Exercises in Statistical Mechanics

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This exercises pool is intended for a graduate course in "statistical mechanics". Some of the problems are original, while other were assembled from various undocumented sources. In particular some problems originate from exams that were written by B. Horovitz (BGU), S. Fishman (Technion), and D. Cohen (BGU).

[Exercise 6774]

Boltzmann equation: Conductivity

Electrons in a metal can be described by a spectrum $\epsilon(\mathbf{k})$, where $\mathbf{k}$ is the crystal momentum, and a Fermi distribution $f_0(\mathbf{k})$ at temperature T.

(a) Find the correction to the Fermi distribution due to a weak electric field $\mathbf{E}$ using the Boltzmann equation and assuming that the collision term can be replaced by $-[f(\mathbf{k}) - f_0(\mathbf{k})]/\tau$ where τ is the relaxation time. Note that $d\mathbf{k}/dt = e\mathbf{E}/\hbar$ and the velocity is $\mathbf{v}_{\mathbf{k}} = \nabla_{\mathbf{k}}\epsilon(\mathbf{k})/\hbar$, i.e. in general $d\mathbf{v}_k/dt$ is $\mathbf{k}$ dependent.

(b) Find the conductivity tensor σ , where $\mathbf{J} = \sigma\mathbf{E}$. In what situation would σ be non-diagonal? Show that σ is non-diagonal if the mass tensor $(\frac{1}{m*})_{i,j} = \frac{1}{\hbar^2}\frac{\partial^2\epsilon(\mathbf{k})}{\partial k_i \partial k_j}$ is not diagonal.

(c) Find σ explicitly for $\epsilon = \hbar^2 k^2/2m*$ in terms of the electron density n. ($m*$ is an effective mass).