Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

[Exercise 7489]

The Kubo formula

Particles with charge e and velocities $\mathbf{v}_i$ couple to an external vector potential by $V_{int} = -\frac{e}{c}\sum_i \mathbf{v}_i \cdot \mathbf{A}$ and the electric field is $\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$. The current density (per unit volume) is $\mathbf{j} = e\sum_i \mathbf{v}_i$.

(a) Identify the response function for an a component field with a given frequency, $E_a(\omega)$, in terms of the conductivity $\sigma(\omega)$ where $\mathbf{j}_a = \sigma(\omega)\mathbf{E}_a$ (assume an isotropic system so that $\sigma(\omega)$ is a scaler). Deduce the energy dissipation rate in terms of $\sigma(\omega)$ and $E_a(\omega)$. Compare with Ohm's law. What is the symmetry of $\text{Re}\sigma(\omega)$ when ω changes sign?

(b) Use the fluctuation dissipation theorem to show the (classical) Kubo formula:

$$\text{Re}\sigma(\omega) = \frac{1}{k_B T}\int_0^\infty \langle j_a(0)\cdot j_a(t)\rangle \cos(\omega t) dt$$

(c) Write the Diffusion constant D in terms of the velocity-velocity correlation function, assuming that this correlation has a finite range in time.

Use Kubo's formula from (b) in the DC limit of zero frequency to derive the Einstein-Nernst formula for the mobility $\mu = \frac{\sigma}{ne} = eD/k_B T$, where n is the particle density. (assume here uncorrelated particles).

(d) The quantum current noise is defined as

$$S(\omega) = \int_0^\infty dt \langle j_a(t)j_a(0) + j_a(0)j_a(t)\rangle \cos(\omega t).$$

Use the quantum FDT to relate this noise to the conductivity. When is the classical result (b) valid? What is the noise at $T = 0$?