Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

[Exercise 7482]

Baruch's D23.

An electrical circuit has in series components with capacitance C, inductance L, resistance R and a voltage source $V_0 \cos\omega t$ with frequency ω.

(a) Write a Langevin equation for this circuit and identify the response function $\alpha_Q(\omega) = \langle Q(\omega)\rangle/(\frac{1}{2}V_0)$. Use this to write the energy dissipation rate.

(b) Use the fluctuation dissipation relation to identify the Fourier transform $\Phi_Q(\omega)$ of the charge correlation function. Evaluate $\langle Q^2(t)\rangle$ and compare with the result from equipartition.

(c) Evaluate the current fluctuations $\langle I^2(t)\rangle$ and compare with the result from equipartition. Under what conditions does one get Nyquist's result $\langle I^2\rangle_{\omega_1\leftrightarrow\omega_2} = \frac{2k_BT}{\pi R}(\omega_2-\omega_1)$?

Hint: $\int_{-\infty}^{\infty} \frac{d\omega/2\pi}{(\omega^2-\omega_0^2)^2+\gamma^2\omega^2} = \frac{1}{2\gamma\omega_0^2}$, $\qquad \int_{-\infty}^{\infty} \frac{\omega^2 d\omega/2\pi}{(\omega^2-\omega_0^2)^2+\gamma^2\omega^2} = \frac{1}{2\gamma}$.