The Lognormal-Like Statistics of a Stochastic Squeeze Process

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GEFENOL 2017

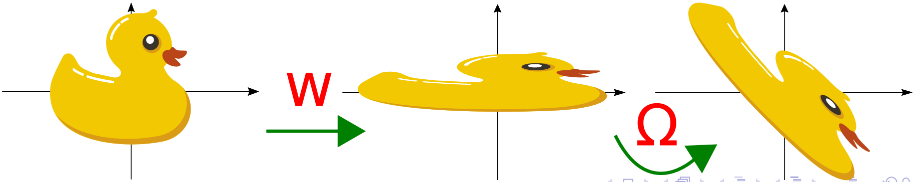
[1] D. Shapira and D. Cohen (arXiv:1701.01381)

The model

A 2-parameter model, described by the Langevin equation (Stratonovich interpretation):

$$\dot{x} = wx - \Omega(t)y$$
$$\dot{y} = -wy + \Omega(t)x$$
$$\langle\Omega(t)\Omega(t')\rangle = 2D\delta(t-t')$$

w generates a squeeze: $x = x_0 e^{wt},\ y = y_0 e^{-wt}$
$\log(r) \sim wt$
$\Omega(t)$ generates rotations
$\log(r) \sim$ const.



Drift and Diffusion for the radial coordinates

In polar coordinates $\mathbf{r}$, φ:

$$\begin{aligned} \dot{\boldsymbol{\varphi}} &= -w\sin(2\boldsymbol{\varphi}) + \Omega(t) \\ \dot{r} &= [w\cos(2\boldsymbol{\varphi})]\, r \end{aligned}$$

$\ln(r)$ performs Brownian motion:

$$\frac{d}{dt}\ln(r(t)) = w\cos(2\boldsymbol{\varphi})$$

After transient time $\ln(r)$ drifts and diffuses:

$$\begin{aligned} \mu \equiv \langle \ln(r) \rangle &= w_r t \qquad \text{[Drift]} \\ \sigma^2 \equiv \mathrm{Var}[\ln(r)] &= 2D_r t \qquad \text{[Diffusion]} \end{aligned}$$

Previous results for $\mathbf{w_r}$ and $\mathbf{D_r}$

For large D, assume that the phase is ergodized over a time scale $\boldsymbol{\tau} \sim 1/D$.

$$\frac{d}{dt}\ln(r(t)) \;=\; w\cos(2\boldsymbol{\varphi})$$

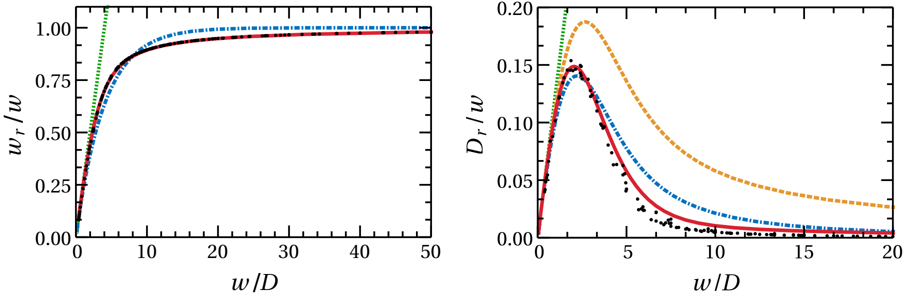
$\ln(r)$ is a sum of $t/\boldsymbol{\tau}$ uncorrelated terms $(w\boldsymbol{\tau})\cos(\boldsymbol{\varphi})$.
Each term has a zero mean, and variance $\sim (w\boldsymbol{\tau})^2$:

$$w_r \sim 0 \qquad D_r \sim (w\boldsymbol{\tau})^2\frac{1}{\boldsymbol{\tau}} \sim \frac{w}{D}$$

A more precise approach[2]:

$$w_r/w \sim \frac{w}{4D} \qquad D_r/w \sim \frac{w}{8D}$$

This agrees with the numerical results for $w/D \ll 1$.



--- Previous result

--- Practical approximation

--- 1st approximation

··· Numerical

— Exact