Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

[Exercise 6778]

Boltzmann equation: particles between two plates

Consider a classical gas of particles with mass m between two plates separated by a distance W. One plate at $y = 0$ is maintained at a temperature T_1, while the other plate at $y = W$ is at a different temperature T_2. A zeroth order approximation to the particle density is,

$$f_0(\mathbf{p}, x, y, z) = \frac{n(y)}{[2\pi m k_B T(y)]^{3/2}} \mathrm{e}^{-\frac{p^2}{2mk_B T(y)}}$$

(a) The steady state solution has a uniform pressure; it does not have a uniform chemical potential. Explain this statement and find the relation between $n(y)$ and $T(y)$.

(b) Show that f_0 does not solve Boltzmann's equation.

Consider a relaxation approximation, where the collision term of Boltzmann's equation is replaced by a term that drives a solution f_1 towards f_0, i.e.

$$[\frac{\partial}{\partial t} + \frac{p_y}{m}\frac{\partial}{\partial y}] f_0(\mathbf{p}, y) = -\frac{f_1(\mathbf{p}, y) - f_0(\mathbf{p}, y)}{\tau}$$

and solve for f_1.

(c) The rate of heat transfer is $Q = n\langle p_y p^2\rangle_1/(2m^2)$; $\langle ...\rangle_1$ is an average with respect to f_1. Justify this form and evaluate Q using the integrals $\langle p_y^2 p^4\rangle_0 = 35(mk_bT)^3$ and $\langle p_y^2 p^2\rangle_0 = 5(mk_bT)^2$. Identify the coefficient of thermal conductivity κ, where $Q = -\kappa\frac{\partial T}{\partial y}$.

(d) Find the profile $T(y)$.

(e) Show that the current is $\langle J_y\rangle = 0$. Explain why this result is to be expected.

(f) For particles with charge e add an external field E_y and extend Boltzmann's equation from (b). Evaluate, for uniform temperature, J_y and the conductivity σ, where $J_y = \sigma E_y$. Check the Wiedemann-Franz law, $\kappa/\sigma T$ =const.