Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

═══ [Exercise 8025]

Thermal flow via a Brownian particle

A Brownian particle in one dimension that has mass $\mathsf{m} = 1$, is in contact with two baths: A hot bath that has temperature T_2 that induces friction with coefficient γ_2, and a cold bath that has temperature T_1 that induces friction with coefficient γ_1. Accordingly the motion of the particle is described by a Langevin equation that includes two friction terms and two independent white noise terms $f_1(t)$ and $f_2(t)$. The purpose of this question is to calculate the rate of heat flow $\dot{Q}$ from the hot to the cold bath.

Note: Each bath exerts on the particle a force that has two components: a systematic "friction" component plus a fluctuating component. The rate of heat flow $\dot{Q}$ equals the rate of work which is done by the force that is exerted on the particle by the hot bath. In steady state, on the average, it equals in absolute value to the rate of work which is done by the force that is exerted on the particle by the cold bath.

(1) Write the Langevin equation for the velocity $v(t)$. Specify the intensity of the noise terms.

(2) Find the steady state value of $\langle v^2 \rangle$.

(3) Express the instantaneous $\dot{Q}$ at time t, given $v(t)$ and $f_2(t)$.

(4) Find an expression for $\langle \dot{Q} \rangle$ at steady state.