Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

[Exercise 7005]

Shot noise

The discreteness of the electron charge e implies that the current is not uniform in time and is a source of noise. Consider a vacuum tube in which electrons are emitted from the negative electrode and flow to the positive electrode; the probability of emitting any one electron is independent of when other electrons are emitted. Suppose that the current meter has a response time τ. If T_e is the average time between the emission of two electrons, then the average current is $\langle I \rangle = e/T_e = \frac{e}{\tau}\eta$, where $\eta = \tau/T_e$ is the transmission probability, $0 \leq \eta \leq 1$.

(a) Show that the fluctuations in I are $\langle (\Delta I)^2 \rangle = \frac{e^2}{\tau^2}\eta(1-\eta)$. Why would you expect the fluctuations to vanish at both $\eta = 0$ and $\eta = 1$? [Hint: For each τ interval n_i is the number of electrons hitting the positive electrode. Therefore, it can be equal to $n_i = 0$ or $n_i = 1$ which results in an average $\langle n_i \rangle = \tau/T_e$; discretize time in units of τ.]

(b) Consider the meter response to be in the range $0 < |\omega| < 2\pi/\tau$. Show that for $\eta \ll 1$ the fluctuations in the frequency domain are $\langle (\Delta I)^2 \rangle = e\langle I \rangle$. What is the condition for this noise to dominate over the Johnson-Nyquist noise in the circuit?

(c) Show that the 3rd order commulant is $\langle (I - \langle I \rangle)^3 \rangle = \frac{e^3}{\tau^3}\eta(1-\eta)(1-2\eta)$.