Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

====== [Exercise 8490]

Baruch's D31.

Consider the reaction A↔B for molecules A,B that evolves via a Langevin type equation

$$\frac{dN_A(t)}{dt} = k_1 N_B(t) - k_2 N_A(t) + A(t) \qquad (*)$$

where the total number of molecules $N = N_A(t) + N_B(t)$ is fixed. k_1, k_2 are reaction constants and $A(t)$ is random with averages $\langle A(t)\rangle = 0$, $\langle A(t)A(t')\rangle = C\delta(t-t')$.

(a) Solve for $\langle N_A(t)\rangle$ with the initial condition $N_A(0) = 0$ and show that its value at long time $\bar{N}_A$ yields $k_1/k_2 = f_A/f_B$, where results of question (A24) are used.

(b) Solve for $\langle[\delta N_A(t)]^2\rangle$ where $\delta N_A(t) = N_A(t) - \bar{N}_A$ and from its long time form and the results of question (A24b) show that $C = \frac{2k_1k_2}{k_1+k_2}N$.

(c) The chemical potential of B is now modified by a term $\delta\mu_B(t) = \delta\mu_B(\omega)\mathrm{e}^{-i\omega t}$. The response $\alpha(\omega)$ is defined by the long time form $\langle N_A(t)\rangle = \alpha(\omega)\delta\mu_B(\omega)\mathrm{e}^{-i\omega t} + \bar{N}_A$. Show that $\delta\mu_B(t) = -k_BT\frac{\delta k_2(t)}{k_2}$ by using the correspondence with question (A24), where $k_2 \to k_2 + \delta k_2(t)$ in Eq. (*). Solve the resulting equation for $\langle N_A(t)\rangle$ and identify $\alpha(\omega)$.

(d) Solve Eq. (*) for the equilibrium fluctuation in the absence of external forces ($\delta\mu_B(t) = 0$)

$$\phi_{N_A}(\omega) = \int \langle \delta N_A(t)\delta N_A(t+\tau)\rangle \mathrm{e}^{i\omega\tau} d\tau \qquad (1)$$

and check the (classical) fluctuation dissipation theorem. [Use the $t \to \infty$ form for the equilibrium correlation].