Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel

This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

[Exercise 8484]

Galvanometer

A galvanometer can be regarded as a spring-held pointer that has mass M, natural oscillation frequency ω_0, and a damping coefficient γ. The position x of the spring indicates the current I. It obeys the equation

$$\ddot{x} + \omega_0^2 x = -\gamma\dot{x} + A(t) + \alpha I$$

where $A(t)$ represents an environmentally induced white noise that has a spectral intensity ν, and α is a coupling constant.

(1) On the basis of the above Langevin equation write a $d\omega$ integral for the variance $\langle x^2\rangle$ in the absence of current.

(2) Based on canonical FDT considerations deduce what is the result of the integral that you wrote in the previous item.

(3) For a constant I, what is the average position $\langle x\rangle$ of the pointer?

(4) Regarding I as a driving source, write what is the conjugate variable, what is the interaction term $\mathcal{H}_{int}$ in the Hamiltonian, and what is the associate susceptibility $\chi(\omega)$.

(5) Write an expression for the average rate of energy absorption $\dot{W}$, given that the current source has a frequency ω and RMS amplitude I_0.

(6) The expression for $\dot{W}$ is formally the same as for a current source that is connected to a parallel RLC circuit. Write expressions for the effective values of R and L and C.

Tip: The equation of a parallel RLC circuit can be written as $G(\omega)V_\omega = I_\omega$ where $G(\omega)$ is a sum of three terms. Capacitors and inductors are described by $I = C\dot{V}$ and by $V = L\dot{I}$ respectively.