Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

[Exercise 8492]

Stochastic picture of sweep in 2-site system

Consider N classical particles in a two site system. The two sites are subjected to a potential difference ε. The temperature of the system is T. Define $n \in [-N, N]$ as the occupation difference. Assume that the thermalization process can be described by a stochastic rate equation

$$\frac{dn}{dt} = -\gamma n + A(t)$$

where $A(t)$ is a noisy term that reflects the fluctuations of the potential difference. Assuming that it has an average value A_ε and a power spectrum $\phi(\omega)$, it follows that n relaxes to an average value $\langle n\rangle_\varepsilon$, with fluctuations that are characterized by a power spectrum $\tilde{C}(\omega)$ and intensity $\nu \equiv \tilde{C}(0)$.

(1) Write what is the interaction energy H_{int} of n with the field ε. Later you will have to be careful with the identification of the conjugate variables.

(2) Using the canonical formalism find what are $\langle n\rangle_\varepsilon$ and $\text{Var}(n)$. Additionally provide approximations for small ε.

(3) Determine what is A_ε such that $\langle n\rangle_\varepsilon$ would be consistent with the canonical result. Assuming small ε deduce that $A_\varepsilon \propto \varepsilon$, and find the pre-factor.

(4) What is the $\chi(\omega)$ that characterizes the response of n to the applied potential in the linear-response regime? Care to identify correctly the conjugate variables; and take into account your answer to item (3).

(5) Consider a quasi-static sweep process, namely, a process during which ε is varied slowly with constant rate $\dot{\varepsilon}$. Use your result for $\chi(\omega)$ in order to express $\langle n\rangle$ in terms of $\langle n\rangle_\varepsilon$ and $\dot{\varepsilon}$.

(6) Deduce from the fluctuation-dissipation relation what is the correlation function $C(\tau)$ that describes the fluctuations. Explain how your answer in item (5) is related to the fluctuation intensity ν.

Advice: Care about factors of "2" in your answers. Failure to provide strictly correct pre-factors will be regarded as an essential error. Exploit item (6) in order to double check your answer in (5).