Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel

This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

═══ [Exercise 6777]

Phase space evolution of confined particle

A thermalized gas particle at temperature T is suddenly confined to positions q in a one dimensional trap. The corresponding state is described by an initial density function $\rho(q,p,t=0) = \delta(q)f(p)$ where $\delta(q)$ is Dirac's delta function and

$$f(p) = \frac{\mathrm{e}^{-p^2/2mk_BT}}{\sqrt{2\pi mk_BT}}. \tag{1}$$

(a) Starting from Liouville's equation with the Hamiltonian $\mathcal{H} = p^2/2m$ derive $\rho(q,p,t)$. For a given time t draw the points in the (p,q) plane where $\rho(q,p,t)$ is finite and emphasize the segment where $f(p)$ is large, $p < \sqrt{mk_BT} \equiv p_0$.

(b) Derive the expressions for the averages $\langle q^2\rangle$ and $\langle p^2\rangle$ at $t > 0$.

(c) Suppose that hard walls are placed at $q = \pm Q$. Repeat the plot of (a) and again emphasize the range $p < p_0$. What happens in this plot at long times $t > 2Qm/p_0 \equiv \tau_0$? What is the meaning of the time τ_0?

(d) A "coarse grained" density $\tilde{\rho}$ is obtained by ignoring variations of ρ below some small resolution in the (q,p) plane; e.g., by averaging ρ over cells of the resolution area. Find $\tilde{\rho}(q,p)$ for the situation in part (c) at long time $t \gg \tau_0$, and show that it is stationary.