

Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horowitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 7481]

FDT for velocities

Fluctuation Dissipation Theorem (FDT) for velocities: Consider an external $F(t) = \frac{1}{2}f_0e^{-i\omega t} + \frac{1}{2}f_0^*e^{i\omega t}$ coupled to the momentum as

$$H = \frac{p^2}{2M} + V(x; \text{env}) - \frac{1}{M}F(t)p$$

where ”env” stands for the environment’s coordinates and momenta.

- (a) Define the velocity response function by $\langle v(\omega) \rangle = \alpha_v(\omega)F(\omega)$ and show that the average dissipation rate is

$$\overline{\frac{dE}{dt}} = \frac{1}{2}\omega|f_0|^2\text{Im}\alpha_v(\omega).$$

- (b) Construct a Langevin’s equation with $F(t)$ and identify $\alpha_v(\omega)$. [Identify also $\alpha_{p/M}(\omega)$ and show that $\text{Im}\alpha_v(\omega) = \text{Im}\alpha_{p/M}(\omega)$.]

Using the known velocity correlations $\phi_v(\omega)$ (for $F = 0$) show the FDT

$$\phi_v(\omega) = \frac{2k_B T}{\omega}\text{Im}\alpha_v(\omega).$$