8025 Thermal flow via Brownian particle

(1) $\ddot{X} = -(\gamma_1 + \gamma_2)\dot{X} + f_1(t) + f_2(t)$ $\qquad \gamma \equiv \gamma_1 + \gamma_2$

$\nu_1 = 2\gamma_1 T_1$, $\nu_2 = 2\gamma_2 T_2$, $\qquad \nu \equiv \nu_1 + \nu_2$

(2) $V(t) = \int^t e^{-\gamma(t-t')} (f_1(t') + f_2(t'))\, dt'$

$\langle V^2 \rangle = \iint^{t\,t} e^{-\gamma(t-t') - \gamma(t-t'')}\, \nu\, \delta(t'-t'')\, dt'\, dt''$

$= \frac{\nu}{2\gamma} = \frac{\gamma_1 T_1 + \gamma_2 T_2}{\gamma_1 + \gamma_2}$

(3) $\dot{Q} = (-\gamma_2 V(t) + f_2(t))\, V(t)$

(4) $\langle f_2(t) V(t) \rangle = \int^t e^{-\gamma(t-t')} \langle f_2(t) f_2(t') \rangle\, dt'$

$= \frac{1}{2}\nu_2 = \gamma_2 T_2$

$\langle \dot{Q} \rangle = -\gamma_2 \langle V^2 \rangle + \langle f_2 V \rangle =$

$= \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} (T_2 - T_1)$