Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

═══ [Exercise 7491]

Onsager

Consider a fluid in two compartments connected with a small hole. Although particles can pass easily through the hole, it is small enough so that within each compartment the fluid is in thermodynamic equilibrium. The compartments have pressure, temperature, volume and particle number P_1, T_1, V_1, N_1 and P_2, T_2, V_2, N_2, respectively. There is an energy transfer rate dE/dt and particle transfer rate dN/dt through the hole.

(a) Identify the kinetic coefficients for dE/dt and dN/dt driven by temperature and chemical potential differences. Rewrite the equations in terms of $\Delta T = T_1 - T_2$ and $\Delta P = P_1 - P_2$ to first order in ΔT and ΔP.

(b) If $\Delta T = 0$ one measures $\epsilon_1 = (dE/dt)/(dN/dt)$. One can also adjust the ratio $\epsilon_2 = \Delta P/\Delta T$ so that $dN/dt = 0$. Show the relation

$$\epsilon_2 = \frac{1}{T}[\frac{E}{V} + P - \frac{N}{V}\epsilon_1]$$

(E/V or P for either compartment).

(c) Assume that the work done during the transfer by the pressure is via reducing the effective volume to zero within the hole. Evaluate ϵ_1 and show that $\epsilon_2 = 0$.