

Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel

This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horowitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 7041]

FDT for RLC circuit

An electrical circuit has in series components with capacitance C , inductance L , resistance R and a voltage source $V_0 \cos \omega t$ with frequency ω .

- Identify the responsefunction $\alpha_Q(\omega) = \langle Q(\omega) \rangle / (\frac{1}{2} V_0)$. Use this to write the energy dissipation rate.
- Use the fluctuation dissipation relation to identify the Fourier transform $\Phi_Q(\omega)$ of the charge correlation function. Evaluate $\langle Q^2(t) \rangle$ and compare with the result from equipartition.
- Evaluate the current fluctuations $\langle I^2(t) \rangle$ and compare with the result from equipartition. Under what conditions does one get Nyquist’s result $\langle I^2 \rangle_{\omega_1 \leftrightarrow \omega_2} = \frac{2k_B T}{\pi R} (\omega_2 - \omega_1)$?

Hint: $\int_{-\infty}^{\infty} \frac{d\omega/2\pi}{(\omega^2 - \omega_0^2)^2 + \gamma^2 \omega^2} = \frac{1}{2\gamma \omega_0^2}$, $\int_{-\infty}^{\infty} \frac{\omega^2 d\omega/2\pi}{(\omega^2 - \omega_0^2)^2 + \gamma^2 \omega^2} = \frac{1}{2\gamma}$.