Exercises in Statistical Mechanics

Based on course by Doron Cohen, has to be proofed
Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel

This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

======= [Exercise 6775]

Coarse grained entropy

Coarse grained entropy. The usual $\rho(p,q,t)$, i.e. the normalized state density in the 6N dimensional phase space (p,q), satisfies Liouville's theorem $d\rho/dt = 0$. We wish to redefine $\rho(p,q,t)$ so that the corresponding entropy increases with time.
Divide phase space to small sub-volumes Ω_ℓ and define a coarse grained density

$$\bar{\rho}(p,q,t) = \bar{\rho}_\ell = \frac{1}{\Omega_\ell}\int_{\Omega_\ell} \rho(p,q,t)dpdq \qquad\qquad (p,q)\in\Omega_\ell$$

so that $\bar{\rho}(p,q,t)$ is constant within each cell Ω_ℓ. Define the entropy as

$$\eta(t) = -\int \bar{\rho}(p,q,t)\ln\bar{\rho}(p,q,t)dpdq = -\sum_\ell \Omega_\ell\bar{\rho}_\ell\ln\bar{\rho}_\ell\,.$$

Assume that at $t=0$ $\rho(p,q,0)$ is uniform so that $\rho(p,q,0)=\bar{\rho}(p,q,0)$.

(a) Show that $\eta(0) = -\int \rho(p,q,t)\ln\rho(p,q,t)dpdq\,.$

(b) Show that $\eta(t)$ increases with time, i.e.

$$\eta(t)-\eta(0) = -\int \rho\left[\ln\frac{\bar{\rho}}{\rho}+1-\frac{\bar{\rho}}{\rho}\right]dpdq \geq 0\,.$$

Hint: Show that $\ln x + 1 - x \leq 0$ for all $x>0$.