E8483: Millikan type experiment

Submitted by: Arthur Shulkin

Question

Consider a Millikan type experiment to measure the charge e of a particle with mass m. The particle is in an electric field E in the z direction, produced by a capacitor whose plates are distance d apart. The experiment is at temperature T and in a poor vacuum, i.e. τ_{col} is short. (τ_{col} is the average time between collisions of the air molecules and the charged particle). The field is opposite to the gravity force and the experiment attempts to find the exact field E^* where $eE^* = mg$ by monitoring the charge arriving at the plates.

1. Write a Langevin equation for the velocity v with a friction coefficient γ describing the particle dynamics.
2. For $E = E^*$ find the time T_D (assuming $T_D \gg 1$) after which the diffusion is observed.
3. For $E \neq E^*$ the equation has a steady state solution $< v_z >= v_d$. Find the drift velocity v_d.
4. Rewrite the equation in terms of v_d and find the long time limit of $< z^2 >$. From the condition that the observation time is $t \ll T_D$ deduce a limit on the accuracy in measuring E^*.
5. If the vacuum is improved (i.e. air density is lowered) but T is maintained, will the accuracy be improved.

Solution

1.

$$\dot{v} = -\frac{\gamma}{m}v + A(t) + \frac{eE}{m} - g \tag{1}$$

2. For time $T_D \gg \tau_{col}$ takes place the relation

$$< r^2 >= \frac{6T}{\gamma}T_D \tag{2}$$

we will assume isotropic media and use the fact $< z^2 >= d^2$

$$< r^2 >=< x^2 > + < y^2 > + < z^2 >= 3 < z^2 >= 3d^3 \tag{3}$$

from the last one can extract an expretion for T_D

$$T_D = \frac{\gamma d^2}{2T} \tag{4}$$

3. For a steady state $\dot{v} = 0$, $E \neq E^*$ and remembering that $< A(t) >= 0$ we get

$$\frac{\gamma}{m} < v >\equiv \frac{\gamma}{m}v_d = \frac{eE}{m} - g \to v_d = \frac{e}{\gamma}(E - E^*) \tag{5}$$

4. According to the last definition of v_d the Langevin equation can be written as follow

$$\dot{v} = -\gamma(v - v_d) + A(t) \tag{6}$$

from the draft term we can recognize the effective velocity as $v - v_d$, therefor

$$< z^2_{effective} >=< (z - v_d t)^2 >=< z^2 > +v_d^2 t^2 = \frac{2T}{\gamma} t + +v_d^2 t^2 \tag{7}$$

where in the final expression the linear term in t is due to diffusion and the quadratic term is due to the drift velocity.

5. Using $t = \frac{d}{v_d}$ and the expression for T_D from (2) $t \ll T_D$ leads to

$$\frac{d}{v_d} = \frac{\gamma d}{e(E - E^*)} \ll \frac{\gamma d^2}{2T} \tag{8}$$

from here we deduce the limit accuracy in measuring E^*

$$\frac{2T}{ed} \ll E - E^* \tag{9}$$

6. By improving the vacuum quality we change the parameter γ, but from (5) we see that the accuracy don't depend on γ therefor no improve will be achived.