Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

═══ [Exercise 8032]

Sub diffusion of Brownian particle

The motion of a brownian particle in 1D is given by the Hamiltonian:

$$H_{total}(x,p;A(t)) = \frac{1}{2m}(p - A(t))^2 + H_{bath}(x)$$

Assume that the equation of motion for the average velocity is:

$$m\frac{\partial\langle v\rangle}{\partial t} = -\eta\langle v\rangle + f(t)$$

In items 5-6-7 assume a zero temperature bath, and define

$$S(t) = \left\langle (x(t) - x(0))^2 \right\rangle$$

1. Relate $f(t)$ to $A(t)$.
2. What is the generalised susceptability $\chi(\omega)$ that relates v to A.
3. Find the power spectrum $\tilde{C}(\omega)$ of the velocity v.
4. Find an explicit expression for the correlation function $C(\tau)$ in the limit of high temperature.
5. In the limit of zero temperature find the coefficient C_0 in $C(\tau) \sim -C_0/\tau^2$.
6. Express $dS(t)/dt$ using the correlation function $C(\tau)$.
7. Given $S(t_0) = S_0$, find what is $S(t)$ for $t > t_0$.