒ Hallmarks of Bloch dynamics.
- Increasing $c/\mathcal{E}$, the Wannier-Stark Ladder is smeared due to the bathes
  ⇒ Implies the destruction of Bloch oscillations.



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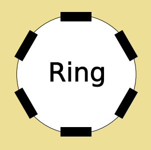

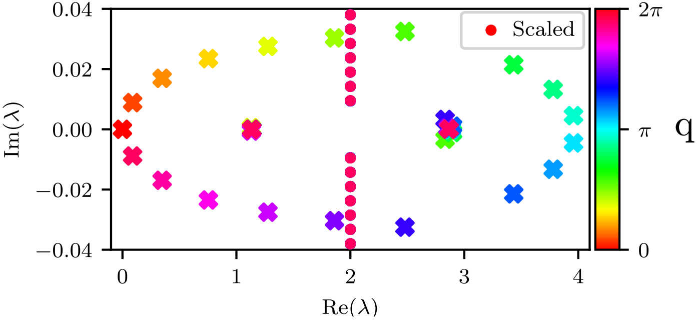

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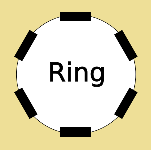

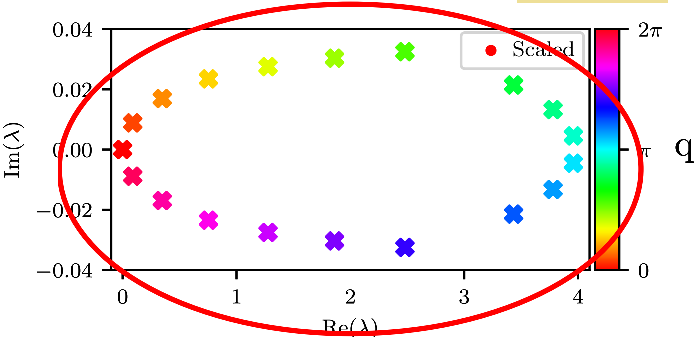

Stochastic-like relaxation modes

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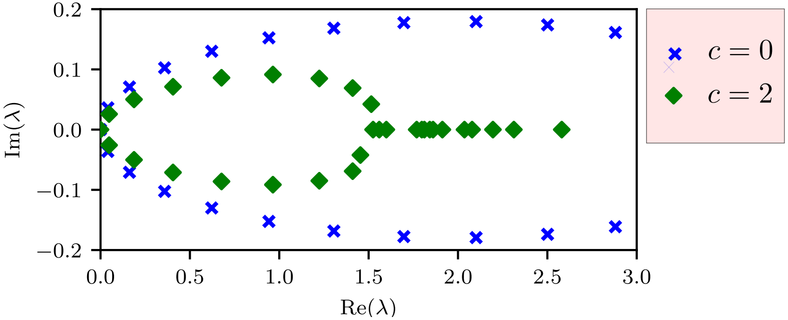
Results - Eigenvalues and disorder

- “Small” disorder: $\mathcal{E}_x \sim \mathcal{E} + [-\sigma_{\mathcal{E}}, \sigma_{\mathcal{E}}]$
- Hatano-Nelson perspective:

  Localized $\leftrightarrow$ Real   Delocalized $\leftrightarrow$ Complex
- Naive perspective: increasing “c” $\rightarrow$ Delocalization
- Observation: Enhanced localization



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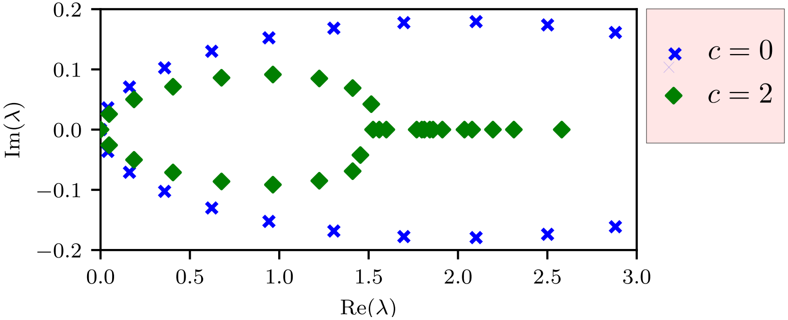

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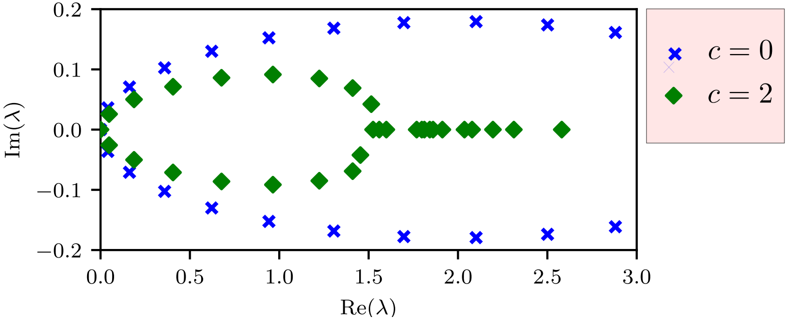

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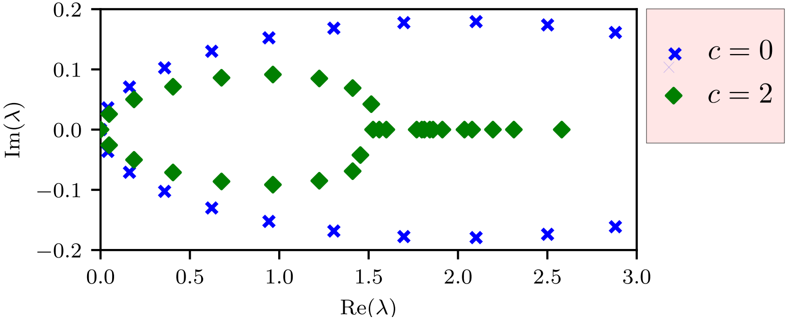


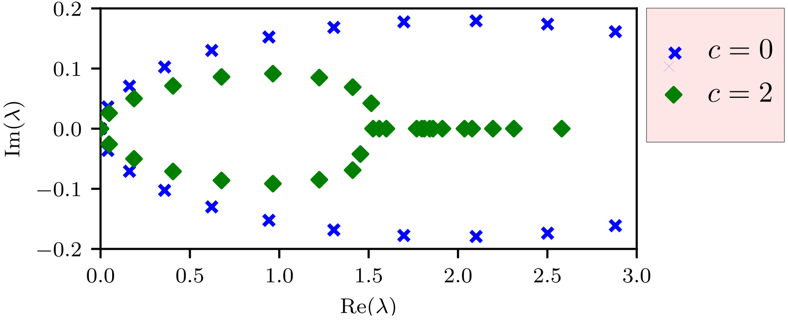
Effective disorder

- Eliminating non-diagonal elements of $\rho \to$
  Effective single-band equation
- Transition rates:

$$w_x^{\pm} = \nu + \nu_x \pm \eta \mathcal{E}_x \equiv w_x \exp(\pm \tilde{\mathcal{E}}_x)$$

$$\nu_x = \frac{\boldsymbol{c}^2}{2} \frac{\boldsymbol{\nu} - \lambda}{(2\boldsymbol{\nu} - \lambda)^2 + \boldsymbol{\mathcal{E}_x}^2 - \boldsymbol{\nu}^2}$$

- Coherent hopping $\to$ Effective c^2-disorder



Summary

1. The NESS current is the sum of stochastic and quasi-coherent terms.
2. It displays non-monotonic dependence on the bias, due to crossover from Drude-type to hopping-type transport.
3. Disorder may increase the current due to convex property.
4. The interplay of stochastic and coherent transition is reflected in the Lindblad spectrum.
5. In the presence of disorder the quasi-coherent transitions enhance the localization of the relaxation modes.