Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel

This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

======= [Exercise 8001]

Random walk with correlations

The total displacement of a particle is a sum over steps $X(t)$, where t is discrete. If we define the velocity as $v(t) = \frac{X(t)}{\tau\ 0}$, where $\tau 0$ is the time between steps, then the random walk is described by the equation $\left(\frac{dx}{dt}\right) = v(t)$.

(a) Given the velocity-velocity correlation function $c(t2-t1) = \langle v(t1)\, v(t2)\rangle$, write down an expression for the spreading $S(t) = \sqrt{\left[\langle (x(t) - x(0))^2\rangle\right]}$.

(b) Find an expression the diffusion coefficient, assuming that $c(\tau)$ is short range.

(d) More generally, show that $\frac{dS(t)}{dt}$ is equal to the $[-t, t]$ integral of $c(\tau)$.

(e) Assume that $c(\tau)$ has zero integral and power law tails $c(\tau) = \frac{-c_0}{\tau^\alpha}$. Determine the sub-diffusive behavior of $S(t)$ depending on the value of α.