Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

======= [Exercise 8483]

Millikan experiment

Consider a Millikan-type experiment whose purpose is to measure the charge e of a particle with mass m. The particle is located beteen plates of capacitor, where the electric field $\mathcal{E}$ is in the "up" direction, while the gravitation g is in the "down" direction. The distance between the plates is L, and the temperature of the system is T. Due to the poor vacuum the particle executes a Brownian motion that is described by a Langevin equation with friction force $-\eta v$. The charge of the electron is estimated via $\delta F = e\mathcal{E} - \mathsf{m}g = 0$. In item (1) the system is prepared with a single particle in the middle. In item (3) assume a uniform gas of N particles. In both cases the current is integrated during a time interval t, and the charge $Q = \int I(t')dt'$ is inspected as "readout".

(1) Assuming that $\delta F = 0$, determine the time t_d such that for $t \ll t_d$ it is not likely to get charge readout.

(2) What is the δF for which the condition $t \ll t_d$ is no longer valid. We shall regard this value, call it δ_1, as the resolution of the measurement.

(3) Assuming that $\delta F = 0$, determine the power spectrum $C(\omega)$ of the current $I(t)$.

(4) Assume that the time of the measurement is t. What is the δF for which the condition $\langle Q\rangle \ll \sqrt{\mathrm{var}(Q)}$ is no longer valid. We shall regard this value, call it δ_N, as the resolution of the measurement.

(5) Express the ratio δ_N/δ_1 as a function of N and t/t_d.

Tips: In the absence of fluctuations $\delta F = 0$ is indicated by having zero readout. In item (3) the "readout" is a current versus voltage ("IV") measurement, and $\delta F = 0$ is indicated by zero current. Due to the fluctuations there is some blurring which determines the resolution δ_N. In order to calculate the fluctuations in item (3) define the one-particle current as the velocity (up to a prefactor).