

Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
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This exercises pool is intended for a graduate course in “statistical mechanics”. Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horowitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

===== [Exercise 7488]

Baruch's D29.

Consider a damped harmonic oscillator with mass M whose coordinate $x(t)$ satisfies

$$\ddot{x} + \gamma\dot{x} + \Omega^2x = A(t)$$

where the power spectrum of $A(t)$, in the quantum case, is

$$\phi_A(\omega) = \frac{\hbar\omega\gamma}{M} \coth \frac{\hbar\omega}{2k_B T}.$$

- (a) Deduce the power spectrum of the velocity $\phi_v(\omega)$.
- (b) For $\gamma \rightarrow 0$ show that $\phi_v(\omega) \rightarrow \delta(\omega - \Omega)$ and that $\langle \frac{1}{2} M v^2 \rangle = \frac{1}{4} \hbar \Omega \coth \frac{\hbar\omega}{2k_B T}$. Explain why is this the expected result.
- (c) Bonus: Evaluate $\langle \frac{1}{2} M v^2 \rangle$ with $\gamma \neq 0$.