

Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel

This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 8481]

Mass on a spring

A balance for measuring weight consists of a sensitive spring which hangs from a fixed point. The spring constant is K . The balance is at temperature T and gravity acceleration is g in the x direction. A small mass m hangs at the end of the spring. There is an option to apply an external force $F(t)$, to which x is conjugate or apply an external vector potential $A(t)$.

- (a) Find the partition function Z .
- (b) Find $\langle x \rangle$ and $\langle x^2 \rangle$ and $\text{Var}(x)$.
- (c) Write a Langevin equation for $x(t)$, with friction γ , and a random force $f(t)$.
- (d) Assuming $\langle f(t)f(0) \rangle = C\delta(t)$, find $\text{Var}(x)$, and deduce what is C by comparing with the canonical result.
- (e) Assuming x is measured in the lab by averaging over time period t_0 , what is the minimal mass that can be meaningfully measured?
- (f) Describe the external force $F(t)$ by a scalar potential and demonstrate FDT.
- (g) Describe the external force $F(t)$ by a vector potential and demonstrate FDT.

Note: $\int \frac{d\omega}{(\omega^2 - \omega_0^2)^2 + \gamma^2 \omega^2} = \frac{\pi}{\gamma \omega_0^2}$.