# Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

## ======= [Exercise 8481]

**Mass on a spring**

A balance for measuring weight consists of a sensitive spring which hangs from a fixed point. The spring constant is is $K$, i.e. the force opposing a length change $x$ is $-Kx$. The balance is at a temperature $T$ and gravity acceleration is $g$. A small mass m hangs at the end of the spring.

(a) Write the partition function and evaluate the average $\langle x\rangle$ and the fluctuation $\langle(x-\langle x\rangle)^2\rangle$. What is the minimal $m$ which can be meaningfully measured?

(b) Write a Langevin equation for $x(t)$ with friction $\gamma$ and a random force $A(t)$. Assuming $\langle A(t)\,A(0)\rangle = C\delta(t)$ evaluate the spectrum $|\tilde{x}(\omega)|^2$ where $\tilde{x}(\omega)$ is the Fourier transform of $\tilde{x} = x - \langle x\rangle$. Evaluate $\langle \tilde{x}^2(t)\rangle$ and from (a) find the coefficient $C$. [You may use $\int d\omega/[(\omega^2 - K/m)^2 + \gamma^2\omega^2] = m\pi/\gamma K$.]

(c) Consider response to a force that couples to the velocity, i.e. the Langevin equation acquires a term $-\partial F/\partial t$. Evaluate the dissipation function $Im\alpha_v(\omega)$ and the power spectrum of the velocity $\phi_v(\omega)$.