# Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

## [Exercise 7010]

**Site occupation during a sweep process**

Consider the occupation $n$ of a site whose binding energy $\varepsilon$ can be controlled, say by changing a gate voltage. The temperature of the environment is $T$ and its chemical potential is $\mu$. Consider separately 3 cases:

(a) The occupation $n$ can be either 0 or 1.
(b) The occupation $n$ can be any natural number $(0, 1, 2, 3, ...)$
(c) The occupation $n$ can be any real positive number $\in [0, \infty]$

We define $\bar{n}$ as the average occupation at equilibrium. The fluctuations of $\delta n(t) = n(t) - \bar{n}$ are characterized by a correlation function $C(\tau)$. Assume that it has exponential relaxation with time constant $\tau_0$. Later we define $\langle n \rangle$ as the average occupation during a sweep process, where the potential is varied with rate $\dot{\varepsilon}$.

(1) Calculate $\bar{n}$, express it using $(T, \varepsilon, \mu)$.

(2) Calculate $\mathrm{Var}(n)$, express the result using $\bar{n}$.

(3) Write an expression for the $\omega$=0 intensity $\nu$ of the fluctuations.

(4) Write an expression for $\langle n \rangle$ during a sweep process.

Irrespective of whether you have solved (1) and (2), in item (3) express the result using $\mathrm{Var}(n)$. In item (4) use the classical version of the fluctuation-dissipation relation, and express the result using $(T, \tau_0, \bar{n}, \dot{\varepsilon})$, where $\bar{n}$ had been given by your answer to item (1). Note that the time dependence is *implicit* via $\bar{n}$.