

exam 2012

Ex 8034

מילואים מרץ/15

$$m\ddot{\theta} + \eta\dot{\theta} = f(t), \quad f(t) = \text{noisy EMF}$$

$$(a) C_f(\omega) = \nu \quad (b) C_f(\omega) = c|\omega|$$

$$\text{Define: } v = \dot{\theta}, \quad x = \sin\theta, \quad t_c = m/\eta$$

$$\tilde{C}_v(\omega) = C_f(\omega) / ((m\omega)^2 + \eta^2) \quad \text{exact}$$

$$C_v(t) = \frac{1}{m} \frac{\nu}{2\eta} e^{-t/t_c} \quad \text{case (a)} \quad \text{exact}$$

$$C_v(t) \sim -\frac{1}{\pi} \frac{c}{\eta^2} \frac{1}{t^2} \quad \text{case (b)} \quad t \gg t_c$$

$$\langle \theta^2 \rangle \approx 2Dt, \quad D = \frac{\nu}{2\eta^2} \quad \text{case (a)} \quad t \gg t_c$$

$$\langle \theta^2 \rangle \approx \frac{2c}{\pi\eta^2} \ln(t/t_c) \quad \text{case (b)} \quad t \gg t_c$$

$$\langle X^2 \rangle = \langle \sin^2\theta \rangle = \frac{1}{2} [1 - e^{-2\eta S(t)}]$$

$$\langle X(t)X(0) \rangle = \frac{1}{2} \langle \cos(\theta(t) - \theta(0)) \rangle = \frac{1}{2} e^{-\frac{1}{2}\eta S(t)}$$

$$C_x(t) \approx \frac{1}{2} \left(\frac{t_c}{t} \right)^\alpha \quad \alpha = \frac{c}{\pi\eta^2} \quad \text{case (b)}$$

$$\text{critical value } \eta_c = \sqrt{c/\pi}$$