Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

======= [Exercise 8483]

Baruch's D24.

Consider a Millikan type experiment to measure the charge e of a particle with mass m. The particle is in an electric field E in the z direction, produced by a capacitor whose plates are distance d apart. The experiment is at temperature T and in a poor vacuum, i.e. τ_{col} is short. (τ_{col} is the average time between collisions of the air molecules and the charged particle). The field is opposite to the gravity force and the experiment attempts to find the exact field E^* where $eE^* = mg$ by monitoring the charge arriving at the plates.

(a) Write a Langevin equation for the velocity $\mathbf{v}$ with a friction coefficient γ describing the particle dynamics. If $E = E^*$ find the time T_D (assuming $\gamma T_D >> 1$) after which a current noise due to diffusion is observed. What is the condition on τ_{col} for the validity of this equation?

(b) When $E \neq E^*$ the equation has a steady state solution $\langle v_z \rangle = v_d$. Find the drift velocity v_d. Rewrite the equation in terms of $\tilde{v}_z = v_z - v_d$ and find the long time limit of $\langle z^2 \rangle$. From the condition that the observation time is $<< T_D$ deduce a limit on the accuracy in measuring E^*.

(d) If the vacuum is improved (i.e. air density is lowered) but T is maintained, will the accuracy be improved?